Project report in Computational Physics

Cellular automata for traffic simulation – Nagel-Schreckenberg model

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Abstract

In this project, traffic is simulated according to the cellular automaton of the Nagel-Scheckenberg model (1992) with different boundary conditions. The sudden occurrence of traffic jams is successfully realised as well as boundary induced phases and phase transitions are observed in the Asymmetric Simple Exclusion Process. The extension to the Velocity Dependent Randomization model leads to metastabile high flow states and hysteresis of the flow. The impact of speed limits on the probability of the formation of traffic jams is investigated. Furthermore, the effects of on- and off-ramps and traffic lights are analysed.

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1 Introduction

The aim of traffic-simulation-algorithms is to gain an understanding of (road-)traffic including it's various phenomena, e.g. the dependence of the different traffic parameters as flow and density or the formation of traffic jams.

With the help of a suitable simulation, one can make predections about the development of real traffic situations and furthermore use the results to optimise traffic plannings.

The first attempts to simulate traffic date back into the 1950s. A very important step foreward was the Nagel-Schreckenberg model (NaSch model) which was invented by Kai Nagel and Michael Schreckenberg in 1992. It was the first model to take into account the imperfect bahaviour of human drivers and was thus the first model to explain the spontanious formation of traffic jams. The NaSch model is the basis of this project.

An interesting application of the (extended) NaSch model is for example the OSLIM project [1] which simulates and predicts the traffic of North-Rhine-Westphalia online and in real time.

2 The Nagel-Schreckenberg model

The basic NaSch model [2] is a probabilistic cellular automaton: It contains a one-lane-road with discrete positions (cells). Also time (rounds) and integral velocities $0, ..., v_{max}$ are discrete. Every round, first each car updates it's velocity dependent on the position of the next car ahead and then every car moves according to it's velocity. The updating consists of 4 steps:

1. *Acceleration:* $v_n \to \min(v_n + 1, v_{max})$
2. *Deceleration:* $v_n \to \min(v_n, d_n - 1)$
3. *Randomization:* $v_n \to \max(v_n - 1, 0)$ with probability p
4. *Movement:* $x_n \to x_n + v_n$

The acceleration step is given by the attempt to drive as fast as possible – within the speed limit v_{max}. Every car has the same target velocity v_{max}. The acceleration is 1. The deceleration step is to avoid crashes: A car will not drive on or pass the position of the car driving ahead with distance d_n. The randomization step leads to an additional deceleration of 1 with probability p and is due to several behaviours of human drivers: The first one is an overreaction at braking and keeping a too large distance to the car in front. Secondly, when d_n increases, one might have a delay in the acceleration process. As a last point, at maximum velocity and free lane, one has a probability of sudden deceleration by distraction. The randomization is the basis for the formation of jams, because otherwise every car would drive with the ideal velocity, the maximum possible velocity without crashing into the car ahead. After the first three steps, the velocity is updated and the cars move.

An illustration of the NaSch model can be found in figure 1. The basic model is irreducible: If any

Figure 1: Illustration of updating and moving in the NaSch model. The number in the cells give the velocity after moving. Left: No randomization: The car in front has free space and accelerates by 1, the second and third car must decelerate to avoid a crash. Right: The randomization leads to an additional deceleration of the second car.

step is skipped, the simulation of traffic will not be successful.

2.1 Boundary conditions

Since the road is finite, one has to include boundary conditions. There are two possibilities (figure 2): Open and closed boundary conditions.

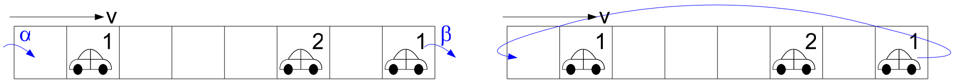

Figure 2: Illustration of the boundary conditions. Left: Open boundary conditions. Right: Closed boundary conditions.

For open boundary conditions, one has two parameters: α is the probability, for a car entering the road, if the first cell is free; β is the probability, that a car can leave the road (to another section), if it is near the end of the road and has enough velocity to reach it in this round. Due to the probabilistic processes at the on- and off-ramp, the traffic density ρ fluctuates in the open boundary case.

Closed boundary conditions mean, that a car, that reaches the ending of the road restarts at the beginning – the road actually does not have a beginning or end. One obtains a circuit with a fixed car density ρ.

2.2 Parameters and transfer to reality

The parameters of the NaSch model are the maximum velocity v_{max}, the probability of random deceleration p, the length of the road (no. of cells) L and the parameters given by one of the boundary conditions, α and β or ρ.

If one transfers the model to reality, one can assume 7.5 m as the space needed for one car and therefore as the length of a cell. One period can be interpreted as 1 s – the reaction time of drivers. For these values one obtains, that a velocity of 1 corresponds to a "real" velocity of 27 $\frac{\text{km}}{\text{h}}$ and furthermore a velocity of 5 – related to 135 $\frac{\text{km}}{\text{h}}$ – is a good approximation for the maximum speed on a motor way.

Examples for the movement of cars according to the model with open boundary conditions for randomization parameters $p = 0.5$ as well as $p = 0$ can be found in figure 3. The randomization $p = 0.5$ leads to formations of jams, while in the $p = 0$ case, the traffic flows at best feasibilities. Jams only occur at the exit.

3 The Asymmetric Simple Exclusion Process (ASEP)

The simplest example for a boundary-induced phase transition is found in the ASEP – a NaSch model under open boundary conditions with maximum velocity $v_{max} = 1$; A car can either drive into the next cell with probability $1 - p$, if the cell is free, or stop. For this model one analyses density and flow states dependent on the parameters α and β (figure 4 for $p = 0.2$ and $L = 100$). The data values are mean values of 1000 rounds. One obtains three phases:

- In the free-flow-phase A, the flow and the density only depend on α and not on β: cars at the exit leave the road with a higher probability than new cars enter the road. Because α is low, flow and density are low. In figure 5 one finds the density profiles along the road for $L = 30$. While the total flow and density are independent of β, the density profile shows that there are jams near the exit at low β.

- In the high-density-phase B, the flow does not depend on α, but on β. The probability of leaving at the exit is low, so a large tailback forms and the flow is low and only dependent on β. The density profiles can be found in figure 6 for $L = 30$. At large α, the jam spreads over the whole road. At small α, the density decreases at the starting point.

- In the maximum flow phase C, the flow is nearly independent of α and β. The profile shows, that the density decreases from the starting point to the exit. The maximum flow is only limited by the bulk rate/randomization parameter p (compare figure 8).

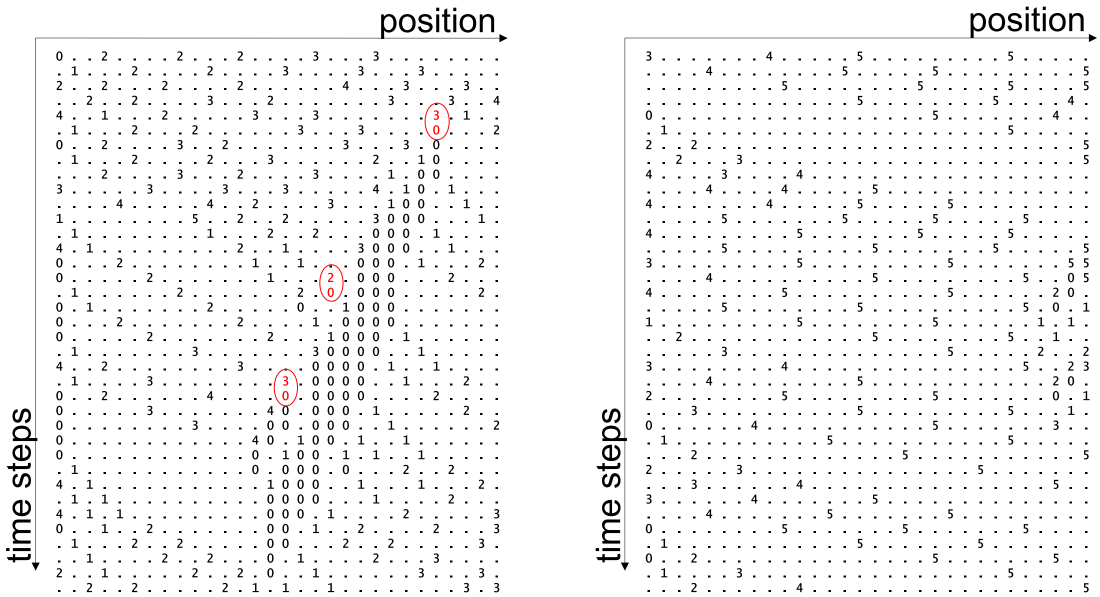

Figure 3: Simulation of the NaSch model for $\alpha = 0.3$, $\beta = 0.8$, $L = 30$ and $p = 0.5$ (left) as well as $p = 0$ (right). Dots stand for free cells. Numbers stand for the velocity of a car in this cell in the last round. With randomization, sudden deceleration leads to jam formation (red circles); In the $p = 0$ case, jams may only occur at the exit.

The obtained results successfully reproduced the phenomena described in [3].

4 Metastability and hysteresis in the Velocity-Dependent-Randomization (VDR) model

In the VDR model the randomization parameter of each car factors it's velocity in. In a very first step, this parameter is calculated (and used in step 3):

0. *Determination of the randomization parameter:* $p_n = p(v_n)$.

At this point the slow-to-start rule is applied: $p_n = \begin{cases} p_0, & \text{if } v_n = 0 \\ p, & \text{if } v_n > 0 \end{cases}$, with $p_0 > p$. If a car stopped completely, it takes a longer time to reaccelerate. This new rule leads to the occurrence of metastable phases and hysteresis of the flow. There are two possibilities to demonstrate this: A circuit (closed boundary condition) with different initial starting conditions or a circuit with controlled on-/off-ramps to increase/decrease the car density continuous.

4.1 Control by initial Conditions

The two extremal initial traffic states (for fixed density ρ) are

- a jam, where all cars start in a row with $v = 0$ (figure 9 left)
- and a maximum flow state, where all cars are equidistantly distributed over the whole road (figure 9 right).

Dependent on these initial conditions, the fundamental diagram, traffic flow j vs. density ρ, is measured (figure 10 left). For low densities, the flow increases proportional to the density. This is the free flow phase, where additional cars can drive with nearly no disturbance. At some critical density, the

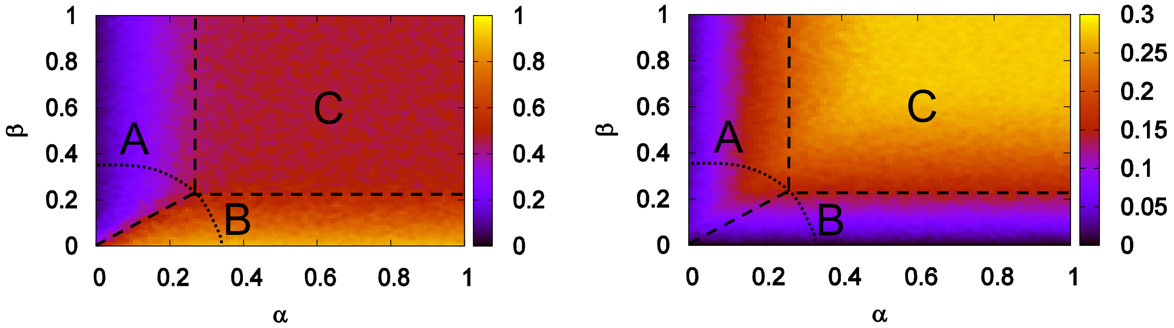

Figure 4: Traffic density (left) and flow(right) in ASEP for $p = 0.2$. One finds the three phases A,B and C. Means of 1000 measurements with road length $L = 100$.

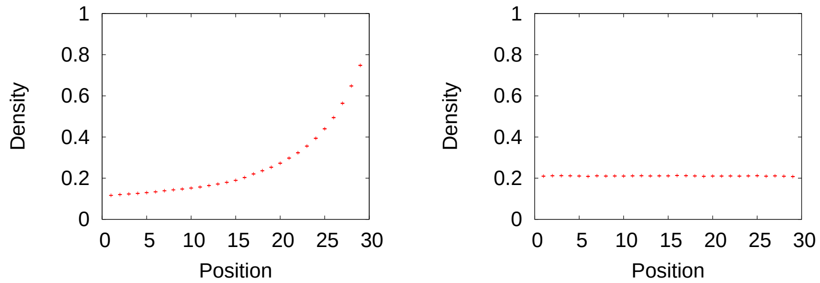

Figure 5: Traffic density profiles in phase A. Left: $\alpha = 0.10$, $\beta = 0.15$. Right: $\alpha = 0.20$, $\beta = 0.95$. Means of 1000 measurements.

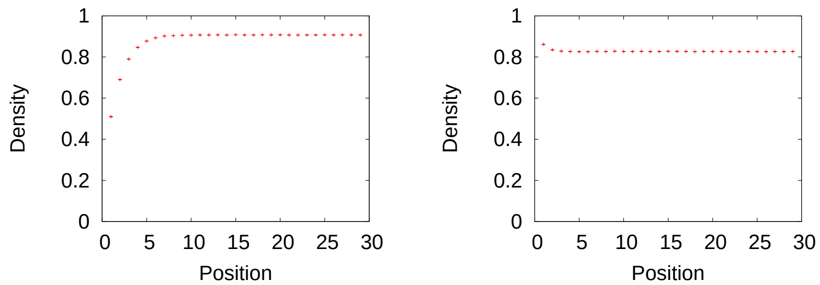

Figure 6: Traffic density profiles in phase B. Left: $\alpha = 0.15$, $\beta = 0.10$. Right: $\alpha = 0.95$, $\beta = 0.20$. Means of 1000 measurements.

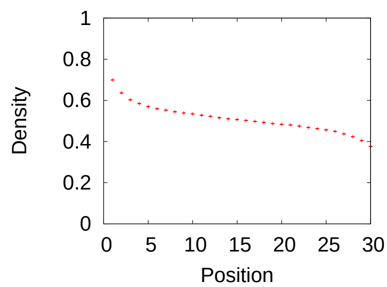

Figure 7: Traffic density profiles in phase C. $\alpha = 0.95$, $\beta = 0.95$. Means of 1000 measurements.

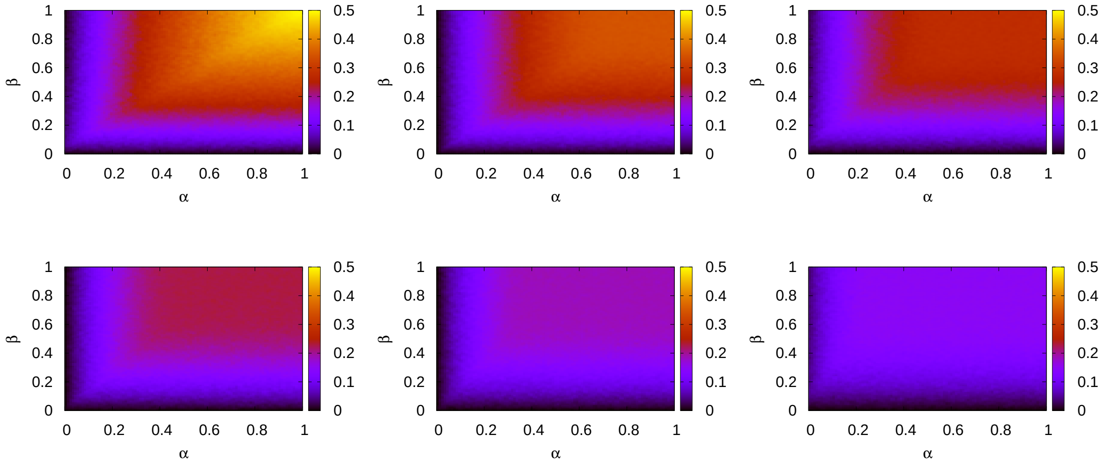

Figure 8: Traffic density and flow in ASEP for different p. From left to right, top to bottom: $p = 0.0$, $p = 0.1$, $p = 0.2$, $p = 0.3$, $p = 0.4$, $p = 0.5$. Means of 1000 measurements with road length $L = 100$.

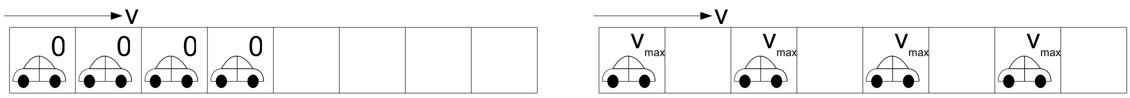

Figure 9: Illustration of the two initial conditions jam (left) und uniform distribution (right)

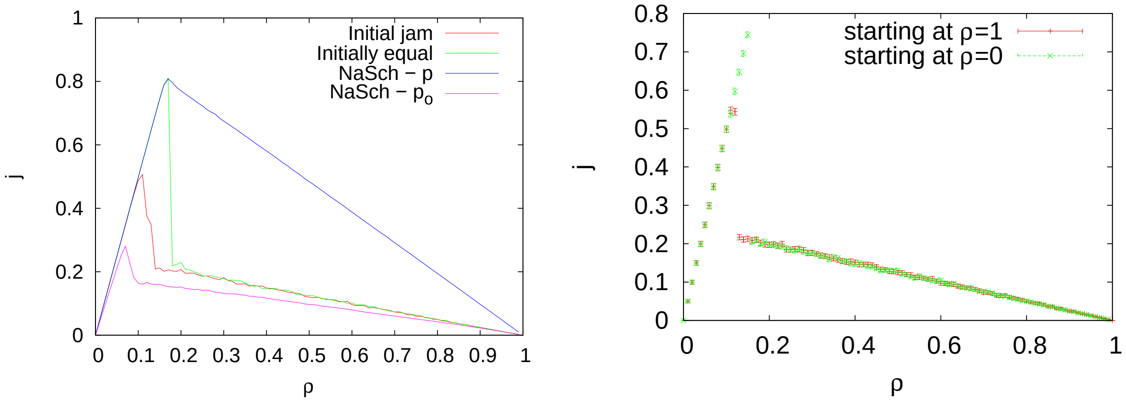

Figure 10: Fundamental diagram of the VDR model. Left: Control by different initial conditions and Nash model. Right: Control by on-/off-ramps. Parameters: $v_{max} = 5$, $p_0 = 0.75$, $p = 1/64$, $L = 100$. Means of 10000 measurements.

initial jam cannot disperse and the flow drops suddenly. For the initial maximum flow state, much higher flows are observed, until it also drops into a jammed state.

One obtains two branches. For the same parameters, two states are possible, while one of them is metastable. This state can, after some consecutive overreactions, collapse into a jammed state.

In contrast to that, the basic NaSch model with randomization parameter p does not lead to a stable jam. After the maximum, the flow decreases linear with the density: Metastability is an effect of the VDR model. Due to the high bulk rate, the NaSch model with parameter p_0 never reaches as high flows as the other three models.

4.2 Control by on- and off-ramp

Using on- or off-ramps, one can move across the branches by controlling the density. Starting at a completely filled (empty) road, cars are removed (added) at the ramp. Hysteresis is observed: Here, one also finds the two branches dependent on the history of ρ with effects of metastability (figure 10 right).

4.3 Lifetime of the metastable phase

The lifetime τ of the metastable state, reached by the initial condition of a maximum flow state, is measured for different maximum velocities dependent on the density ρ (figure 11). At this place, a jam

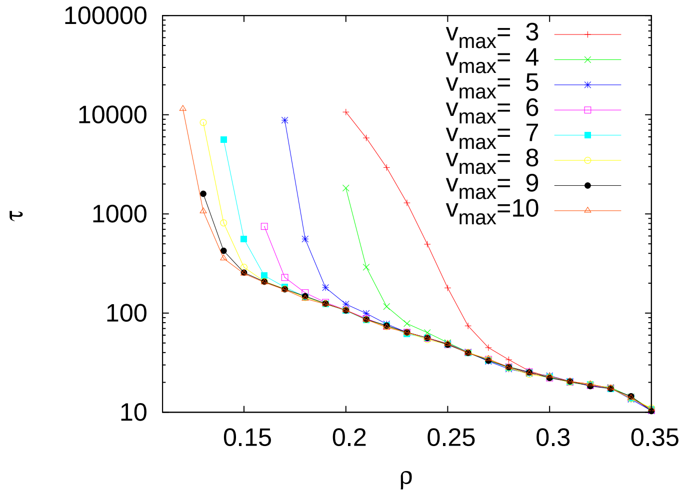

Figure 11: Lifetime τ (log-scale) of the metastable branch for different v_{max} and ρ. Means of 1000 measurements.

is defined as three completely stopped cars in a row.

For high densities, the lifetime is very low and independent of the maximum velocity, because nearly no car reaches the maximum velocity. With decreasing density, the lifetimes increase more than exponentially. For a given density, a small v_{max} can lead to lifetimes orders of magnitude larger than at higher v_{max}. In the metastable phase, the probability of jam occurrence is much smaller, if the maximum velocity is reduced. In reality, this can help to avoid traffic jams by adjusting the speed limit, if densities in the metastable region are detected.

5 Further applications

There are numerous possibilities to extend the NaSch and the VDR model to take different road situations into account. Here, the effects of on- and off-ramps and the effects of traffic lights are studied. Further possible extensions could be the presence of high-distraction regions on the road (e.g. due to construction zones). These regions can be simulated by raising the randomazation parameter for those cells. Other extensions are the so-called anticipation models which include every drivers reaction not only to the distance to the car directly ahead, but also to the behavior of the second car ahead. Furthermore, one can get more realistic models by simulating roads with more than one driving-lane, including lane changes and heterogeneous maximum velocity behaviours for different cars.

5.1 The effects of on- and off-ramps

Here, the effects of on- and off-ramps are studied. The length of each ramp is 25 cells, which corresponds to 187.5 m – similar to real on-/off-ramps. In this example, the on-ramp starts at the 80th cell, the off-ramp at the $L-80$th cell, while the road is comprised of $L = 3000$ cells. The density is kept constant, meaning a car is only added to the road at the on-ramp, if a car can be removed at the off-ramp in the same round. The ramps act in every fifth round.

Figure 12 shows the fundamental diagram for the NaSch model with and without the on- and off-ramps. One can see, that in the medium density regime, the flow is decreased and forms a plateau-like

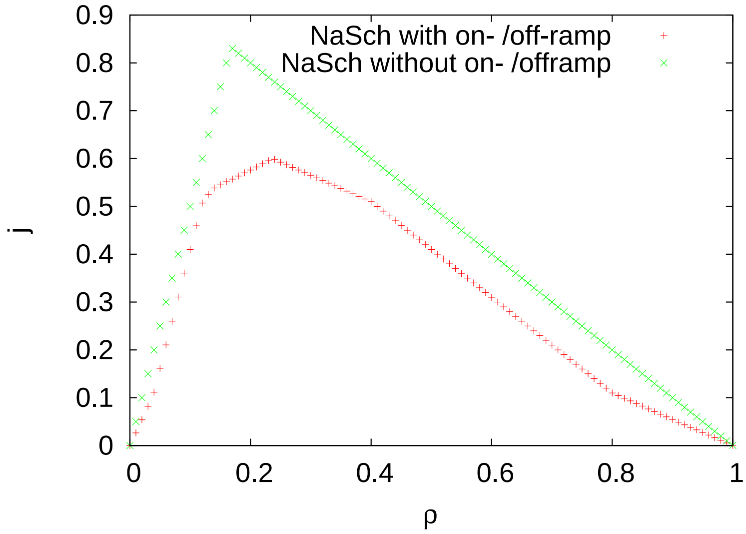

Figure 12: The fundamental diagram for the NaSch model with and without on- and off-ramps. The randomization parameter is set to $p = 0$.

shape / is independent of the density. This can be understood quite intuitively, since the adding cars at the on-ramp will lead to a jam-like state just before the ramp.

It was shown, that the density will indeed form a real plateau [3]. This behaviour was not reproduced, since it would demand for a more complex way of calculating the flow. This is because one cannot simply measure the flow through one cell, since it will obviously be different, depending on where it is measured (meaning before or after the on-ramp).

5.2 The effects of traffic lights

The effects of traffic lights are studied in the NaSch and the VDR model. The lights change from green to red and vice versa every five rounds. Figure 13 shows the effects in both models. In the basic NaSch model, jams form in front of the red traffic lights, but vanish again in the green phases. The VDR model shows a different behaviour. Here, the jams persist and start to move backwards against the driving direction of the cars, even in the green phases. This is due to the slow-to-start rule.

The more realistic VDR model explains the effect of several jams, forming in front of traffic lights. With those results, one can use the VDR model to optimise the length of the green- and red-phases on roads with many traffic lights, as found in inner city situations. Furthermore, one could include other effects like roundabouts and try to find an optimal solution to avoid traffic at road junctions.

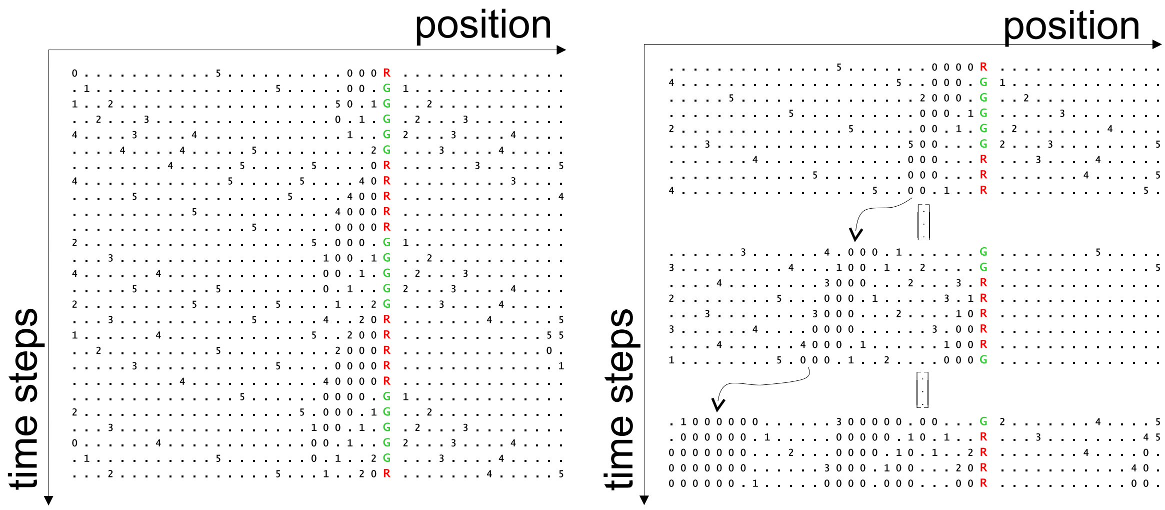

Figure 13: Traffic-flow on a road with traffic lights. Left: In the basic NaSch model with $p_0 = p$; Right: In the VDR model with $p_0 > p$ (slow-to-start-rule).

6 Summary

The NaSch model successfully simulated the spontaneous formation of traffic jams and reproduced the different flow and density phases observed in real traffic situations. The VDR model, an extension to the NaSch model, adding a slow-to-start rule, reproduced effects of metastability and hysteresis.

The models can be used to understand, predict and optimise different traffic situations. Density-dependend speed limits, the effects of on- and off-ramps and traffic lights where presented and examined as examples for the various possible extensions and fields of applications.

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References

[1] Ministerium für Bauen und Verkehr des Landes Nordrhein-Westfalen, *Verkehrsinformationssystem autobahn.NRW*, `http://www.autobahn.nrw.de`, 16.03.2011

[2] K. Nagel and M. Schreckenberg *A cellular automaton model for freeway traffic*, J. Phys. I France, 2221-2229 (1992)

[3] Andreas Schadschneider, *Statistical Physics of Traffic Flow*, Physica A285, 101 (2000).