



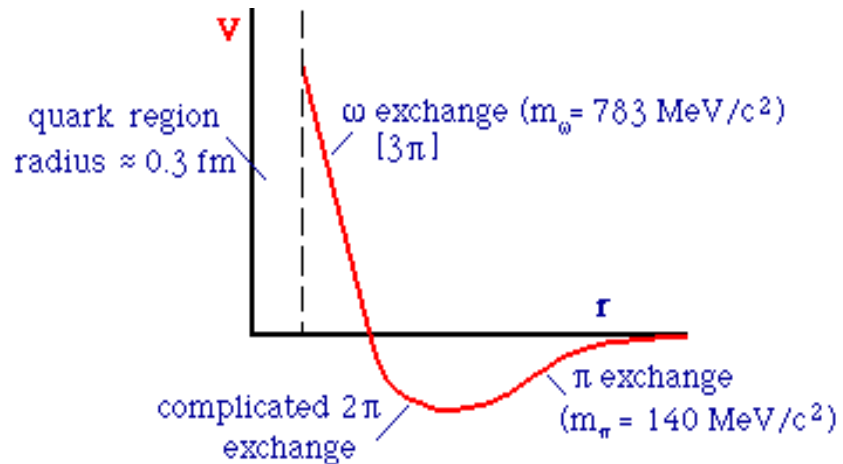
The extraordinary case of the lightest scalar resonances: what do we know and why should we care?

J. R. Peláez

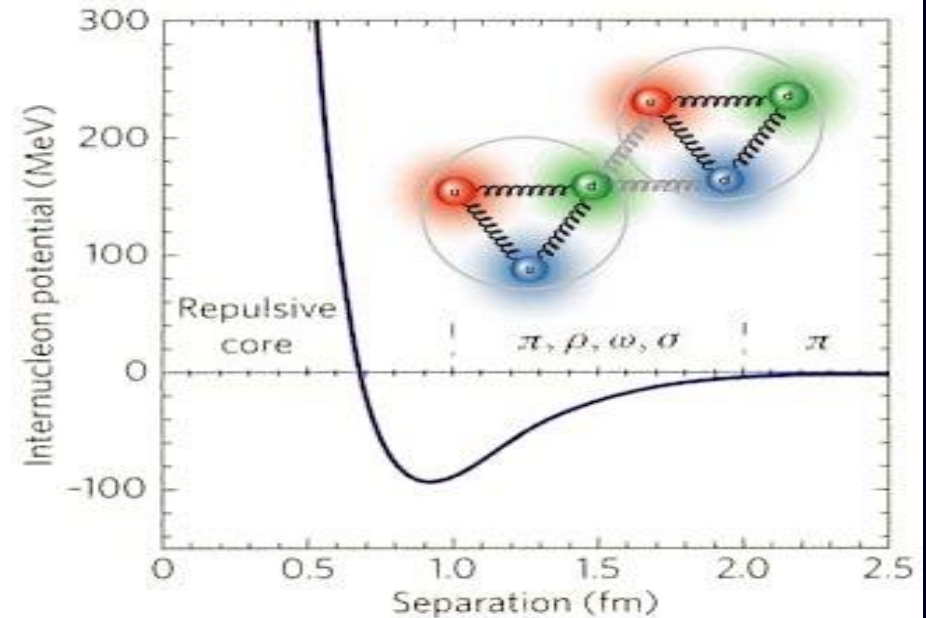
Why should we care? NN attraction

- $I=0, J=0$ $\pi\pi$ exchange very important for nucleon-nucleon **attraction!!**

Crude Sketch of NN potential:



From C.N. Booth



Scalar-isoscalar field already proposed by Johnson & Teller in 1955

Name given by Schwinger in 1957. Multiplet of SU(2) isospin with pions

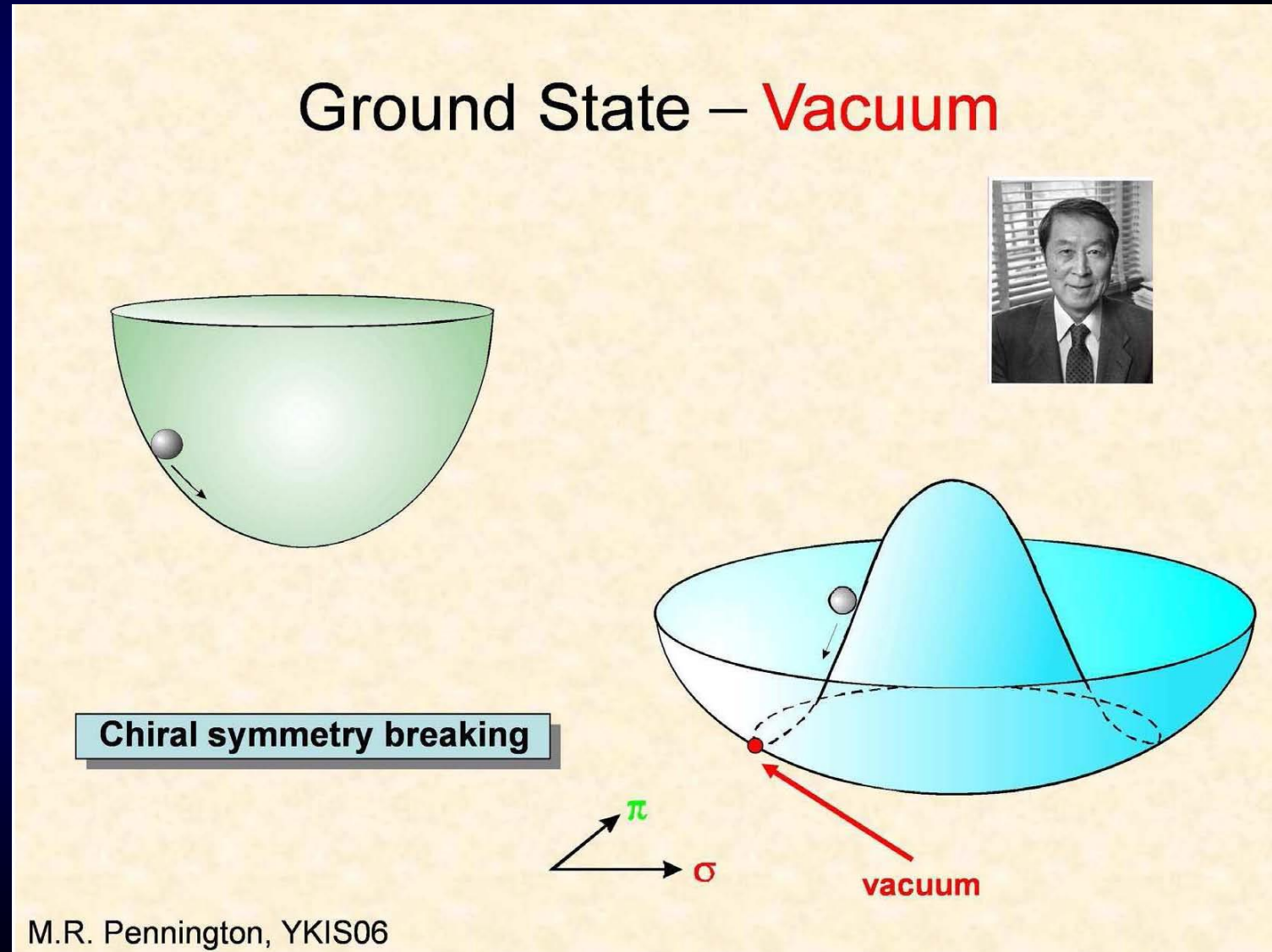
We would not be here if the σ was slightly different!!!

(On a first and CRUDE approximation. Many Anthropic papers: Donoghue, Epelbaum, Hanhart, Meissner, JRP, Oberhummer...)

Why should we care? Spontaneous Chiral Symmetry Breaking

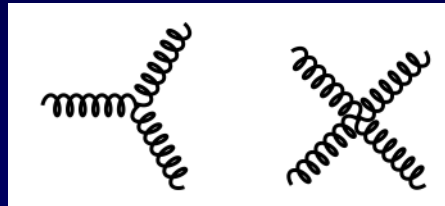
- In the 60's: "Linear sigma model" (Gell-Mann) and Nambu Jona Lasinio models of SPONTANEOUS CHIRAL SYMMETRY BREAKING. Pions are Goldstone Bosons!!
- f_0 's relevant due to their vacuum quantum numbers. Particularly the lightest one: $f_0(500)/\sigma$

The "Linear sigma model"
nowdays is a
QUALITATIVE APPROXIMATION
at low energies



Why should we care?: Glueballs & Spectroscopy

- Glueballs: Feature of non-abelian QCD nature
The lightest one expected with f_0 quantum numbers $I=0, J=0$



- From lattice QCD glueball around 1.5 GeV (give or take 0.2 GeV)
- Several f_0 states have been “observed”: $f_0(500)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1700)$.

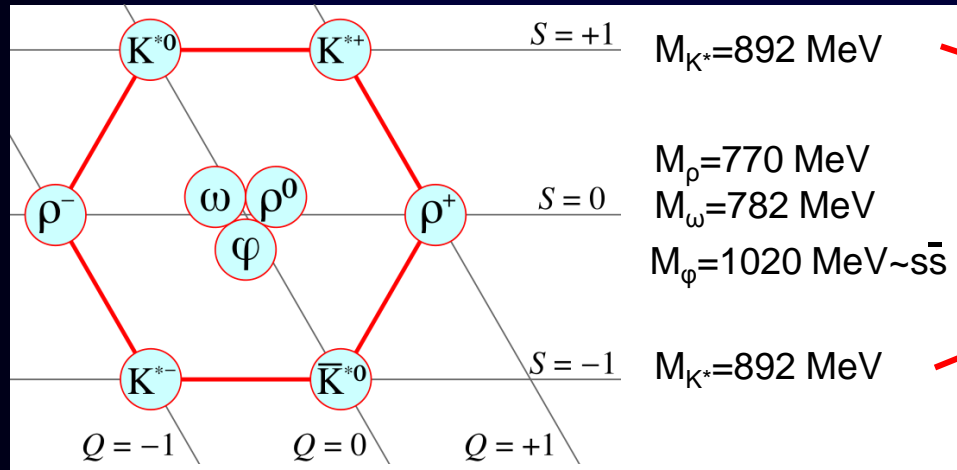
One of them the glueball? Not quite, most likely mixing occurs.

We have to understand the spectroscopy and nature of the other scalars as well

Ordinary mesons: Spectroscopy

From naive quark model: **quark–antiquark states**

- With only 3 light quarks, grouped in SU(3) **nonets**

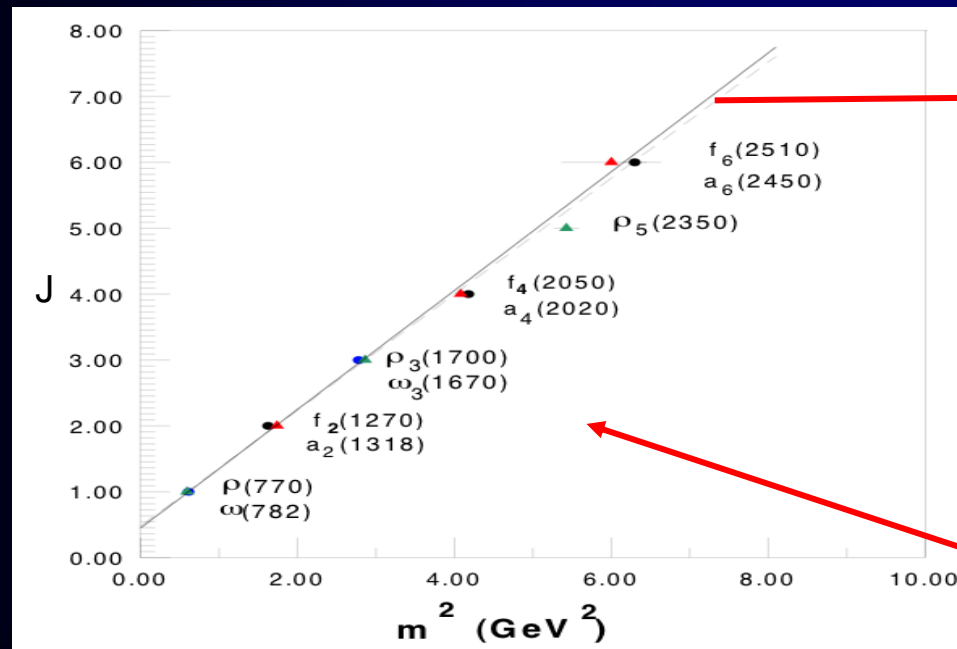


$q\bar{q}$ Mass hierarchy:

These heavier because
 $m_s \gg m_u \sim m_d$

**Not for
light scalars!**

- Follow linear (J, M^2) Regge trajectories



**Linear (J, M^2) trajectories with
Universal slope $\sim 0.8-1 \text{ GeV}^{-2}$**
(Also for baryons)

Rigid rotating rod, Stringy picture
Color flux tube... CONFINEMENT

Note no scalars there

**Let's classify
scalars!!**

Let us first see HOW MANY SCALARS EXIST (in the PDG) below 2 GeV:

- Isospin=0: $\sigma/f_0(500)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1700)$ 5 states.

Half century-long controversy

Settled.

(Even at PDG)

Mild fading
controversy

- Isospin=1: $a_0(980)$, $a_0(1450)$. 3x2=6 states

- $I=1/2$, $S=\pm 1$: $\kappa/K_0^*(800)$, $K_0^*(1430)$ 4x2=8 states

40 yr-long controversy

Almost Settled.

(only waiting for PDG)

19 states... enough to form TWO NONETS
And something more.

The lightest ones should form the lightest nonet.
But some **were or still are controversial**

Let's revisit the longstanding
controversy about the
EXISTENCE of the σ and κ
and their present status!!

The σ longstanding controversy (Following PDG)

- The σ , controversial since the 60's.

“not well established” 0^+ state in PDG until 1974

Removed from 1976 until 1994.

Back in PDG in 1996, renamed “ $f_0(600)$ ”

Huge Revision in 2012



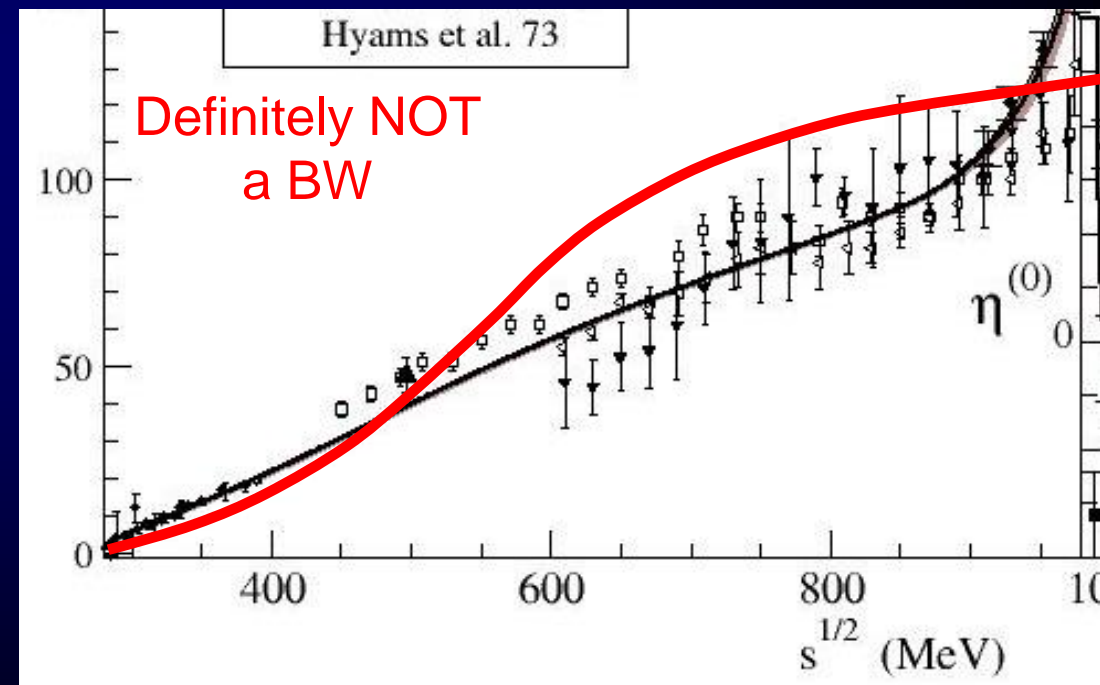
- The reason: The σ is EXTREMELY WIDE and has no “BW-resonance peak”.

Usually quoted by its pole:

$$\sqrt{s_{pole}} \approx M - i \Gamma / 2$$

Poles are process independent, peaks are not

- The $\kappa/K(800)$: similar situation, but with strangeness and still OUT of PDG “summary tables”.

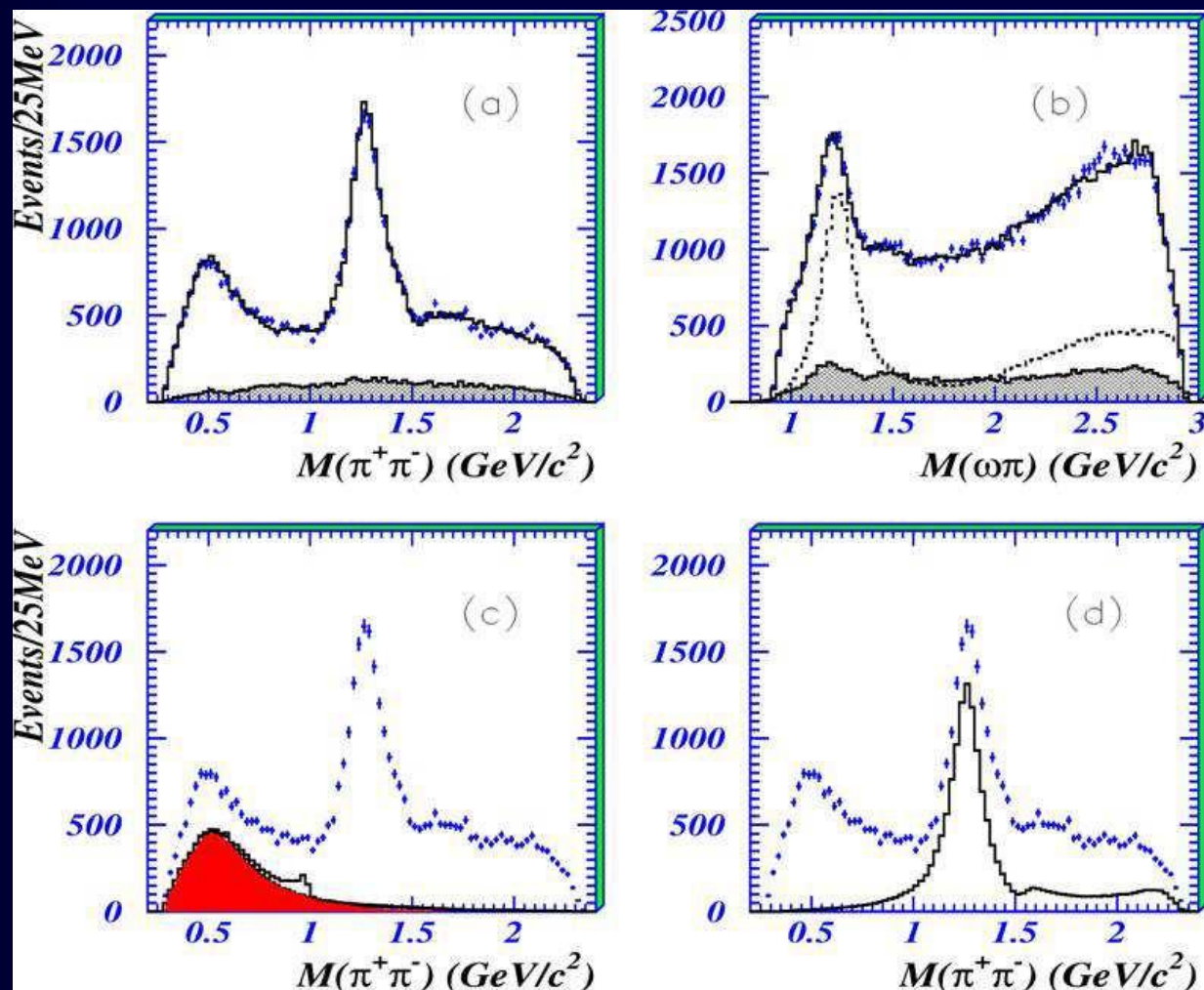


- Narrower $f_0(980)$ and $a_0(980)$ scalar well established

The σ/κ longstanding controversy (Following PDG)

Strong support from starting in early 2000's from production in decays.

Example: σ from BES: $J/\psi \rightarrow \omega \pi^+ \pi^-$ (Phys. Lett. B598: 149-158,2004.) (also E791, CLEO, now LHCb, etc...)



But production analyses rely on model dependent Assumptions (isobars, BW form, K-matrix... breaking analyticity, Watson's theorem...) and are not really good for precision, although they get a pole not too far from real one.

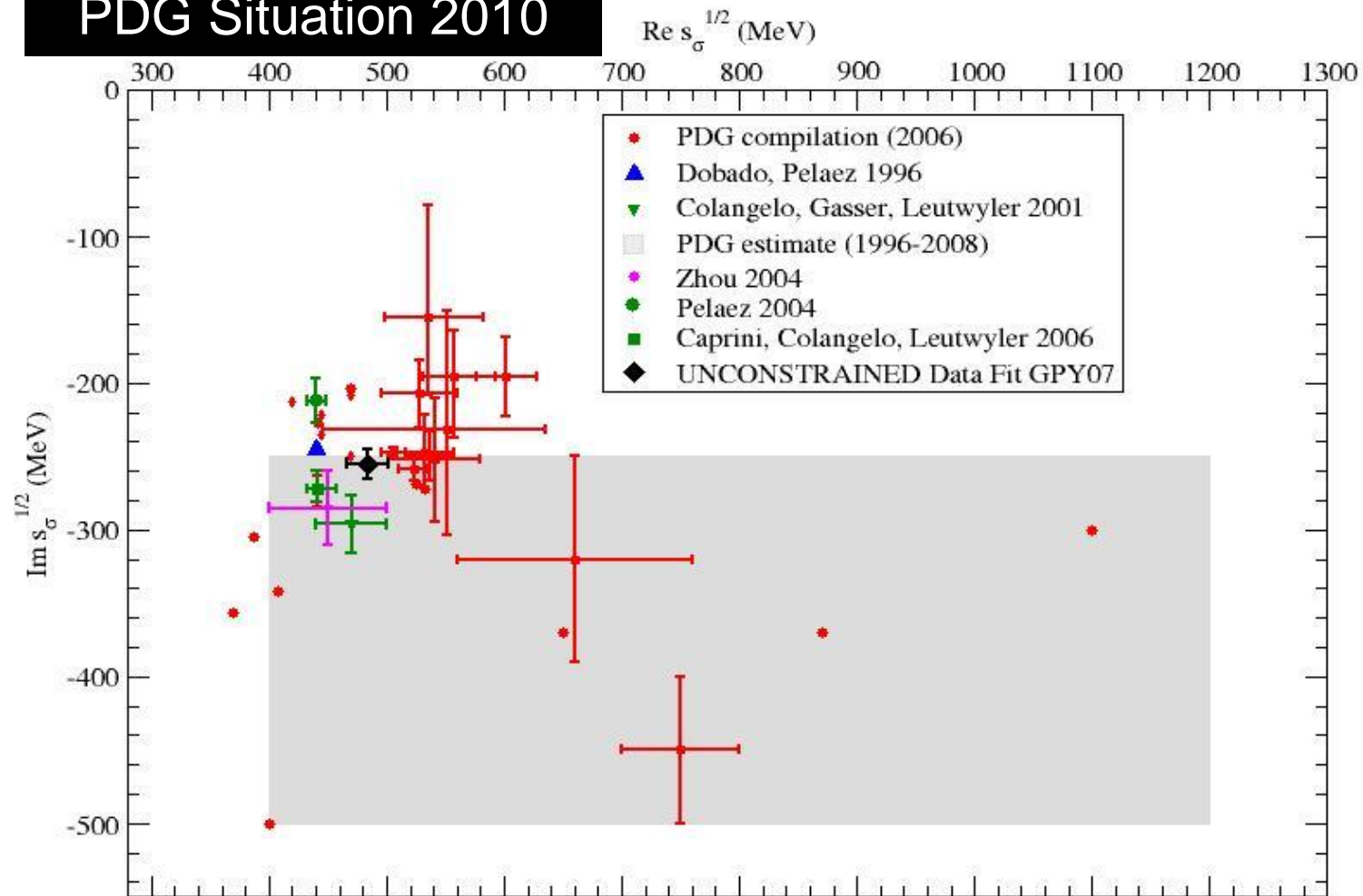
Example: $M - i\Gamma/2 = (541 \pm 39) - i(252 \pm 42)\text{MeV}$

Very wide Resonance = pole deep in complex plane

Need correct analytic continuation

SIMPLE MODELS (like BW, or worse) created a mess

PDG Situation 2010



Need for dispersive formalism (analyticity) and chiral symmetry also relevant.

What is a dispersion relation.? Very Briefly and for $\pi\pi$

CAUSALITY:

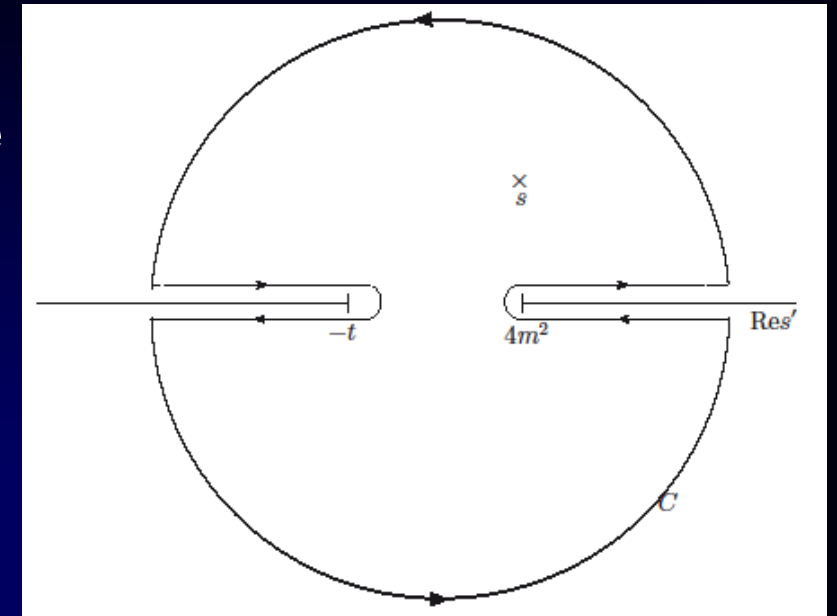
Partial waves $t(s)$ are ANALYTIC in complex s plane with cuts due to thresholds (also in crossed channels)

Cauchy Theorem determines $t(s)$ at ANY s , from an INTEGRAL on the contour

If $t \rightarrow 0$ fast enough at high s , curved part vanishes

$$t(s) = \frac{1}{\pi} \int_{th}^{\infty} \frac{\text{Im } t(s')}{s - s'} ds' + LC$$

Otherwise, determined up to polynomial (subtractions)



- Good for:
- 1) Calculating $t(s)$ where there is not data
 - 2) Constraining data analysis
 - 3) ONLY MODEL INDEPENDENT extrapolation to complex s -plane

The real improvement: Analyticity and Effective Lagrangians

● Solutions of Roy-like equations.

70's Roy, Basdevant, Pennington, Petersen...

00's Ananthanarayan, Caprini, Colangelo, Gasser, Leutwyler, Moussallam, Decotes Genon, Lesniak, Kaminski...

Left cut implemented with precision . Use data on all waves + high energy + ChPT for subtraction constants

$$\sigma_{pole} \approx 441_{-8}^{+16} - i272_{-12.5}^{+9} \text{ MeV}$$

Caprini, Colangelo, Leutwyler (2006)

● Data Analyses constrained with Roy & Forward Dispersion Relations.

García-Martín, Kaminski, JRP, Ruiz de Elvira, Yndurain 00's

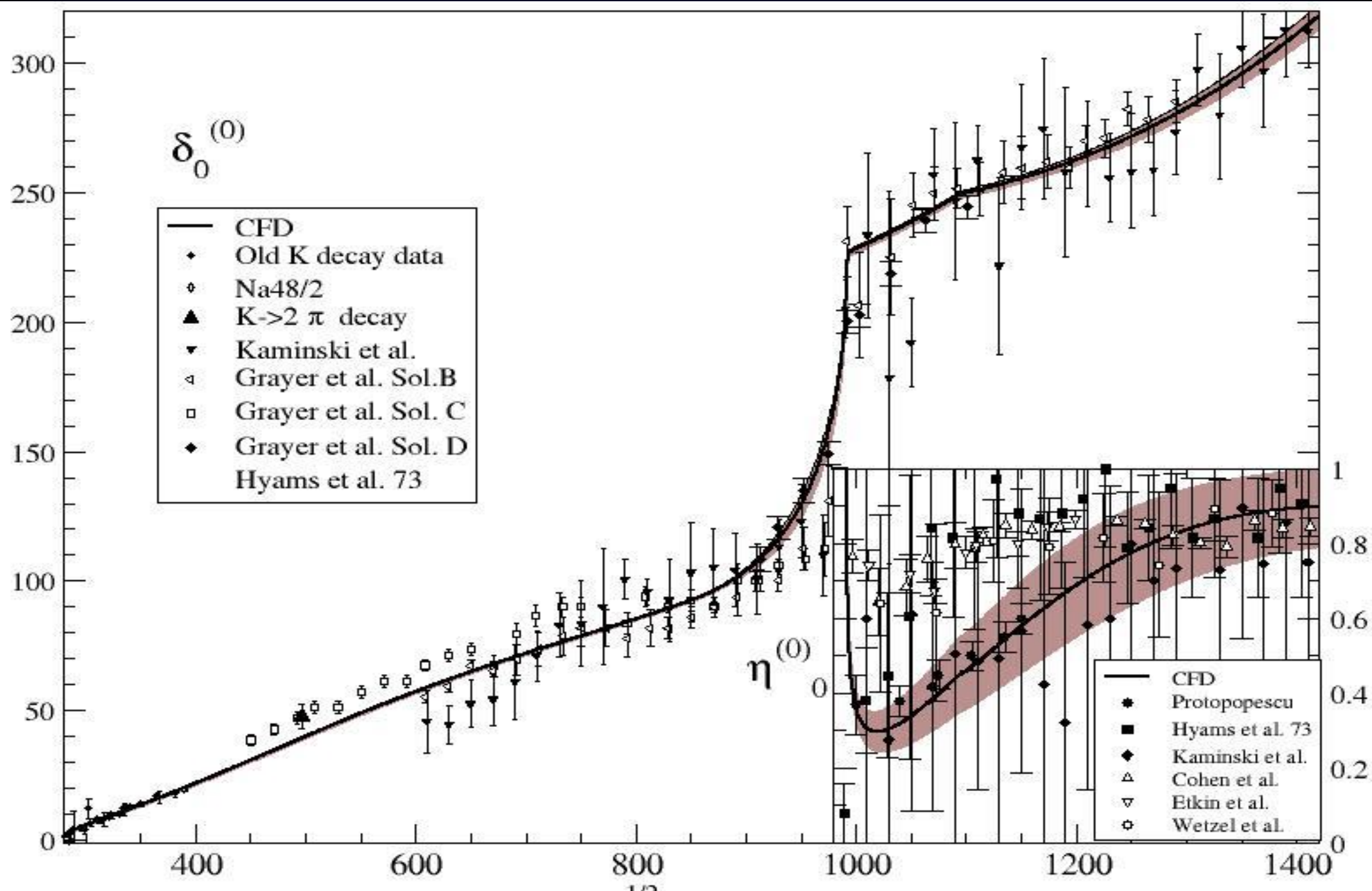
Left cut implemented with precision . Use data on all waves at all energies. NO ChPT.

These two methods good for precision. Game changers for PDG

● Unitarized ChPT

90's Truong, Dobado, Herrero, JRP, Oset, Oller, Ruiz Arriola, Nieves, Meissner,...

Use ChPT amplitudes inside dispersion relation. Relatively simple, although different levels of rigour.
Generates all scalars. Crossing (left cut) approximated... , not good for precisión but good for understanding parameters



Some relevant DISPERSIVE POLE Determinations which the PDG took into account in their 2012 edition

- GKPY equations = Roy like with one subtraction+FDRs

García Martín, Kaminski, JRP, Yndurain PRD83,074004 (2011)

R. Garcia-Martin , R. Kaminski, JRP, J. Ruiz de Elvira, PRL107, 072001(2011).

Includes latest NA48/2 constrained data fit. One subtraction allows use of data only
NO ChPT input but good agreement with previous Roy Eqs.+ChPT results.

$$(457_{-15}^{+14}) - i(279_{-7}^{+11}) \text{ MeV}$$

- Roy equations

B. Moussallam, Eur. Phys. J. C71, 1814 (2011).

An S0 Wave solution up to KK threshold with input from previous Roy Eq. works

$$(442_{-8}^{+5}) - i(274_{-5}^{+6}) \text{ MeV}$$

- Analytic K-Matrix model

G. Mennesier et al, PLB696, 40 (2010)

$$(452 \pm 13) - i(259 \pm 16) \text{ MeV}$$

The consistency of dispersive approaches, and also with previous results implementing UNITARITY, ANALYTICITY and chiral symmetry constraints by many people ...

(Ananthanarayan, Caprini, Bugg, Anisovich, Zhou, Ishida Surotsev, Hannah, JRP, Kaminski, Loiseau, Lesniak, Oller, Oset, Dobado, Tornqvist, Schechter, Fariborz, Saninno, Van Beveren, Rupp, Zou, Zheng, etc....)

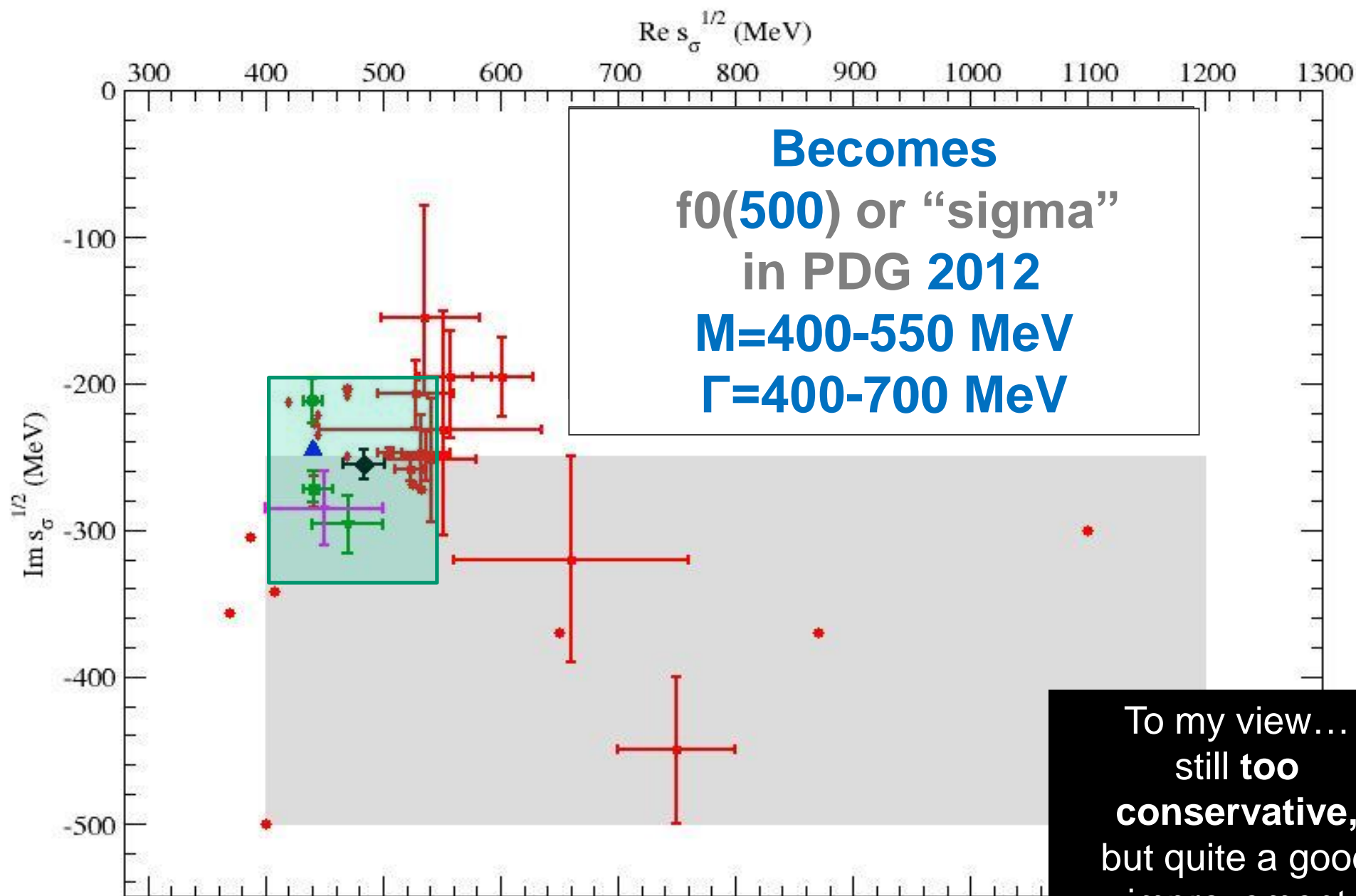
... led the PDG to neglect those works not fulfilling these constraints also restricting the sample to those consistent with NA48/2, together with results from heavy meson decays Finally quoting in the 2012 PDG edition...

$M=400-550 \text{ MeV}$
 $\Gamma=400-700 \text{ MeV}$

Accordingly THE NAME of the resonance was changed to...

$f_0(500)$

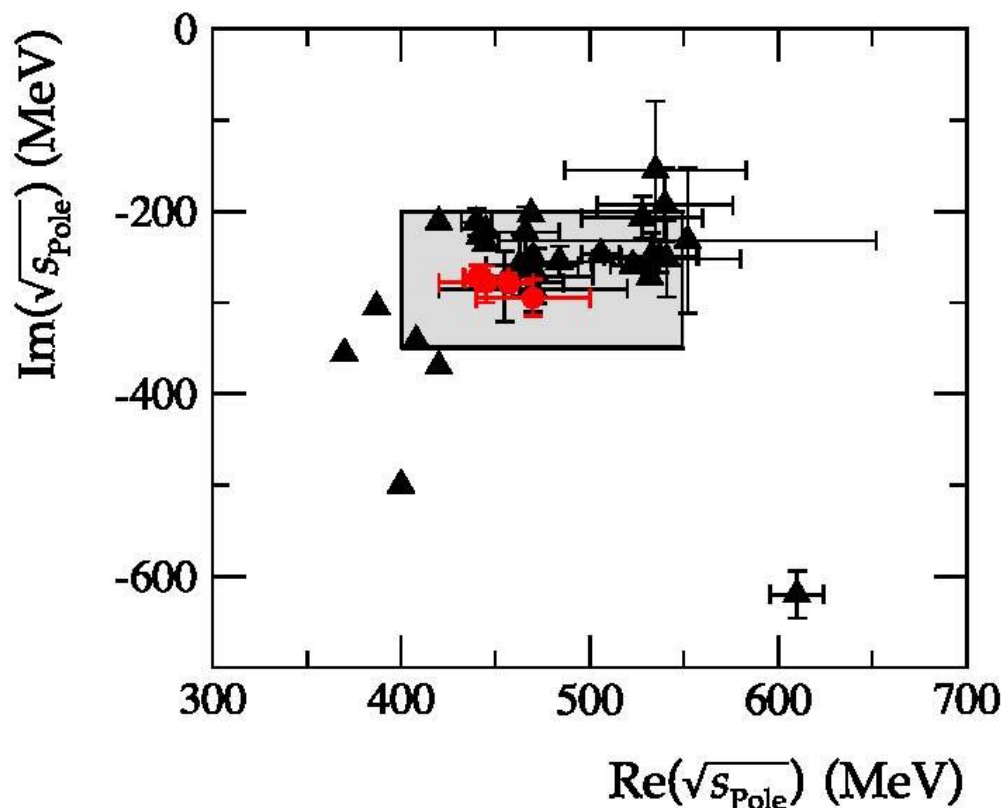
DRAMMATIC AND LONG AWAITED CHANGE ON “sigma” RESONANCE @ PDG!!



Actually, in
PDG 2012:
“Note on
scalars”

One might also take the more radical point of view and just average the most advanced dispersive analyses, Refs. [8–11], shown as solid dots in Fig. 1, for they provide a determination of the pole positions with minimal bias. This procedure leads to the much more restricted range of $f_0(500)$ parameters

$$\sqrt{s_{\text{Pole}}^\sigma} = (446 \pm 6) - i(276 \pm 5) \text{ MeV} .$$



So... the sigma
issue is settled
Even in the PDG!

- 8. G. Colangelo, J. Gasser, and H. Leutwyler, NPB603, 125 (2001).
- 9. I. Caprini, G. Colangelo, and H. Leutwyler, PRL 96, 132001 (2006).
- 10. R. Garcia-Martin, R. Kaminski, JRP, J. Ruiz de Elvira, PRL107, 072001(2011).
- 11. B. Moussallam, Eur. Phys. J. C71, 1814 (2011).

Some relevant DISPERSIVE POLE Determinations of the $f_0(980)$

(according to PDG2010 to 2012 changes)

● GKPY equations = Roy like with one subtraction

García Martín, Kaminski, JRP, Yndurain PRD83,074004 (2011)

Garcia-Martin , Kaminski, JRP, Ruiz de Elvira, PRL107, 072001(2011)

$$(996 \pm 7) - i(25_{-6}^{+10}) \text{ MeV}$$

● Roy equations

$$(996_{-14}^{+4}) - i(24_{-3}^{+11}) \text{ MeV}$$

B. Moussallam, Eur. Phys. J. C71, 1814 (2011).

**Thus, PDG12 made a small correction for the $f_0(980)$ mass
& more conservative uncertainties**

$$M = 980 \pm 10 \text{ MeV} \rightarrow M = 990 \pm 20 \text{ MeV}$$

No changes on the $a_0(980)$ mass and width at the PDG 2012
nor ever since

- Still “omitted from the summary table” since, “needs confirmation”

But all sensible implementations of unitarity, chiral symmetry, describing the data find a pole between 650 and 770 MeV with a 550 MeV width or larger.

As for the sigma, the most sounded determination comes from a SOLUTION of a Roy-Steiner dispersive formalism, consistent with UChPT Decotes Genon et al 2006

Fortunately, the PDG mass and width averages are dominated by the Roy-Steiner SOLUTION

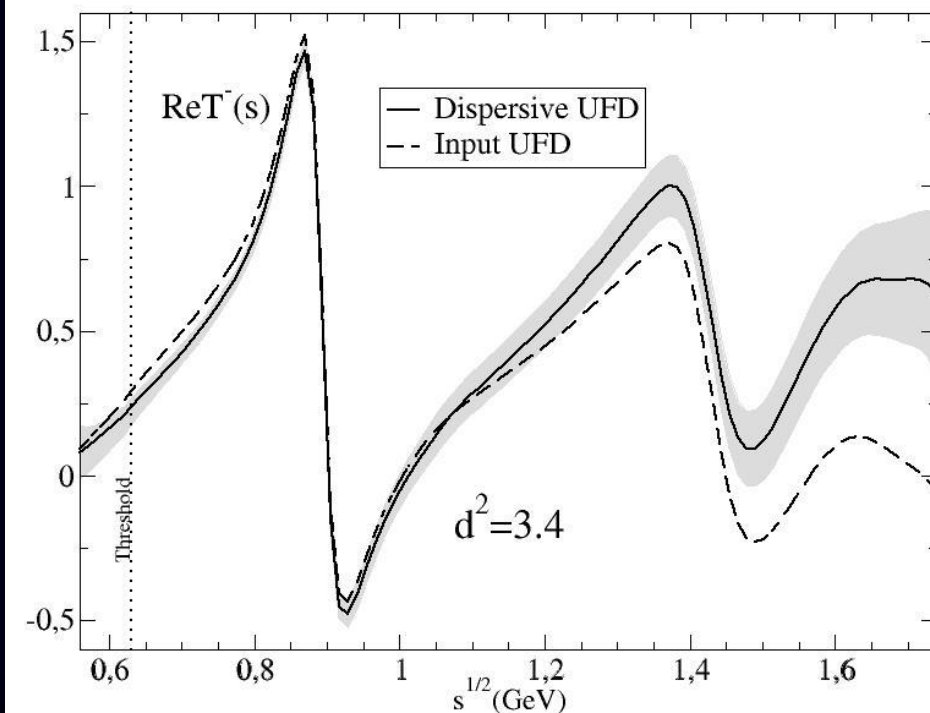
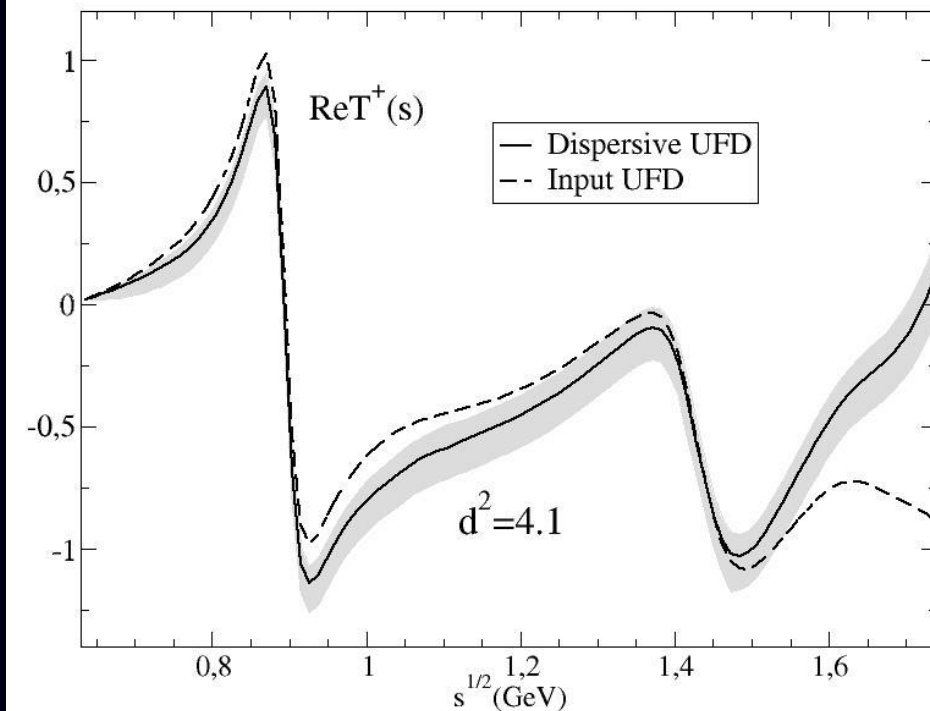
$M-i\Gamma/2=(682\pm29)-i(273\pm12)$ MeV @PDG2015

$K_0^*(800)$ Situation similar to the sigma before the 2012 revision

PDG willing to revise it but, as it happened with the sigma.... require additional independent dispersive DATA analysis, we were asked by different groups to make this additional dispersive analysis

Dispersive analysis of πK scattering DATA

(not a solution of dispersion relations)
This similar to what we did
for the σ and $\pi\pi$



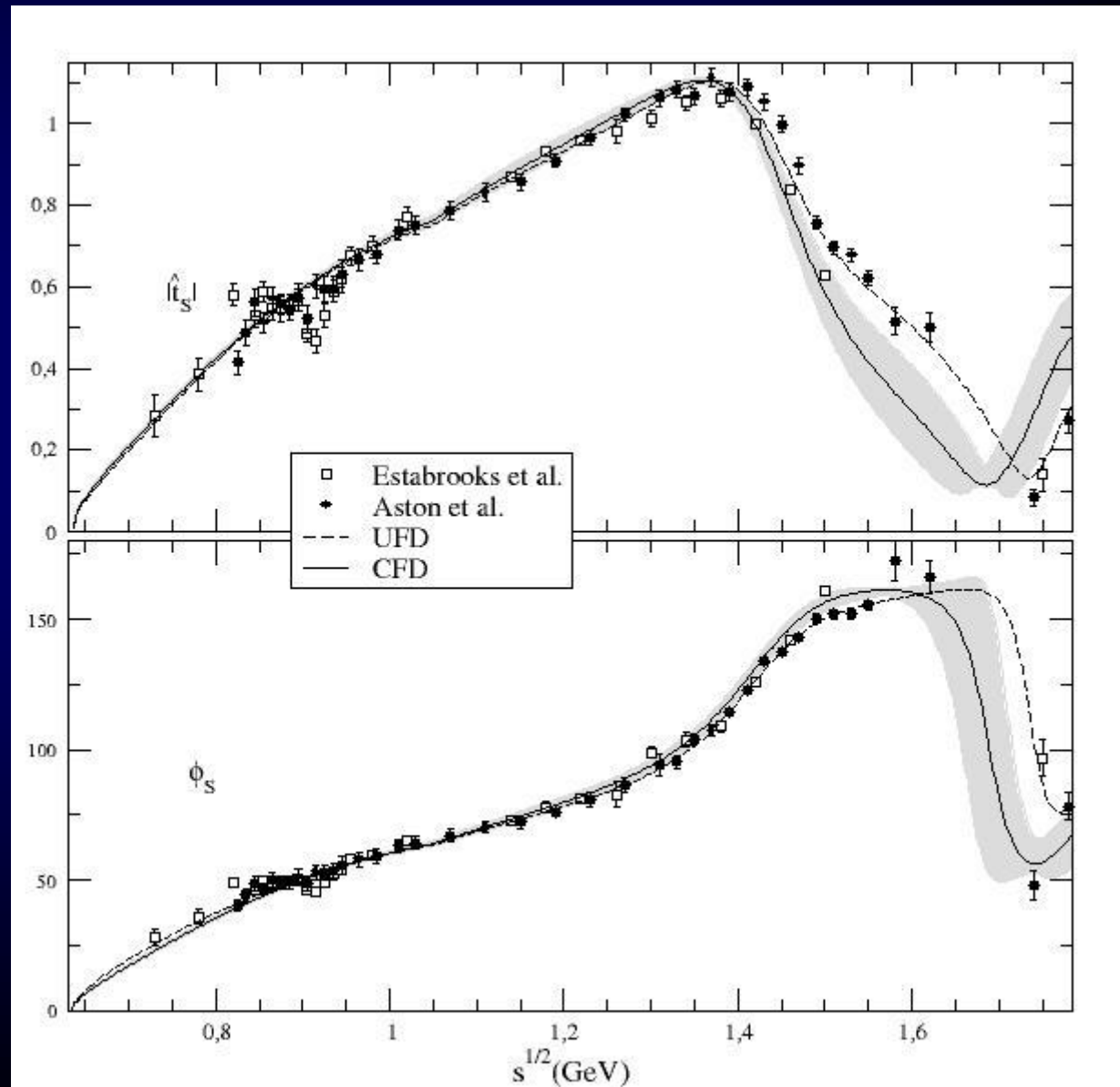
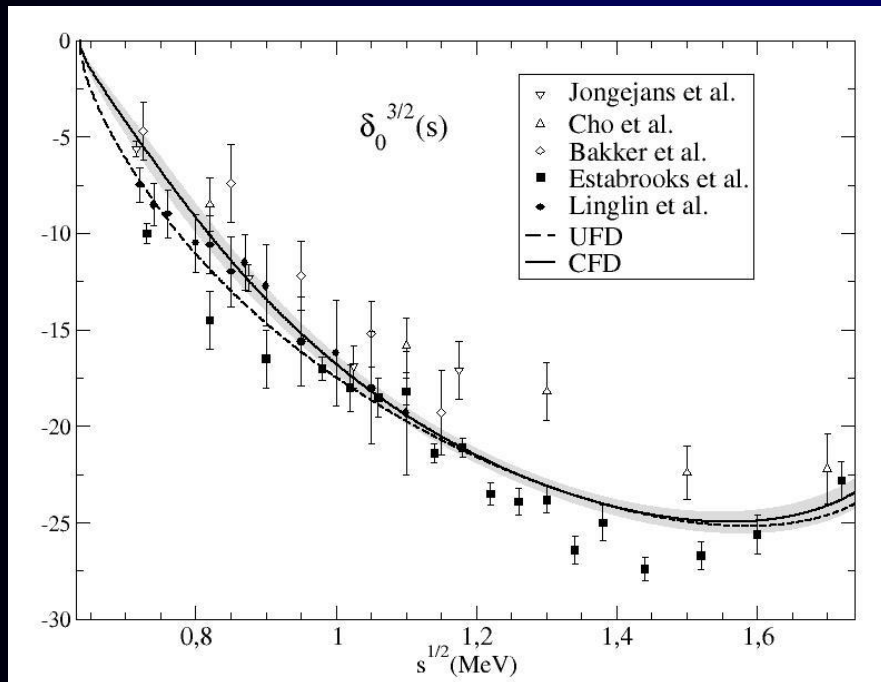
Forward Dispersion relations:
Not well satisfied by data

So we use
Forward Dispersion Relations
as CONSTRAINTS on fits

From Unconstrained (UFD) to Constrained Fits to data (CFD)

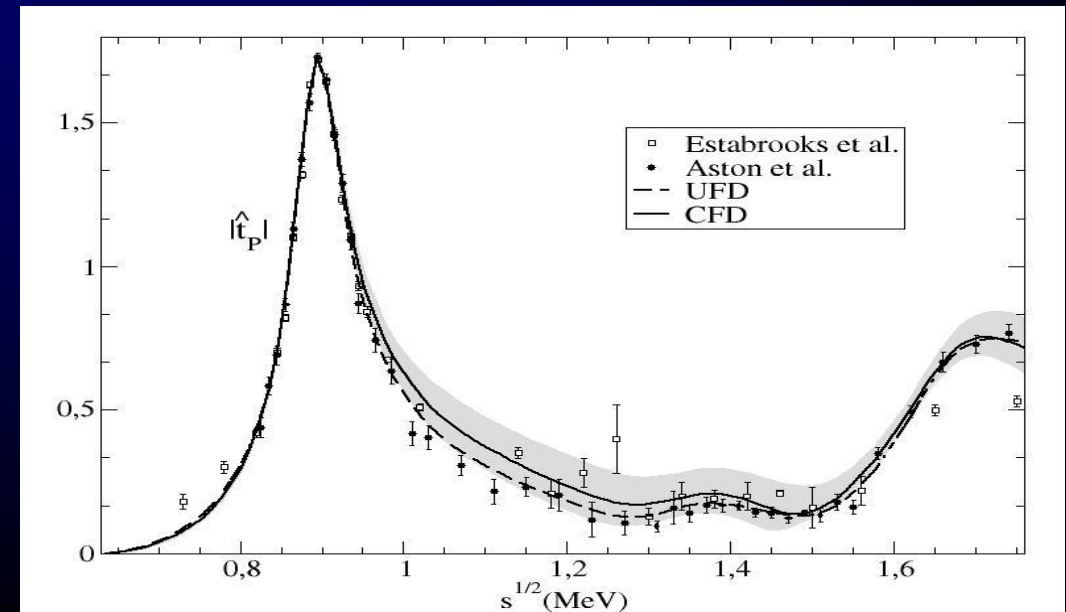
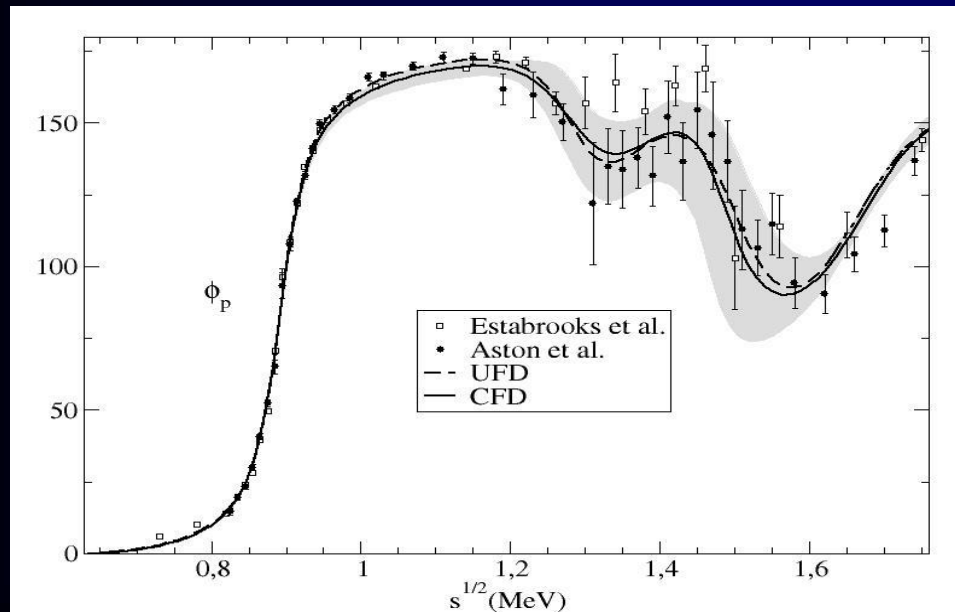
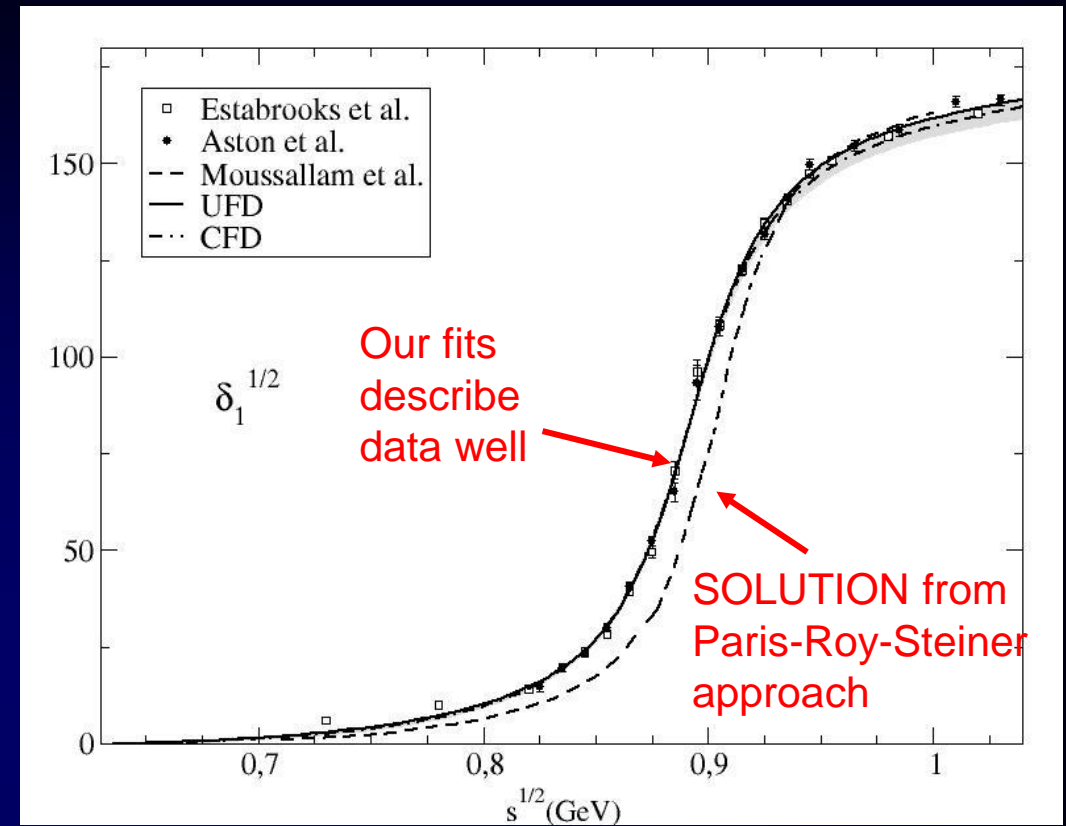
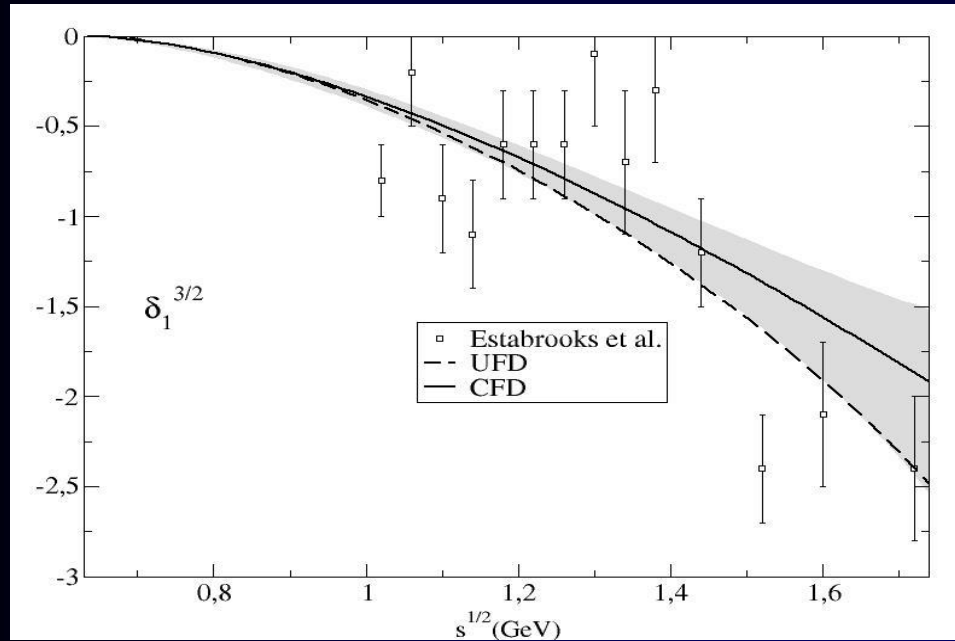
S-waves. The most interesting for the kappa

Largest changes from UFD to CFD
at higher energies



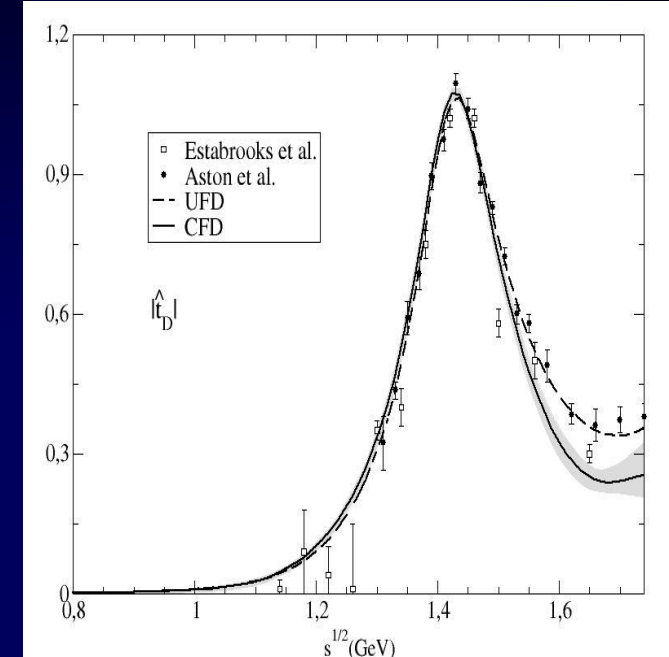
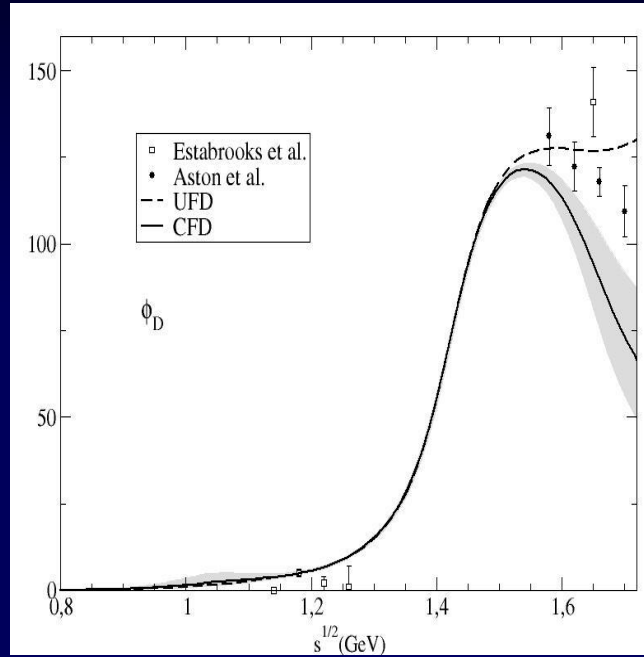
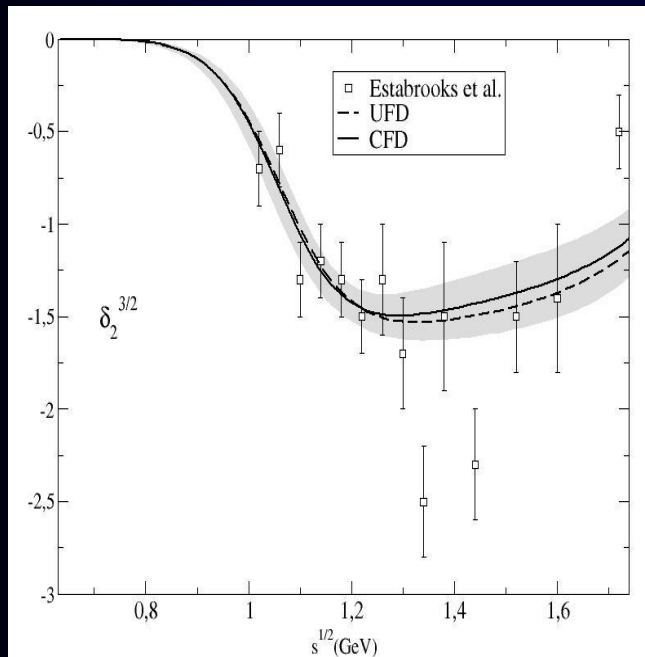
From Unconstrained (UFD) to Constrained Fits to data (CFD)

P-waves: Small changes



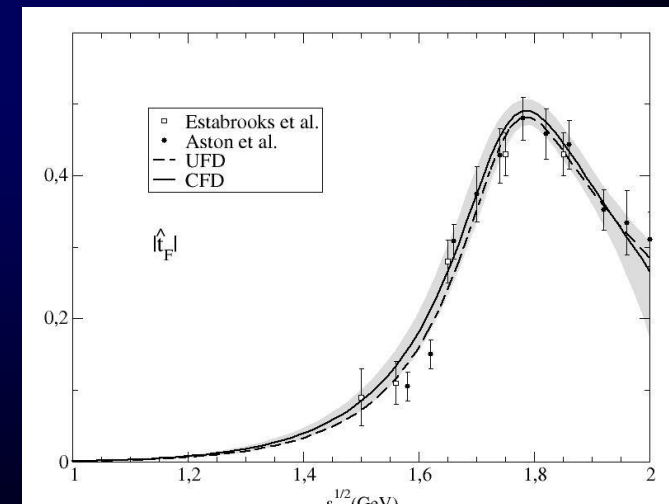
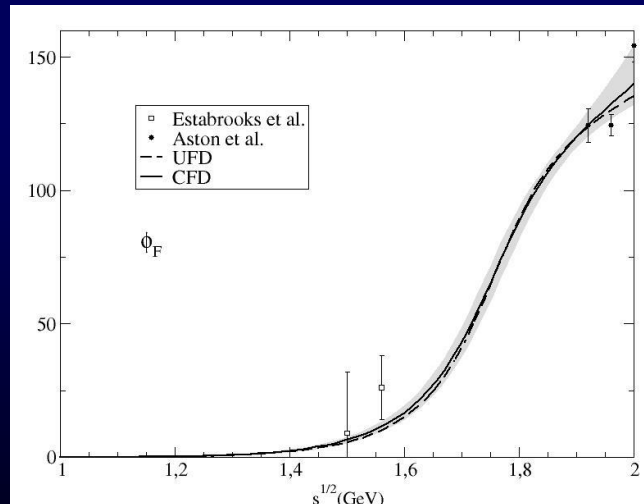
From Unconstrained (UFD) to Constrained Fits to data (CFD)

D-waves: Largest changes of all, but at very high energies

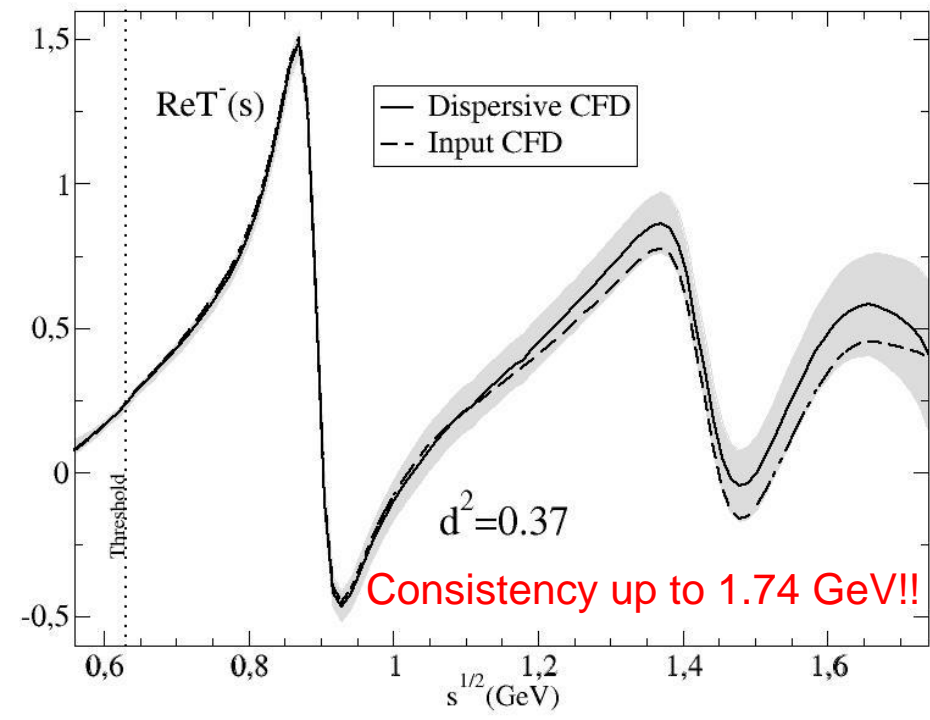
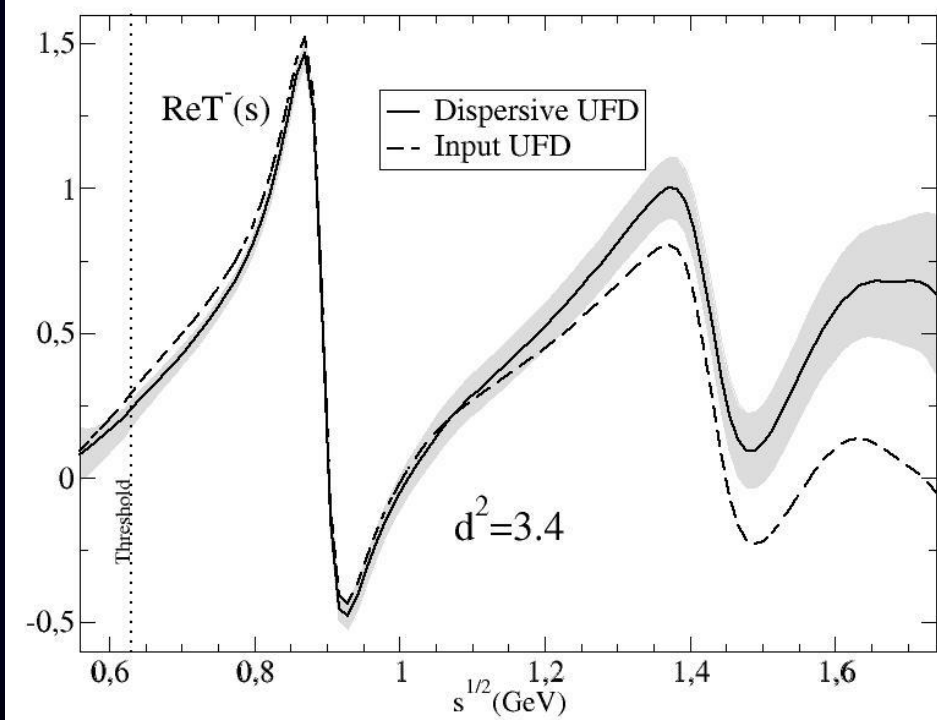
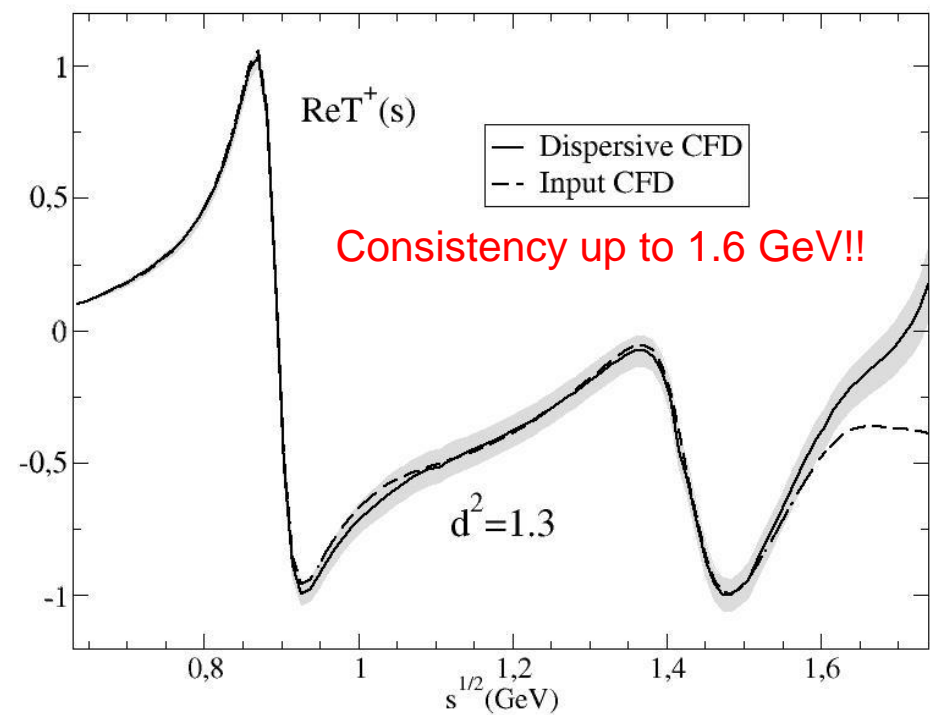
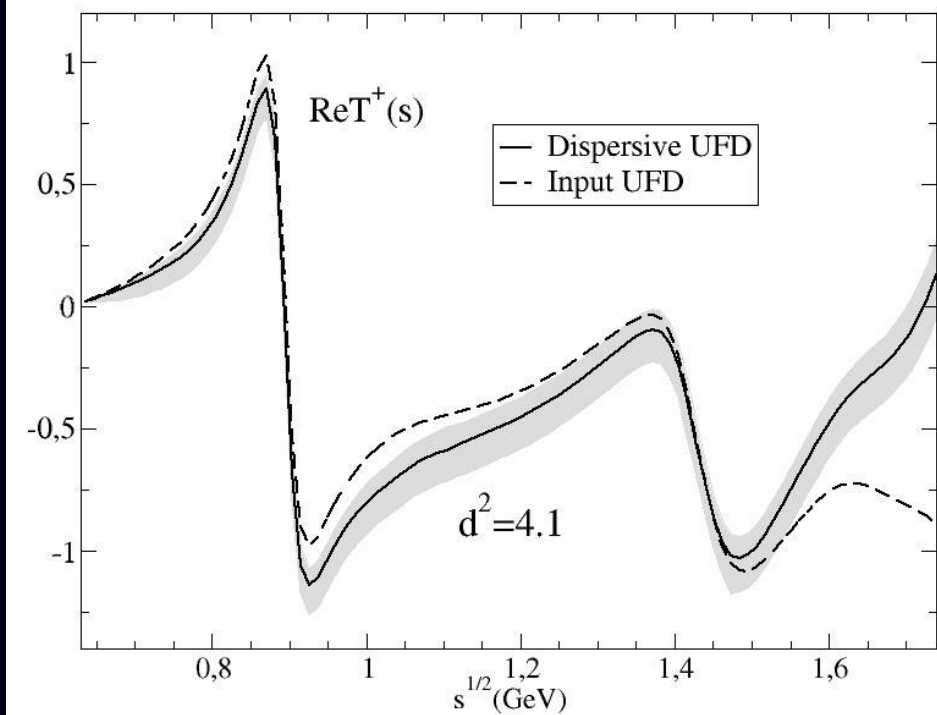


F-waves:

Imperceptible changes



Regge parameterizations allowed to vary: Only πK -p residue changes by 1.4 deviations



Our Kappa pole from CFD

Preliminary and STILL MODEL DEPENDENT. But THERE IS A POLE
Extracted from conformal parameterization

$$M-i\Gamma/2=(680\pm15)-i(334\pm15) \text{ MeV}$$

$$M-i\Gamma/2=(682\pm29)-i(273\pm12) \text{ MeV} \text{ @PDG2015}$$

BUT still in progress:

We are planning to extract it in a model Independent way with rigorous analytic methods and also imposing Roy-Steiner dispersion relations, as done for the sigma. IN PROGRESS

We expect this second dispersive determination will finally settle the $\kappa/K_0^*(800)$ issue at the PDG.

Thus, we have identified how many light scalars exist...

- Isospin=0: $\sigma/f_0(500)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1700)$ 5 states.
- Isospin=1: $a_0(980)$, $a_0(1450)$. $3 \times 2 = 6$ states
- $I=1/2$, $S=\pm 1$: $\kappa/K_0^*(800)$, $K_0^*(1430)$ $4 \times 2 = 8$ states

LET'S CLASSIFY THEM !!

The extraordinary spectroscopy of light scalars

- Very naive Quark Model, constituent M_q mass= 350 MeV

$q\bar{q}$ mesons: $P=(-1)^{L+1}$ $C=(-1)^{L+S}$

Vectors, 1^- 3S_1 $L=0$ Mass= $2M_q \sim 700$ MeV The $\rho(770)$!!

Scalars, 0^{++} 3P_0 $L=1, \dots$ Lightest $q\bar{q}$ scalar expected
heavier than $\rho(770)$!!

Naively $\sigma(500)$ does not look $q\bar{q}$
same for $\kappa(800)$ versus $K^*(892)$

- Tetraquark? $M_\sigma = 4M_q = 1400$ MeV..... not naively
Possible if strong binding, as in diquark-antidiquark, or instanton interactions, etc...
- Molecule of GB? $M_\sigma = 2 M_\pi = 280$ MeV. Somewhat better, but not quite.
Interaction not enough to bind molecule.

1) Mixture? 2) QM is NOT QCD

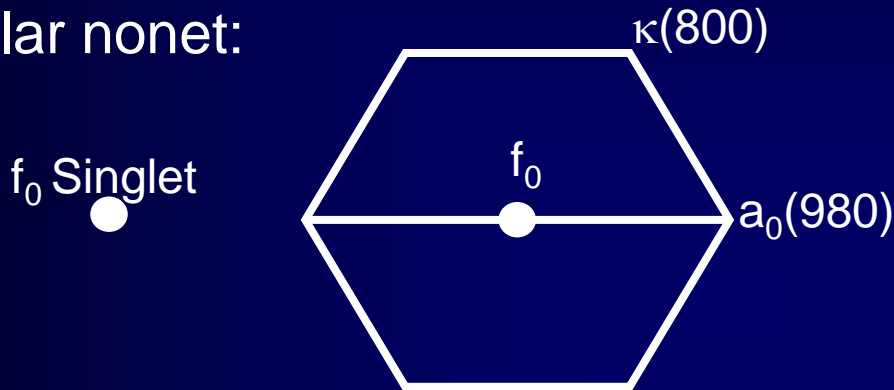
The extraordinary spectroscopy of light scalars

● Scalar SU(3) multiplets identification controversial

Too many resonances for many years.
But there is an emerging picture...



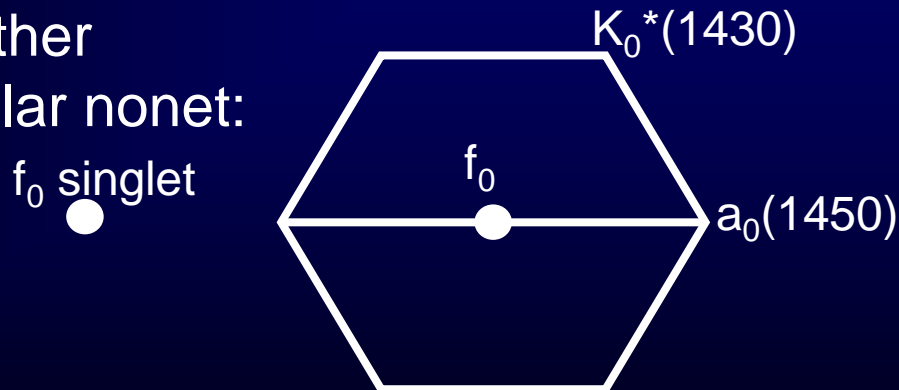
A Light scalar nonet:



Non-strange heavier!!
Inverted hierarchy problem
For quark-antiquark

$f_0(500)$ and $f_0(980)$ are
really OCTET/SINGLET mixtures

+ Another
heavier scalar nonet:



+ glueball



1) Mixture?

2) QM is NOT QCD

3) Tetraquarks/molecules?

The extraordinary spectroscopy of light scalars: tetraquarks/molecules?

M.R. Pennington, YKIS06

Scalar meson multiplets

$q\bar{q}$

$q\bar{q}q\bar{q}$

molecule

$\bar{s}s$ ————— f_0
 $\bar{s}n$ ————— K_0
 $\bar{n}n$ ————— a_0/f_0

$\bar{s}s\bar{n}n$ ————— a_0/f_0
 $\bar{s}n\bar{n}n$ ————— K_0
 $\bar{n}n\bar{n}n$ ————— f_0

KK

$K\pi$

$\pi\pi$

Jaffe

Maiani, Piccinini, Polosa, Riquer

N_c large \rightarrow stable

N_c large \rightarrow meson continuum

Subtle
difference
sometimes
not
differentiated
in literature

Natural
980 MeV
Mass
is just $2M_K$

2) QM is NOT QCD

We do not know how to solve QCD....
and lattice is not able (yet) to get a sigma (but almost gets a kappa)

HOWEVER WE HAVE
A QCD LOW-ENERGY EFFECTIVE THEORY
Let's use it!!

DOF: Pseudo-Goldstone Bosons of the spontaneous chiral symmetry breaking

$$\mathbf{SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)}$$

$$N_f = 2 \longrightarrow \pi's$$

$$N_f = 3 \longrightarrow \pi's, K's \text{ and } \eta$$

- Systematic expansion in powers of masses and momenta (model independent)

$$L_{\text{eff}} = L_2 + L_4 + L_6 + \dots,$$

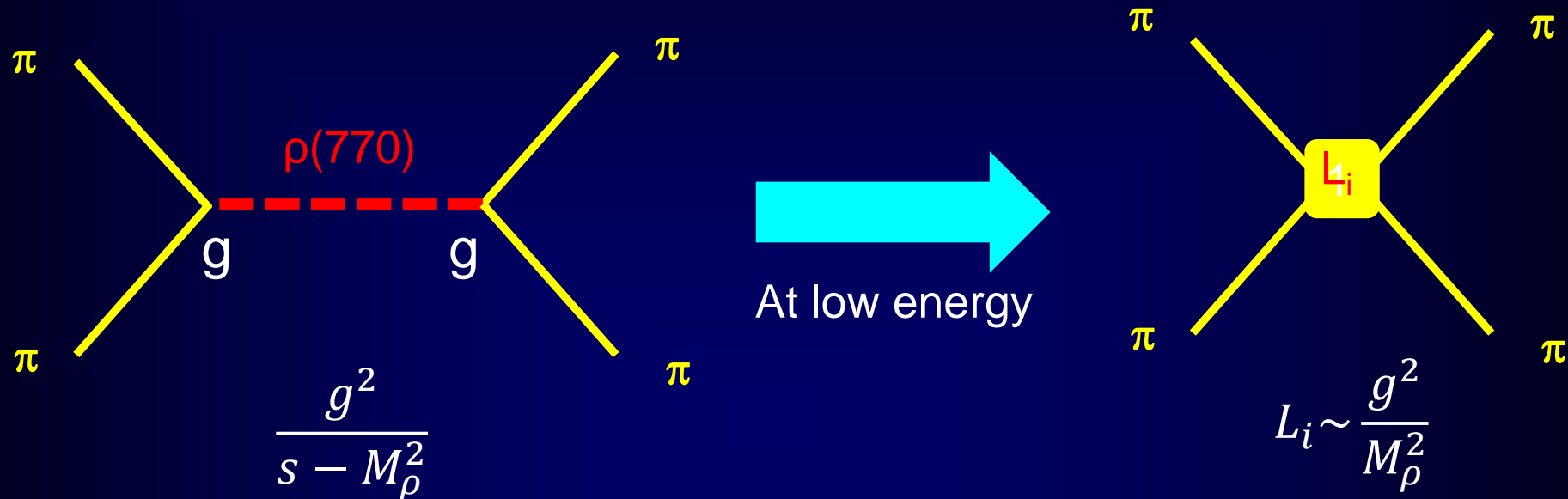
- To LO just one parameter $f_\pi \sim Nc^{1/2}$
- Parameters: Low Energy Constants (LECs). To NLO:

$$N_f = 2 \longrightarrow 4 \text{ } l's \text{ (one loop) and } 7 \text{ } r's \text{ (two loops)}$$

$$N_f = 3 \longrightarrow 8 \text{ } L's \text{ (one loop)}$$

many more at NNLO.

- LECs saturated by first resonance exchange:



But only VECTOR DOMINANCE seen!!

NO SCALAR DOMINANCE*
despite scalars lighter and wider!

Why? **Another “extraordinary” property**

*Actually there are some subleading contributions but with masses around 1 GeV

Standard ChPT

- Meson-meson scattering in standard ChPT:

$$t(s) = t_2(s) + t_4(s) + t_6(s) + \dots$$

- $t(s) \sim s / f_\pi^2 \sim 1/N_c$

- Advantages of ChPT:

- SYSTEMATIC EXPANSION, MODEL INDEPENDENT

- *Limitations:*

- *only low energy region*

- ***BUT NO RESONANCES. Only pions, kaons and etas***

But we can reproduce resonances
If ChPT is used as input for Dispersion relations:

UNITARIZED ChPT

The real improvement: Analyticity and Effective Lagrangians

● Solutions of Roy-like equations.

70's Roy, Basdevant, Pennington, Petersen...

00's Ananthanarayan, Caprini, Colangelo, Gasser, Leutwyler, Moussallam, Decotes Genon, Lesniak, Kaminski...

Left cut implemented with precision . Use data on all waves + high energy + ChPT for subtraction constants

$$\sigma_{pole} \approx 441_{-8}^{+16} - i272_{-12.5}^{+9} \text{ MeV}$$

Caprini, Colangelo, Leutwyler (2006)

● Data Analyses constrained with Roy & Forward Dispersion Relations.

García-Martín, Kaminski, JRP, Ruiz de Elvira, Yndurain 00's

Left cut implemented with precision . Use data on all waves at all energies. NO ChPT.

Not for precision, but for connecting with ChPT and QCD parameters

● Unitarized ChPT

90's Truong, Dobado, Herrero, JRP, Oset, Oller, Ruiz Arriola, Nieves, Meissner,...

Use ChPT amplitudes inside dispersion relation. Relatively simple, although different levels of rigour.
Generates all scalars. Crossing (left cut) approximated... , not good for precisión but good for understanding parameters

The Inverse Amplitude Method: Dispersive Derivation for ELASTIC scattering

- Unitarity for physical s

$$\text{Im} \frac{1}{t} = -\sigma$$

and

$$\text{Im} t_4 = \sigma |t_2|^2$$

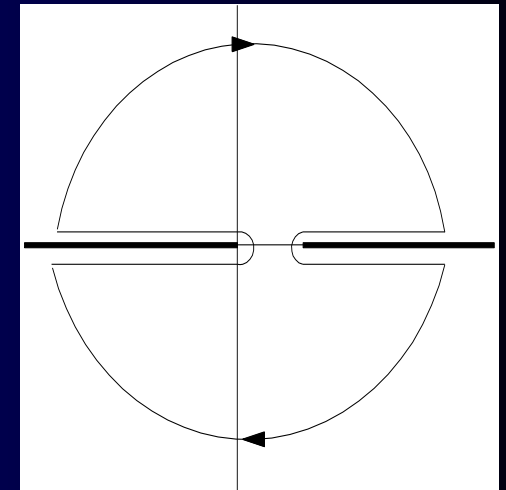
- Define $G \equiv \frac{t_2^2}{t}$

$$\text{Im} t_4 = \sigma t_2^2 = -\text{Im} G$$

- Write dispersion relations for G and t_4

$$t_{IJ}^{(4)} = b_0 + b_1 s + b_2 s^2 + \frac{s^2}{\pi} \int_{s_{th}}^{\infty} \frac{\text{Im} t_{IJ}^{(4)}(s') ds'}{s'^3 (s' - s - i\epsilon)} + LC(t_{IJ}^{(4)}).$$

$$G(s) = G_0 + G_1 s + G_2 s^2 + \frac{s^3}{\pi} \int_{s_{th}}^{\infty} \frac{\text{Im} G(s') ds'}{s'^3 (s' - s - i\epsilon)} + LC(G) + PC,$$



Subtraction Constants
from ChPT expansion
OK since $s=0$
 $G(0)=t_2(0)-t_4(0)$

PHYSICAL cut
EXACTLY Opposite
to each other

Up to NLO ChPT
Opposite to each other

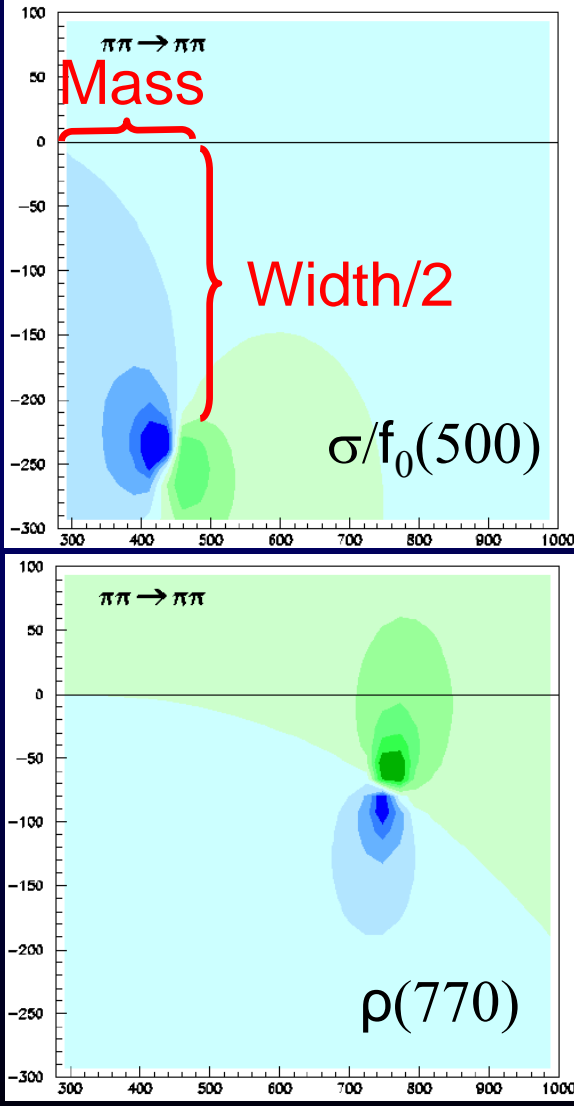
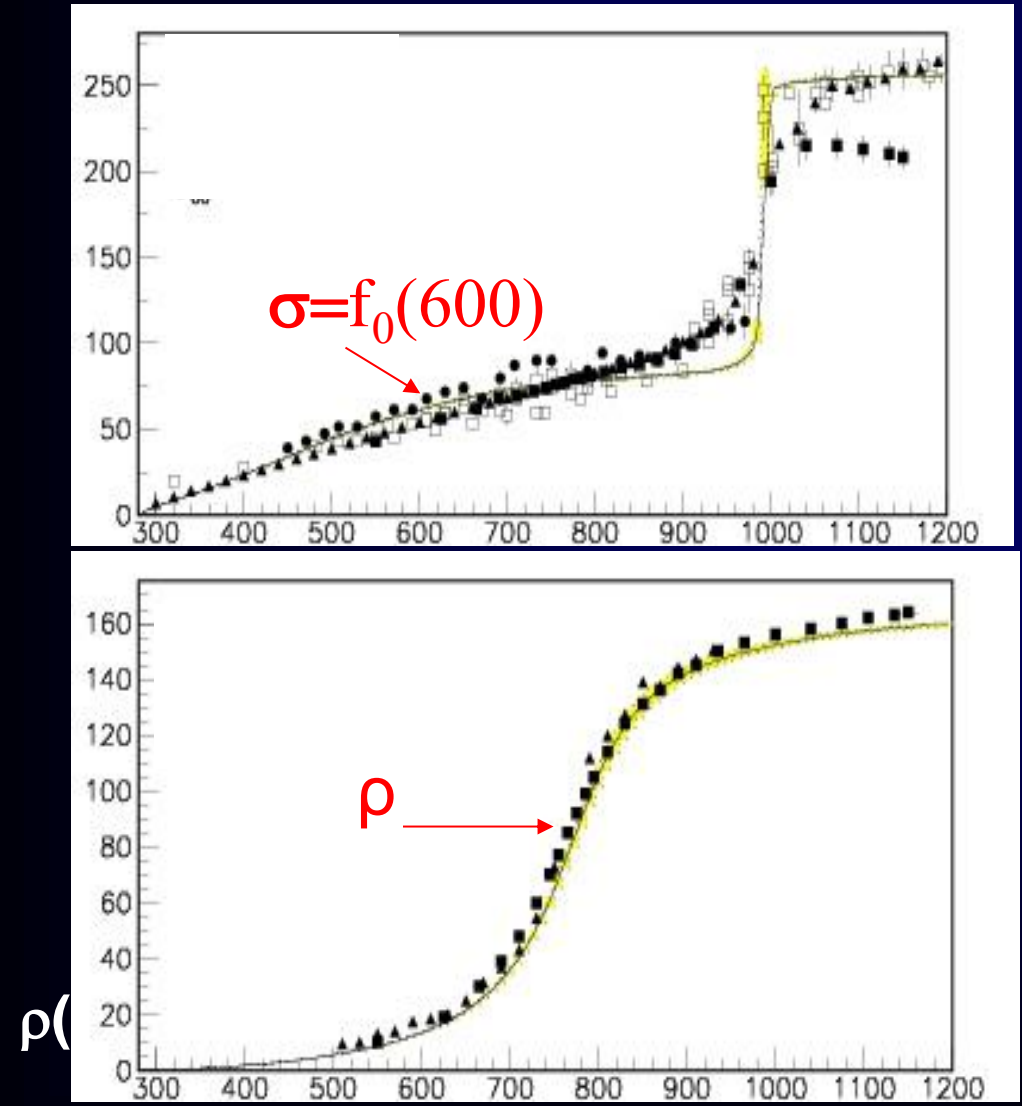
PC is $O(p^6)$ and
negligible

IAM

$$t \approx \frac{t_2^2}{t_2 - t_4 + PC}$$

- Very simple systematic extension to higher orders
- Simultaneously: ANALYTICITY
Unitarity+Chiral expansion

- Generates Poles of Resonances:
 $\sigma/f_0(500)$, $\rho(770)$, $\kappa/K_0(800)$, $K^*(892)$,
Similar results with coupled channels
Oller, Oset, JRP, Gómez-Nicola + $f_0(980)$, $a_0(980)$

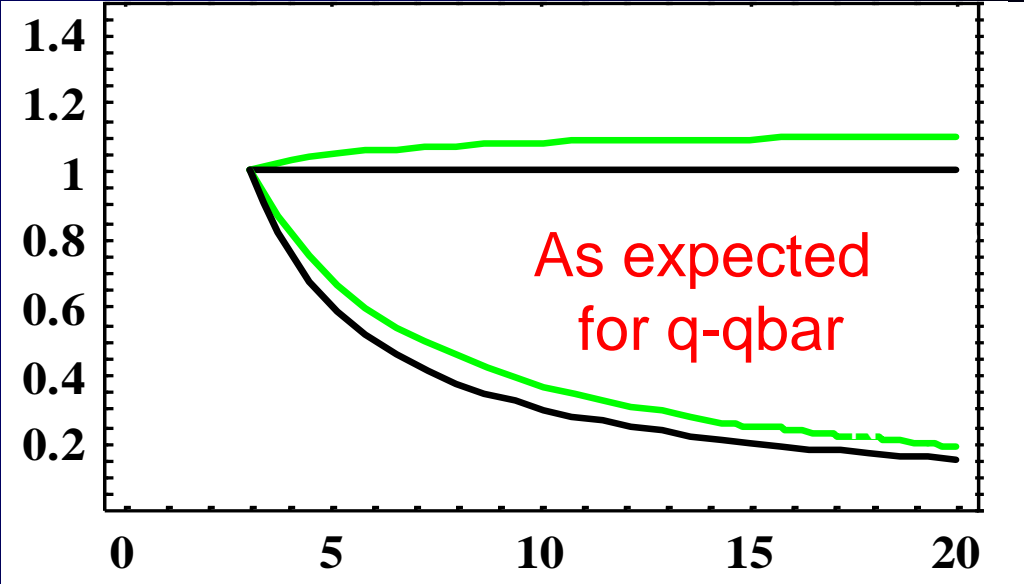
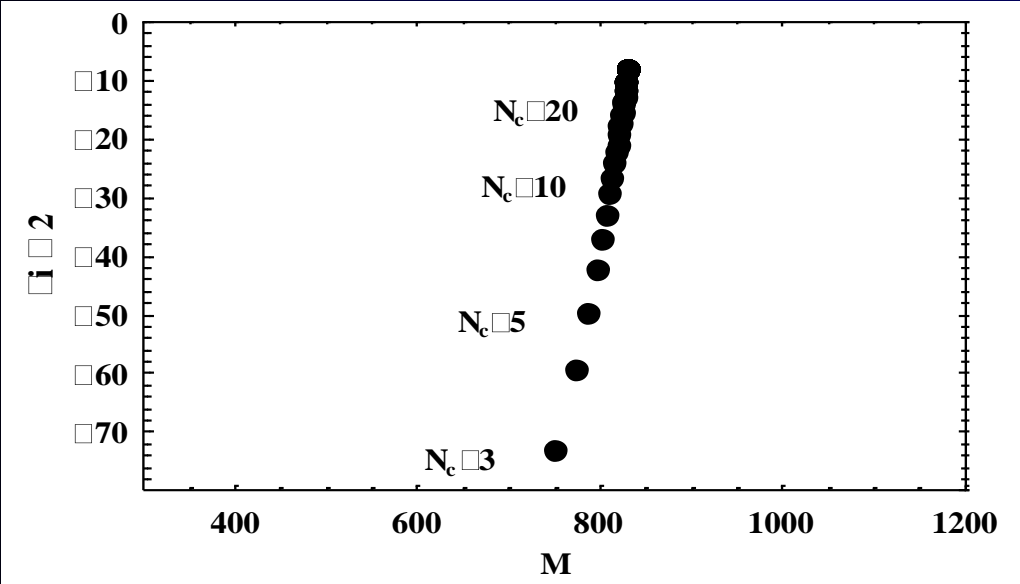


From $1/N_c$
dependence of
ChPT
parameters

↓

UChPT predicts
 $1/N_c$
behavior of
resonance
poles

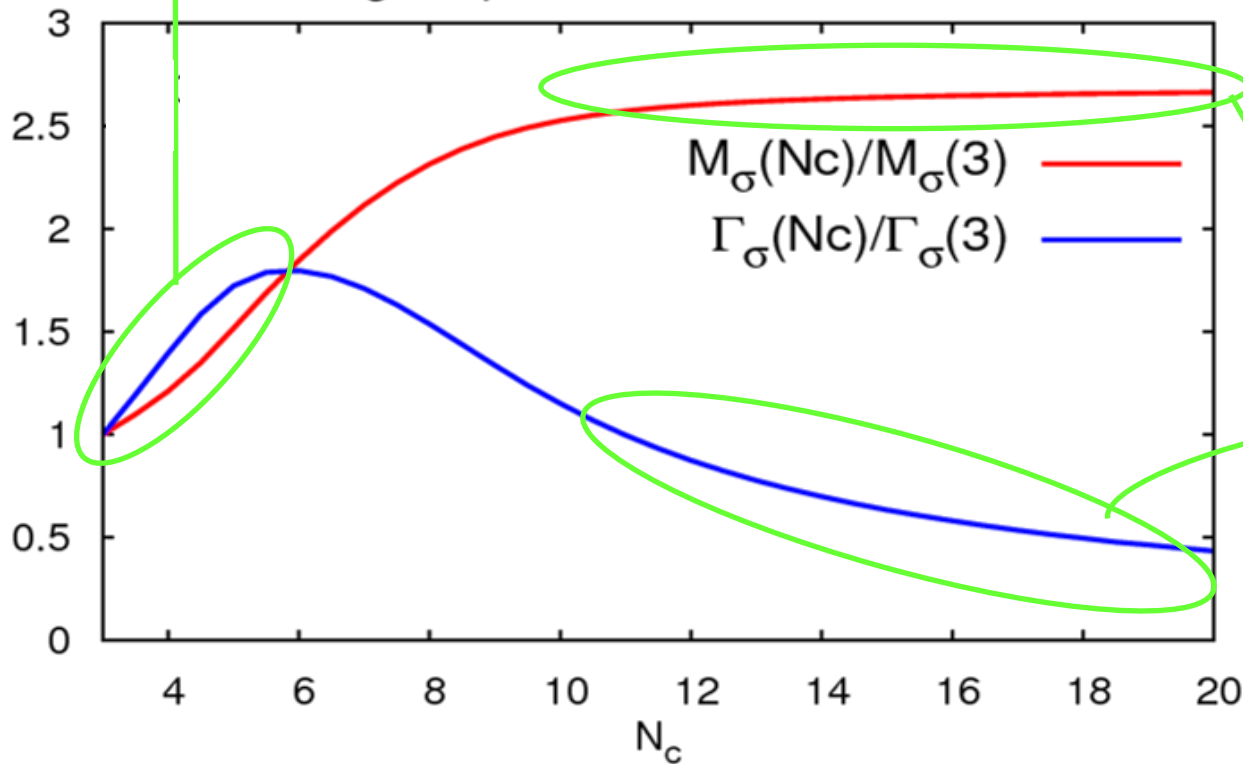
The $\rho(770)$



We can identify quark-antiquark mesons

Similarly for the $K^*(892)$

What about scalars ?

Sigma pole evolution with N_c 

Near $N_c = 3$ DOES NOT
BEHAVE AS $q\bar{q}$

But for $N_c \sim 10$ to 12

M becomes constant $\sim 1\text{ GeV}$
 Γ starts decreasing

↓
Hint of mixing with
a HEAVIER
 $q\bar{q}$ component?

Dominant non- $q\bar{q}$ component near $N_c=3$ ROBUST

Hints of possible heavier $q\bar{q}$ subcomponents

Dominant behavior also found in other UChPT variants (Uehara,Zheng,Oller, Nieves, Pich...)

Subdominant $q\bar{q}$ component around 1 GeV also suggested in, ChQM, SD-eqs, sum-rules....
Relevant for fixing “semi-localduality” problem in non- $q\bar{q}$ mesons

Other approaches supporting a non-ordinary nature of the lightest scalars

- Tetraquark models

Jaffe, Fariborz, Schechter, Sannino, Giacosa, Riquer, Polosa, t'Hooft, Maiani, Isidori,...

- Extended or unitarized LSM.

Schechter, Fariborz, Black, Sannino, Giacosa, Scadron,...

- Unitarized Quark Models: Pole doubling, Relatively similar pole trajectories

Van Baveren, Rupp, Bugg...

- Sum rules

Nielsen, Navarra, Lee, Hosaka, Jido, Oka...

- Lattice

Alford, Jaffe, Kunihiro et al., Mathur et al. Dudek et al., Bali et al.

- Schwinger Dyson /Bether Salpeter form quarks and gluons:

Roberts, Fisher, Eichmann, Williams

- Unitarized Chiral Perturbation Theory/Chiral Unitary Approach and the N_c behavior

JRP, Oller, Oset, Nieves, Ruiz Arriola, ...

- Non-ordinary Regge behavior of the $f_0(500)$

Nebreda, JRP, Szczepaniak, Carrasco...

For a FANTASTIC review...

From controversy to precision on the sigma meson: a review on the status of the non-ordinary $f_0(500)$ resonance.

J.R.P. [arXiv:1510.00653](https://arxiv.org/abs/1510.00653)

Extraordinary scalars: Regge Theory and Chew-Frautschi Plots

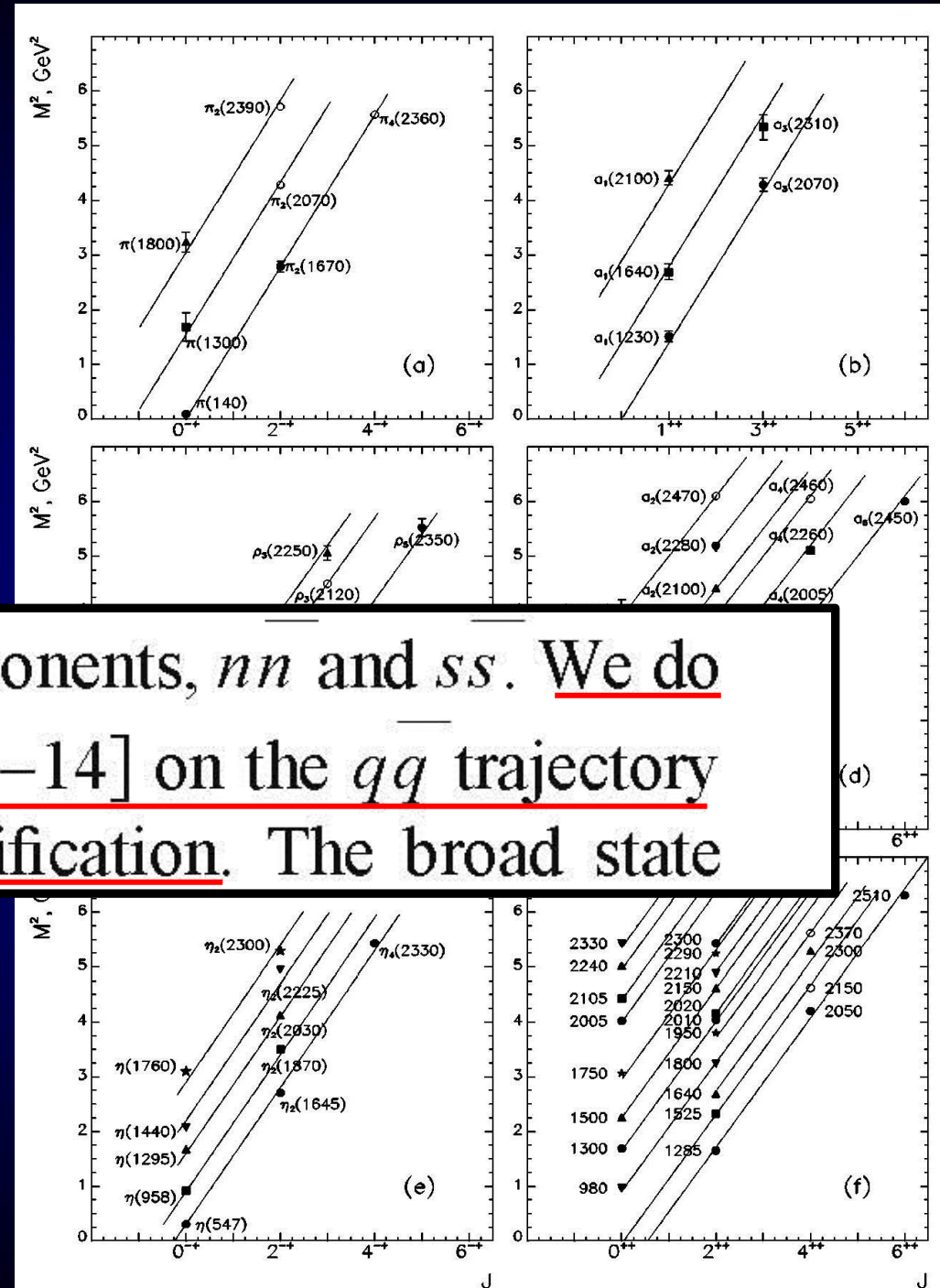
All hadrons are classified in almost linear (J, M^2) trajectories

Roughly, this can be explained by a quark-antiquark pair confined at the ends of a string-like/flux-tube configuration.

ALL OF THEM? Not quite

are doubled due to two flavor components, nn and ss . We do not put the enigmatic σ meson [11–14] on the $q\bar{q}$ trajectory supposing σ is alien to this classification. The broad state

Actually DIFFERENT INTERACTIONS MAY GIVE RISE TO DIFFERENT REGGE TRAJECTORIES



Parametrization of pole dominated amplitudes

We want to **CALCULATE** (Not fit) the TRAJECTORIES OF RESONANCES

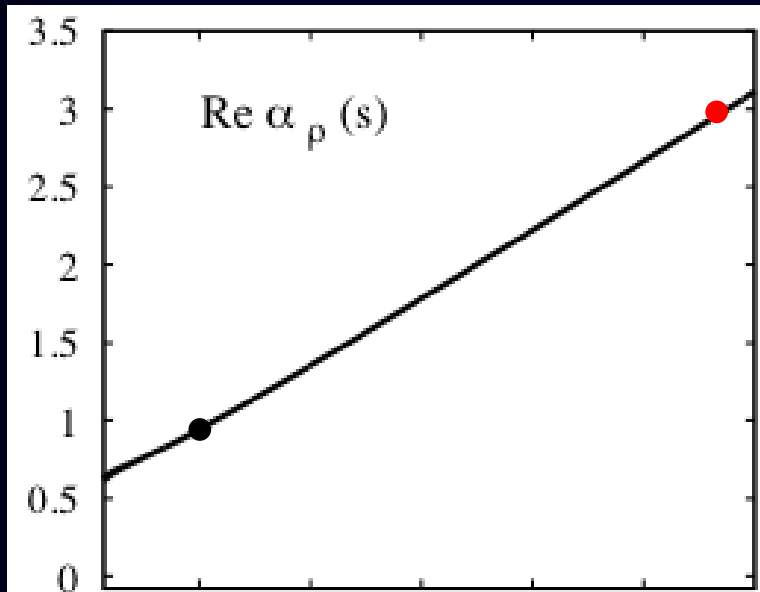
For an elastic resonances, the trajectory and residue should satisfy a system of integral equations:

$$\begin{aligned}\operatorname{Re}\alpha(s) &= \alpha_0 + \alpha' s + \frac{s}{\pi} PV \int_{4m_\pi^2}^{\infty} ds' \frac{\operatorname{Im}\alpha(s')}{s'(s' - s)}, \\ \operatorname{Im}\alpha(s) &= \rho(s)b_0 \frac{\hat{s}^{\alpha_0 + \alpha' s}}{|\Gamma(\alpha(s) + \frac{3}{2})|} \exp(-\alpha' s [1 - \log(\alpha' \tilde{s})]) \\ &\quad + \frac{s}{\pi} PV \int_{4m_\pi^2}^{\infty} ds' \frac{\operatorname{Im}\alpha(s') \log \frac{\hat{s}}{\hat{s}'} + \arg \Gamma(\alpha(s') + \frac{3}{2})}{s'(s' - s)}\end{aligned}$$

We solve these eqs. Imposing the value of an “observed” pole
LET US CHECK THE METHOD WORKS

Results: ρ case ($l = 1, J = 1$)

We get a prediction for the ρ Regge trajectory, which is almost real



This is a “prediction” for the whole tower of $\rho(770)$ Regge partners:

$\rho(1690)$

$\rho(2350)$

....

the LINEAR behavior
is a RESULT

Almost LINEAR $\alpha(s) \sim \alpha_0 + \alpha' s$

intercept $\alpha_0 = 0.520 \pm 0.002$

slope $\alpha' = 0.902 \pm 0.004 \text{ GeV}^{-2}$

Previous studies from FITS:

[1] $\alpha_0 = 0.5$

[1] $\alpha' = 0.83 \text{ GeV}^{-2}$

[2] $\alpha_0 = 0.52 \pm 0.02$

[2] $\alpha' = 0.9 \text{ GeV}^{-2}$

[3] $\alpha_0 = 0.450 \pm 0.005$

[4] $\alpha' = 0.87 \pm 0.06 \text{ GeV}^{-2}$

Remarkably consistent with the literature!!,
(taking into account our approximations)

[1] A. V. Anisovich et al., Phys. Rev. D 62, 051502 (2000)

[2] J. R. Pelaez and F. J. Yndurain, Phys. Rev. D 69, 114001 (2004)

[3] J. Beringer et al. (PDG), Phys. Rev. D 86, 010001 (2012)

[4] P. Masjuan et al., Phys. Rev. D 85, 094006 (2012)

The method identifies many other ordinary states...

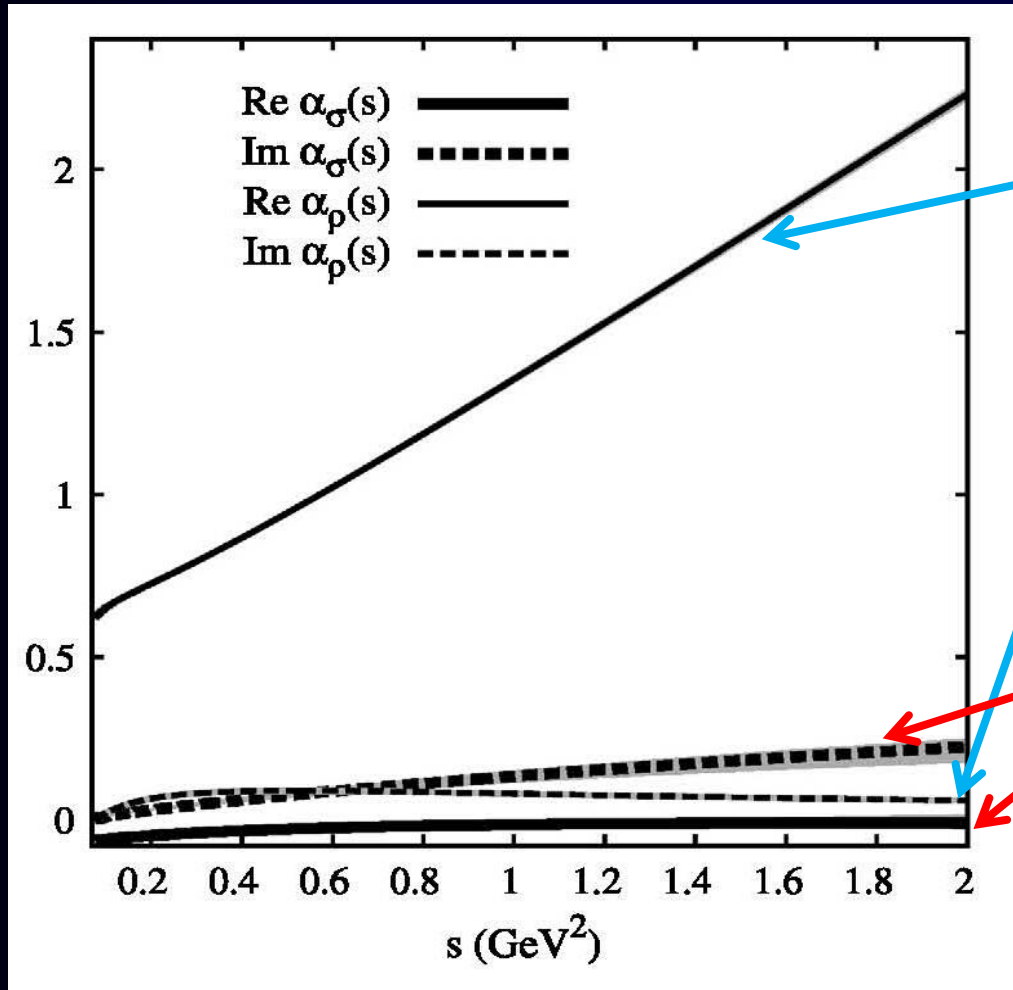
$\rho(770)$, $f_2(1275)$, $f_2'(1525)$, $K^*(892)$, $K_1(1400)$, $K^*(1430)$

J.A.. Carrasco J. Nebreda, JRP, A.Szczepaniak, Phys.Lett. B749 (2015) 399

JRP, A. Rodas, in preparation

What about scalars?

From their poles only, using a dispersive formalism



The $\rho(770)$ trajectory comes out almost-real and linear, consistent with ordinary trajectories

The $f_0(500)$ trajectory is not even real and much smaller (another scale at play)

Similar for the κ

JRP, A. Rodas, in preparation

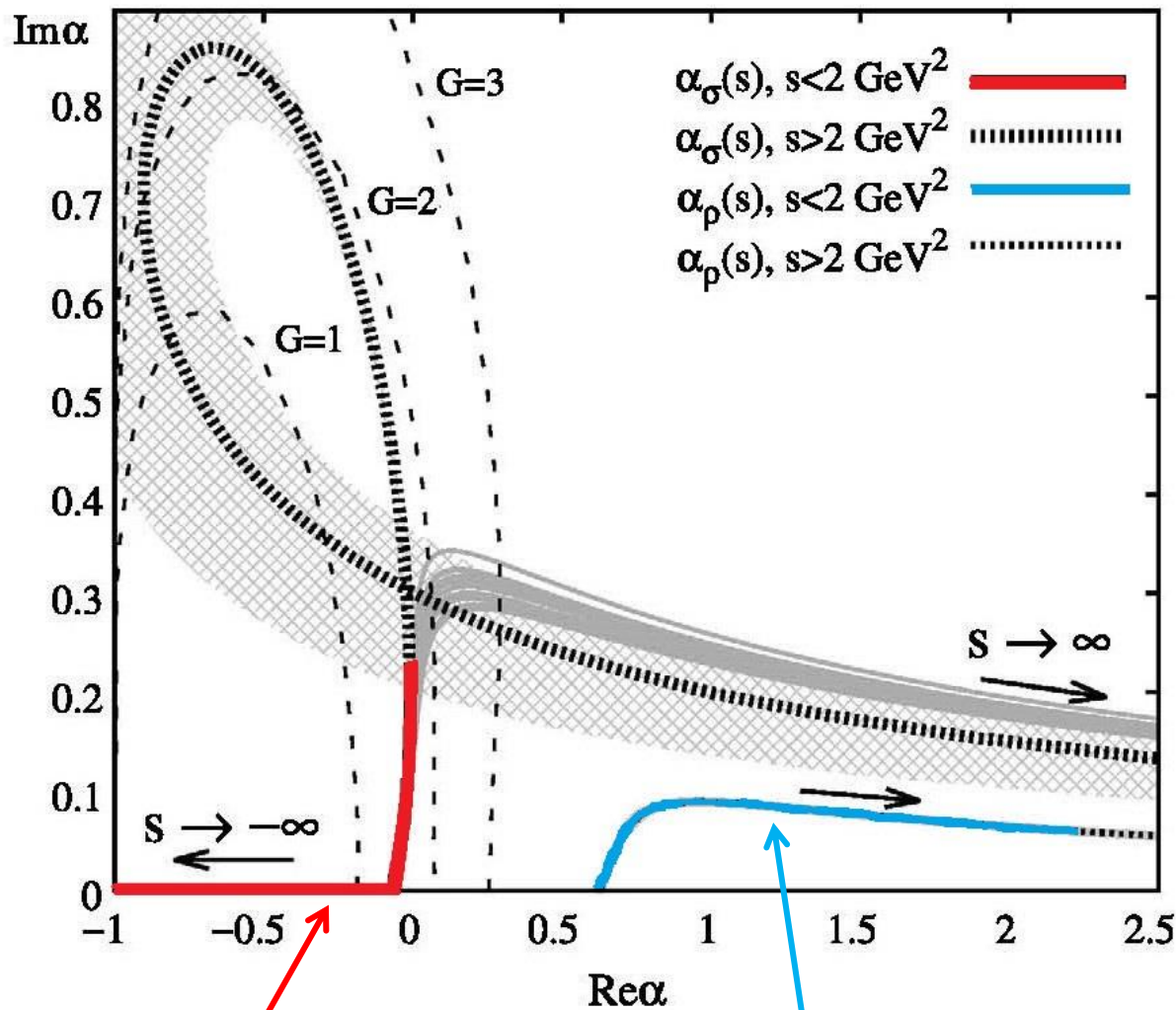
No evident Regge partners for the $f_0(500)$, explaining why it is not in linear fits and disfavors a predominant $q\text{-}\bar{q}$ nature

If not-ordinary...

What then?
Can we identify the dynamics of the σ and κ
trajectories?

Not quite yet... but...

Plotting the trajectories in the complex J plane...



Non-ordinary σ
trajectory

Ordinary ρ trajectory

Striking similarity with
Yukawa potentials at low energy:

$$V(r) = -Ga \exp(-r/a)/r$$

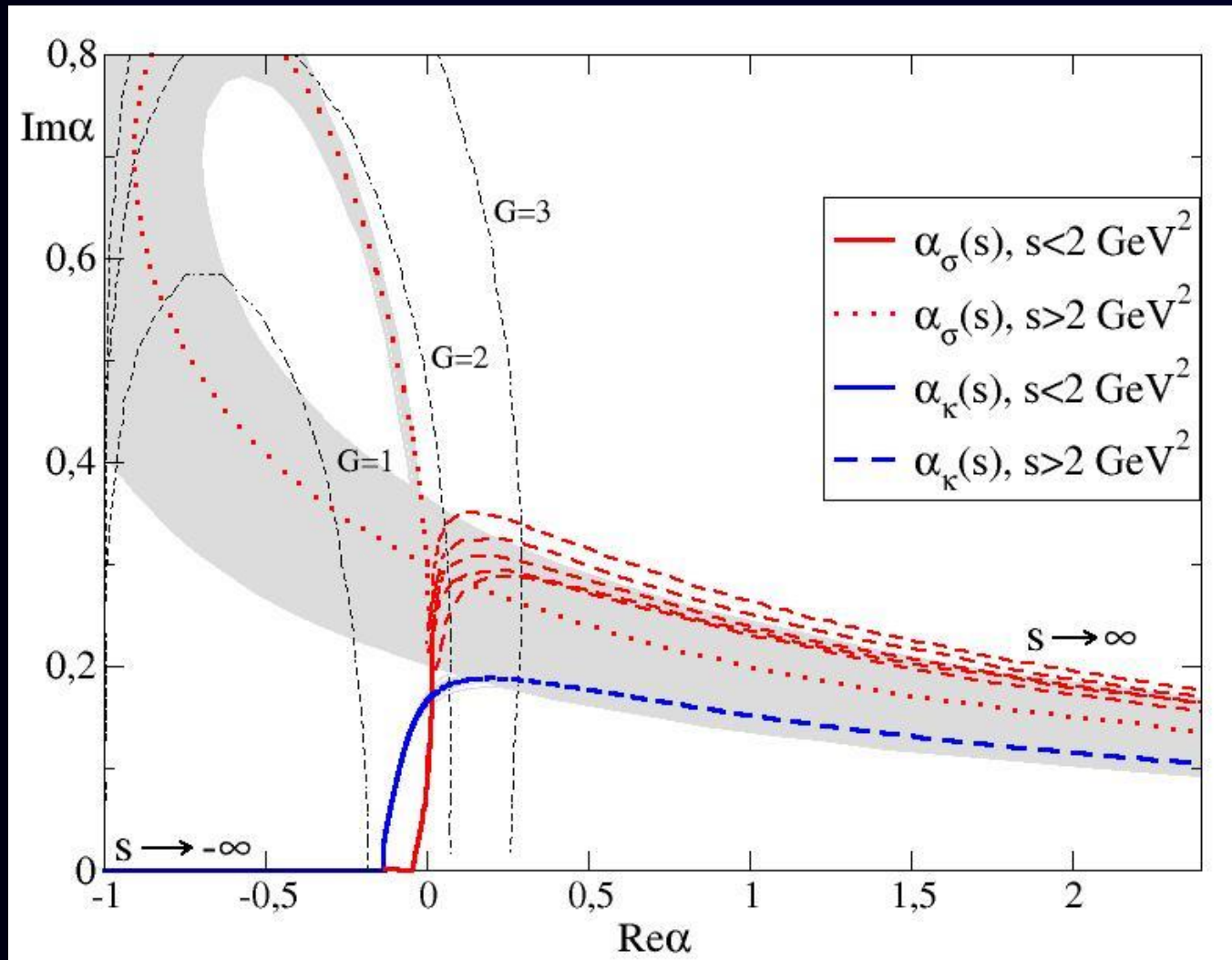
Our result is mimicked with
 $a = 0.5 \text{ GeV}^{-1}$
to compare with S-wave $\pi\pi$
scattering length 1.6 GeV^{-1}

“a” rather small !!!

The extrapolation of our trajectory also follows a Yukawa but deviates at very high energy

For the kappa we find a very similar behavior:

JRP, A. Rodas, in preparation



Compared to:
 $V(r) = -Ga \exp(-r/a)/r$

Similar order of
 magnitude for range

$$a_{\pi\pi} = 0.5 \text{ GeV}^{-1}$$

$$a_{\pi K} = 0.33 \text{ GeV}^{-1}$$

$$a_{\pi\pi} / a_{\pi K} \sim 1.52$$

Maybe a_{MM} scales as
 inverse of reduced mass

$$\mu_{\pi K} / \mu_{\pi\pi} = 1.57$$

Summary

Part 1: Existence and parameters

- After 60 years of controversy, a low-mass and very wide $\sigma/f_0(500)$ has been recognized (even @PDG) with relatively precise parameters
- The use of good data and MODEL INDEPENDENT DISPERSIVE methods were essential to establish its parameters
- The $\kappa/K_0^*(800)$ is now in a similar situation as the $\sigma/f_0(500)$ in 2010. We are working to have an additional DISPERSIVE DETERMINATION that will confirm its parameters. Expect changes @PDG soon.

Part 2: Nature and classification

- Using unitarized ChPT we find that the light scalars do NOT follow predominantly a quark-antiquark behavior. The sigma may have a subdominant quark-antiquark component with a mass around 1 GeV
- Using dispersive approach we can CALCULATE the Regge trajectories of elastic resonances. The ρ , K^* , f_2 , f_2' and K_1 result in the usual linear trajectories.
- But the $\sigma/f_0(500)$ and $\kappa/K_0^*(800)$ **do not fit into conventional linear Regge trajectories**. They behave similarly and have scales typical of meson physics