The spectrum of nucleon and Delta resonances in a dynamical coupled-channel model

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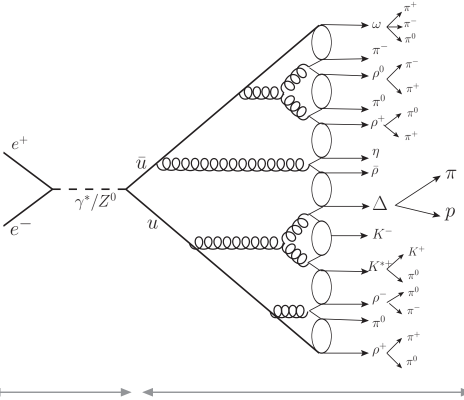
November 20, 2014

Fundamental forces
in the standard model particle physics

Hadron production in e^+e^- collisions:



electroweak interaction

strong interaction

$\rightarrow$ fundamental forces in nature

Strong force:

- Fundamental particles: quarks (q)
  (almost free at high energies)
- Observed particles: hadrons
  (low and medium energies)
  - Mesons ($q\bar{q}$ states)
  - **Baryons** (qqq, $\bar{q}\bar{q}\bar{q}$ states)

    ↳ protons & neutrons $\rightarrow$ matter

(+ exotic states ...)

Big question:

How do quarks and gluons form hadrons?

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in the standard model particle physics

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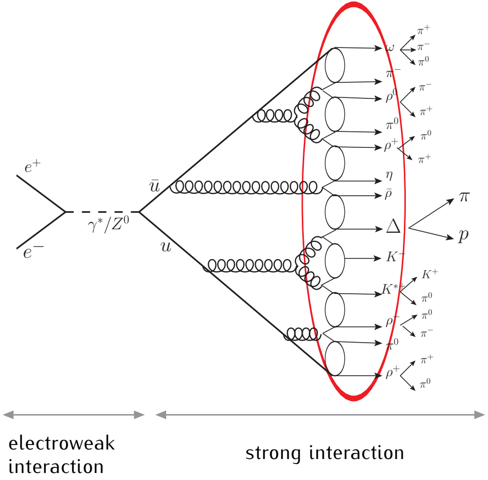

→ fundamental forces in nature

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Strong interactions

Interaction between colored **quarks**, mediated by **gluons**

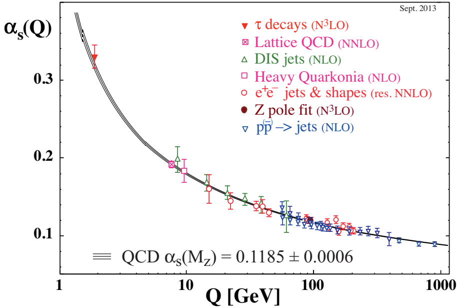
- Quantum Chromodynamics (QCD): gauge filed theory of the strong interactions

$$\mathcal{L}_{QCD} = \sum_q \bar{\Psi}_{q,a}(i\not{\partial}\delta_{ab} - g_s\gamma^\mu t^C_{ab}\mathcal{A}^C_\mu - m_q\delta_{ab})\Psi_{q,b} - \frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu}$$

$\Psi_{q,b}$: quarks
$\mathcal{A}^C_\mu$: gluon fields
$F^A_{\mu\nu}$: field tensors

$\uparrow$

Running coupling constant $\alpha_s = \frac{g_s^2}{4\pi}$



picture from PDG

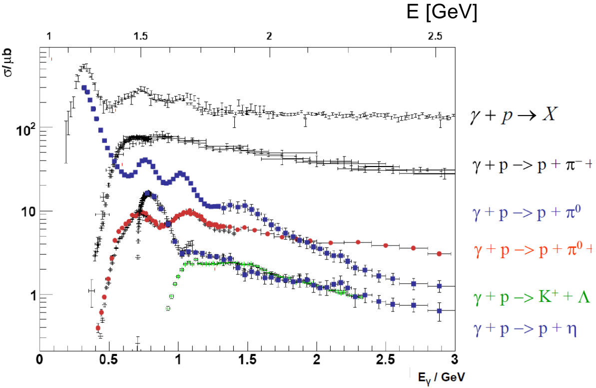
- High energies: asymptotic freedom
- Low and intermediate energies:
  - no perturbative QCD
  - confinement of quarks in **hadrons**

$\hookrightarrow$ other theoretical approaches

(e.g. EFT, quark models, lattice QCD)

Experimental tests of strong interactions at medium energies

→ measurements of hadronic cross sections and asymmetries



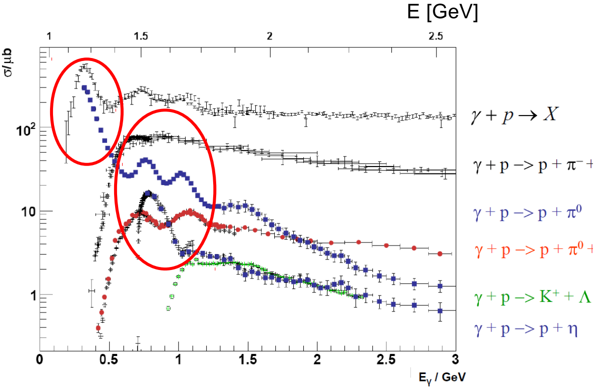
source: ELSA; data: ELSA, JLab, MAMI

What are those bumps??

- energy and angular momentum **excitations** of baryons?
- background processes?
- something else?

Experimental tests of strong interactions at medium energies

$\rightarrow$ measurements of hadronic cross sections and asymmetries



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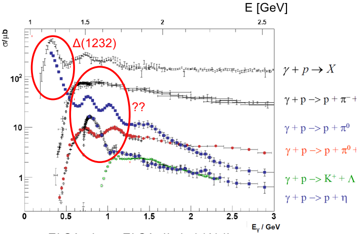
The excited hadron spectrum:
Connection between experiment and QCD in the non-perturbative regime

Excited hadron spectrum: testing ground for theories of the strong force at low and medium energy

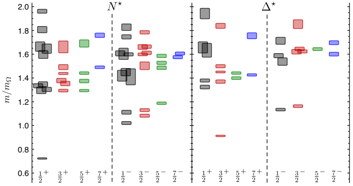
Experimental study of hadronic reactions



source: ELSA; data: ELSA, JLab, MAMI

$\Longleftrightarrow$

Theoretical predictions of excited hadrons
e.g. from lattice calculations:
(with some limitations)



$m_\pi = 396$ MeV [Edwards *et al.*, Phys.Rev. D84 (2011)]

$\Rightarrow$ Partial wave decomposition:
decompose data with respect to a conserved quantum number:
total **angular** momentum and parity J^P

Missing resonance problem

Theoretical description of a scattering process

$$S = \mathbb{1} + iT$$

- Lippmann-Schwinger equation: $T = V + VGT$ — V: interaction potential, G: Green's function

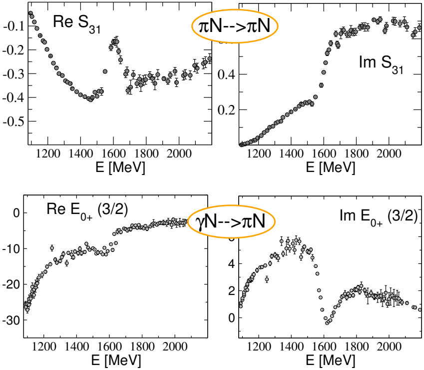
choose basis: $\langle L'S'p'|T^{IJ}|LSp\rangle \to$ partial wave amplitude $L_{2I\,2J}$

$$\langle L'S'p'|T^{IJ}|LSp\rangle = \langle L'S'p'|V^{IJ}|LSp\rangle + \int_0^\infty dq\, q^2 \langle L'S'p'|V^{IJ}|LSq\rangle\, G\, \langle LSq|T^{IJ}|LSp\rangle$$

- construct V, e.g. with polynomials, effective Lagrangians ...
- T should respect unitarity and analyticity

Excited states / Resonances

$J^P = 1/2^+$, $I = 3/2$



Points: SAID 2006 and CM12

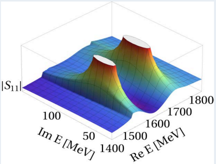
Breit-Wigner parameterization:

$$\mathcal{M}^{Res}_{ba} = -\frac{g_b g_a}{E^2 - M^2_{BW} + iE\Gamma_{BW}}$$

- M_{BW}, Γ_{BW} channel dependent
- background? overlapping resonances? thresholds?

Resonances: poles in the T-matrix

- Pole position E_0 is the same in all channels
- thresholds: branch points



Re(E_0) = “mass”
-2Im(E_0) = “width”
residues$\rightarrow$ branching ratios

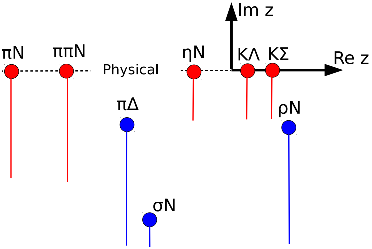
Thresholds of inelastic channels

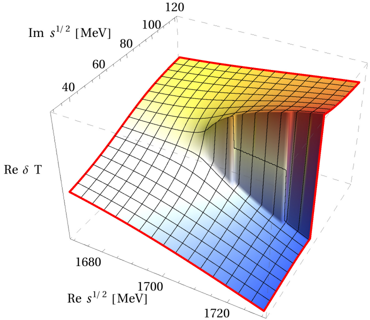
opening of **inelastic channels** $\Rightarrow$ **branch point** and new **Riemann sheet**

3-body $\pi\pi N$ channel:

- parameterized effectively as $\pi\Delta$, σN, ρN
- $\pi N/\pi\pi$ subsystems fit the respective phase shifts

↳ branch points move into complex plane





Example: ρN branch point at
$M_N + m_{rho} = 1700 \pm i75$ MeV

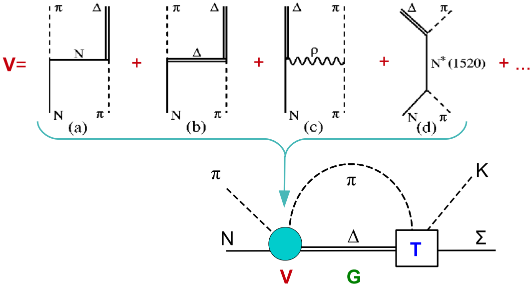
Inclusion of branch points important to avoid false resonance signal!

A dynamical coupled-channel approach: the hadronic Jülich model

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

The scattering equation in partial wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq \quad q^2 \quad \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$



- potentials V constructed from effective $\mathcal{L}$
- s-channel diagrams: T^P
  genuine resonance states
- t- and u-channel: T^{NP}
  dynamical generation of poles
  partial waves strongly correlated

A dynamical coupled-channel approach: the hadronic Jülich model

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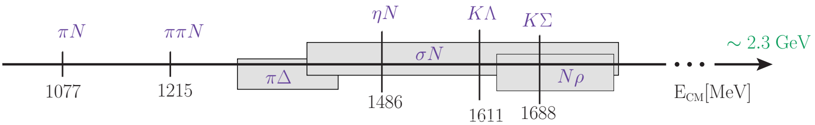
- $J \leq 9/2$



- Unitarity (2 body) and analyticity respected

Analysis of pion-induced reactions

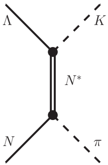
- calculate observables from T-matrix
- fit **free parameters** of T to data or partial wave amplitudes

$$\sigma = \frac{1}{2}\frac{4\pi}{p^2}\sum_{JLS,L'S'}|\tau^{JL'S'}_{LS}|^2$$

with $\tau_{fi} = -\pi\sqrt{\rho_f\rho_i}\,T_{fi}$

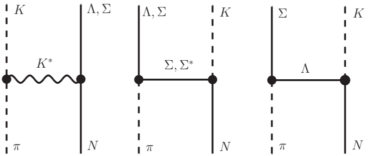
ρ: phase factor

s-channel: resonances (T^P)



m_{bare} + $f_{\pi NN^*}$

t- and u-channel exchange: “background“ (T^{NP})



cut offs Λ in form factors $\left(\frac{\Lambda^2 - m_{ex}^2}{\Lambda^2 + \vec{q}^2}\right)^n$

(couplings fixed from SU(3))

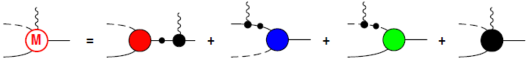
$\Rightarrow$ search for poles in the complex energy plane of T

All exchange diagrams | data base

Photoproduction
Different approaches

- **Field theoretical** approaches : ANL-Osaka, DMT, **Jülich-Athens-Washington**, ...

Example: Gauge invariant formulation by Haberzettl, Huang and Nakayama
Phys. Rev. C56 (1997), Phys. Rrev. C74 (2006), Phys. Rev. C85 (2012)



↓
complicated, involved construction/calculation

Focus of the present analysis:

extraction of resonance parameters

⇒ flexible, **phenomenological** parameterization of photo excitation

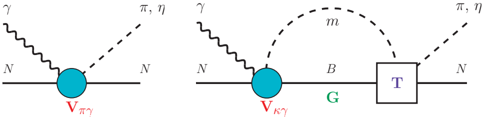
- Advantage: easy to implement, analyze large amounts of data
- Disadvantage: no information on microscopic reaction dynamics

Photoproduction in a semi-phenomenological approach

Multipole amplitude

$$M^{IJ}_{\mu\gamma} = V^{IJ}_{\mu\gamma} + \sum_{\kappa} T^{IJ}_{\mu\kappa} G_\kappa V^{IJ}_{\kappa\gamma}$$

(partial wave basis)

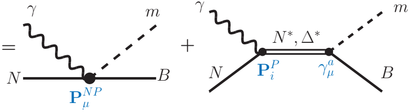


$m = \pi, \eta$, $B = N, \Delta$

$T_{\mu\kappa}$: Jülich hadronic T-matrix → Watson's theorem fulfilled by construction
→ **analyticity of T**: extraction of resonance parameters

Phenomenological potential:

$V_{\mu\gamma}(E,q) =$



$$= \frac{\tilde{\gamma}^a_\mu(q)}{m_N} P^{\mathrm{NP}}_\mu(E) + \sum_i \frac{\gamma^a_{\mu;i}(q) P^{\mathrm{P}}_i(E)}{E - m^b_i}$$

$\tilde{\gamma}^a_\mu$, $\gamma^a_{\mu;i}$: hadronic vertices → correct threshold behaviour, cancellation of singularity at $E = m^b_i$

i: resonance number per multipole; μ: channels πN, ηN, $\pi\Delta$

Polynomials

Photoproduction of pseudoscalar meson

- Photocouplings of resonances
- high precision data from ELSA, MAMI, JLab...$\rightarrow$ resolve questionable/find new states

Photoproduction amplitude of pseudoscalar mesons:

Chew, Goldberger, Low, and Nambu, Phys. Rev. 106, 1345 (1957)

$\vec{q}$: meson momentum
$\vec{k}$ ($\vec{\epsilon}$): photon momentum (polarization)

$$\hat{\mathcal{M}} = F_1 \vec{\sigma}\cdot\vec{\epsilon} + iF_2 \vec{\epsilon}\cdot(\hat{k}\times\hat{q}) + F_3 \vec{\sigma}\cdot\hat{k}\hat{q}\cdot\vec{\epsilon} + F_4 \vec{\sigma}\cdot\hat{q}\hat{q}\cdot\vec{\epsilon}$$

F_i: complex functions of the scattering angle, constructed from multipole amplitudes $M^{IJ}_{\mu\gamma}$

$\Rightarrow$ 16 polarization observables:
asymmetries composed of **beam**, **target** and/or **recoil** polarization measurements

$\Rightarrow$ **Complete Experiment**: unambiguous determination of the amplitude

8 carefully selected observables, including

Chiang and Tabakin, PRC 55, 2054 (1997)

- **single** and **double** polarization observbales
- measurement of **beam**, **target** and **recoil** polarization

↳ easier to realize in K than in π or η photoproduction

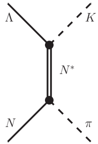
$\hookrightarrow$ Caveat: in reality more observables needed (data uncertainties)

Data analysis and Fit Results

Combined analysis of pion- and photon-induced reactions
Simultaneous fit

Fit parameter:

s-channel: resonances (T^P)



- $\pi N \to \pi N$
  $\pi^- p \to \eta n,\ K^0\Lambda,\ K^0\Sigma^0,\ K^+\Sigma^-$
  $\pi^+ p \to K^+\Sigma^+$

$m_{bare} + f_{\pi NN^*}$

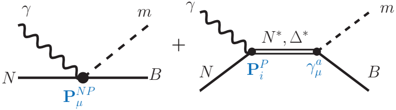
$\Rightarrow$ 128 free parameters

11 N^* resonances $\times$ (1 m_{bare}+ couplings to πN, ρN, ηN, $\pi\Delta$, $K\Lambda$, $K\Sigma$))
+ 10 Δ resonances $\times$ (1 m_{bare}+ couplings to πN, ρN, $\pi\Delta$, $K\Sigma$)

- $\gamma p \to \pi^0 p,\ \pi^+ n,\ \eta p$

  $\Rightarrow$ 456 free parameters

  couplings of the polynomials



↳ calculations on the JUROPA supercomputer: parallelization in energy ($\sim$ 300 – 400 processes)

Data base

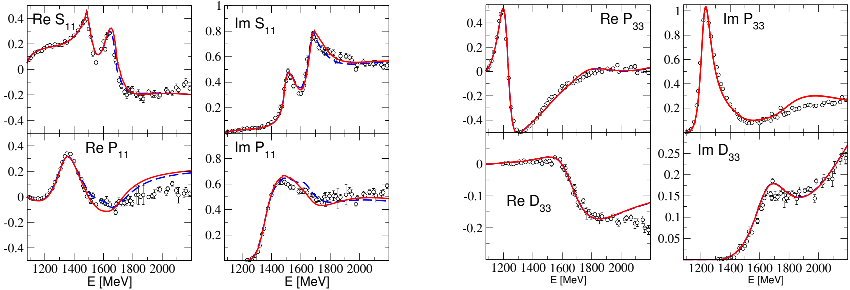
simultaneous fit to π- and γ-induced reactions

	Fit A	Fit B
$\pi N \to \pi N$	PWA SAID 2006 [Arndt *et al.*, PRC 74 (2006)]	
$\pi^- p \to \eta n$	$d\sigma/d\Omega$, P	
$\pi^- p \to K^0 \Lambda$	$d\sigma/d\Omega$, P, β	
$\pi^- p \to K^0 \Sigma^0$	$d\sigma/d\Omega$, P	
$\pi^- p \to K^+ \Sigma^-$	$d\sigma/d\Omega$	
$\pi^+ p \to K^+ \Sigma^+$	$d\sigma/d\Omega$, P , β	
	$\sim$ 6000 data points	
$\gamma p \to \pi^0 p$	$d\sigma/d\Omega$, Σ, P, T, $\Delta\sigma_{31}$, G, H	
$\gamma p \to \pi^+ n$	$d\sigma/d\Omega$, Σ, P, T, $\Delta\sigma_{31}$, G, H	
$\gamma p \to \eta p$	$d\sigma/d\Omega$, P, Σ	$d\sigma/d\Omega$, P, Σ, T, F
	29,761 data points	30,049 data points

- More single/double polarization:
  E, $C_{x'L}$, $C_{z'L}$,
  T, P, H (ELSA 2014)
  $(\gamma p \to \pi^0 p)$
  $\Rightarrow$ predictions

$\pi N \to \pi N$ partial wave amplitudes — selected results, preliminary

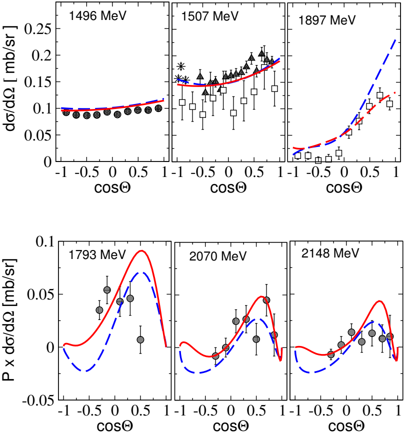
Fit A and Fit B



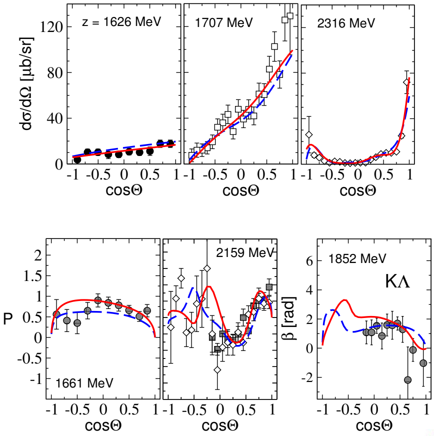
- Notation: $L_{2I\,2J}$
- Input to fit: energy-dependent partial wave analysis, GWU/SAID 2006
  up to $J = 9/2$ ($\sim H_{39}$)

$\pi N \to \eta N, K\Lambda$ selected results, preliminary

$\pi^- p \to \eta N$

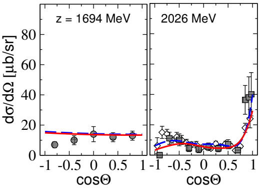


$\pi^- p \to K^0 \Lambda$

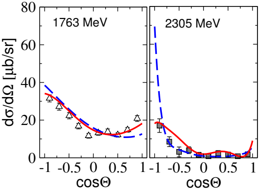


$\pi N \to K\Sigma$ selected results, preliminary

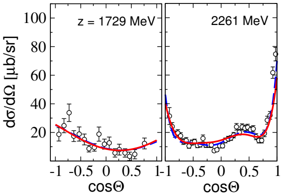
$\pi^- p \to K^0\Sigma^0$

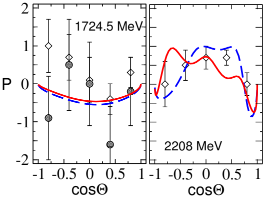


$\pi^- p \to K^+\Sigma^-$

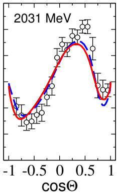


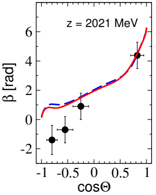
$\pi^+ p \to K^+\Sigma^+$





No polarization data!

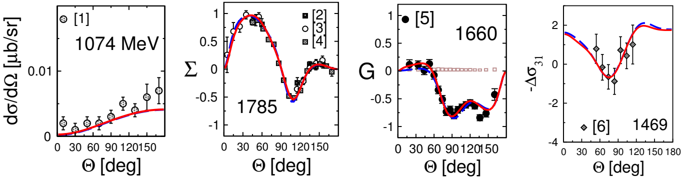




Pion photoproduction: selected fit results

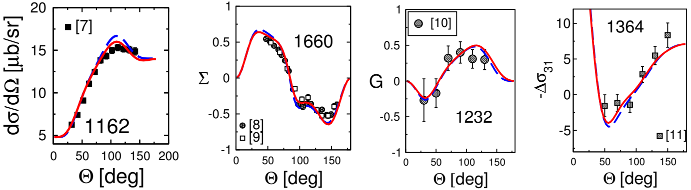
preliminary

- $\gamma p \to \pi^0 p$



[1] Schmidt 2001 (MAMI)
[2] Elsner 2009 (ELSA)
[3] Sparks 2010 (ELSA)
[4] Bartalini 2005 (GRAAL)
[5] Thiel 2012 (ELSA)
[6] Ahrens 2002 (MAMI)

- $\gamma p \to \pi^+ n$



[7] Ahrens 2004 (MAMI)
[8] Bartalini 2002 (GRAAL)
[9] Ajaka 2000 (GRAAL)
[10] Ahrens 2005 (MAMI)
[11] Ahrens 2006 (MAMI)

Double polarization in $\gamma p \to \pi^0 p$

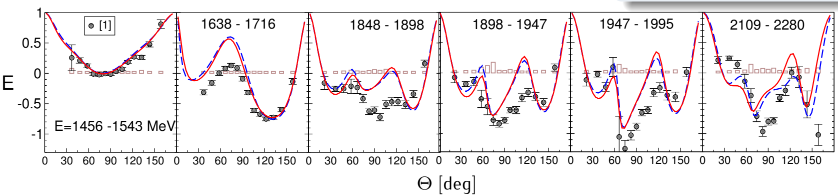
Data NOT included in fit

selected results, preliminary

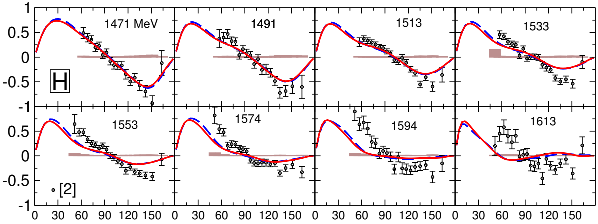
Polarization

Beam	Target	Recoil
$+1$	$-z$	0
-1	$-z$	0

[1] Gottschall *et al.* 2013 (ELSA) PRL. 112 1, 012003



[2] Hartmann *et al.* 2014 (ELSA) PRL 113, 062001



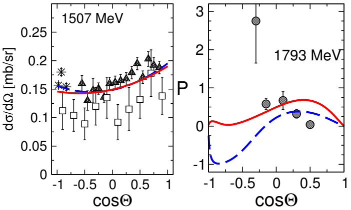
Polarization

Beam	Target	Recoil
$\perp'$	x	0
$\parallel'$	x	0

new ELSA T,P data

Eta photoproduction: $\gamma p \to \eta p$

- Inelastic channels → possibility resolve "missing resonance" problem
- Data quality in $\pi^- p \to \eta n$:



data points $\pi^- p \to \eta n$: 732
$\gamma p \to \eta p$: 6164

data situation much better in $\gamma p \to \eta p$

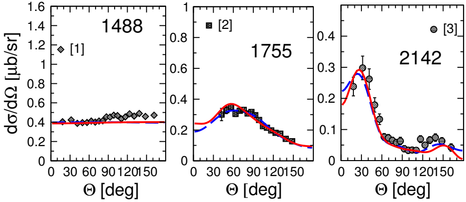
⇒ Fix $N^* N\eta$ coupling from photoproduction (to a large extent)

Fit results $\gamma p \to \eta p$

selected results, preliminary

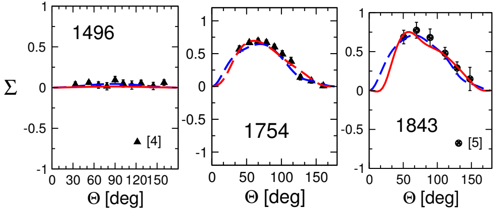
T, F not included (blue dashed line) — T, F included (red solid line)

- Differential cross section



[1] McNicoll *et al.* 2010 (MAMI), [2] Williams *et al.* 2009 (JLab), [3] Credé *et al.* 2009 (ELSA)

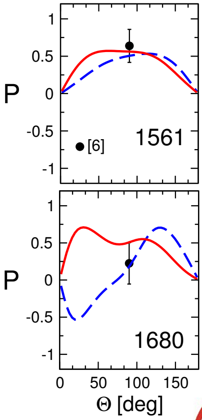
- Beam asymmetry



[4] Bartalini *et al.* 2007 (GRAAL), [5] Elsner *et al.* 2007 (ELSA)

- Recoil polarization

– only 7 data points in total –

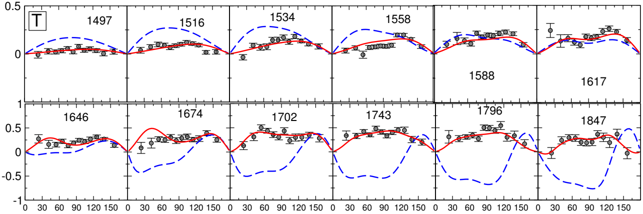


[6] Heusch *et al.* (Caltech), PRL25, 1381 (1970)

T and F in $\gamma p \to \eta p$

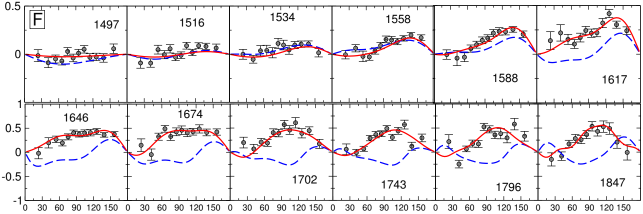
preliminary

Data: Akondi *et al.* (A2 at MAMI) PRL 113, 102001 (2014)



prediction
fit

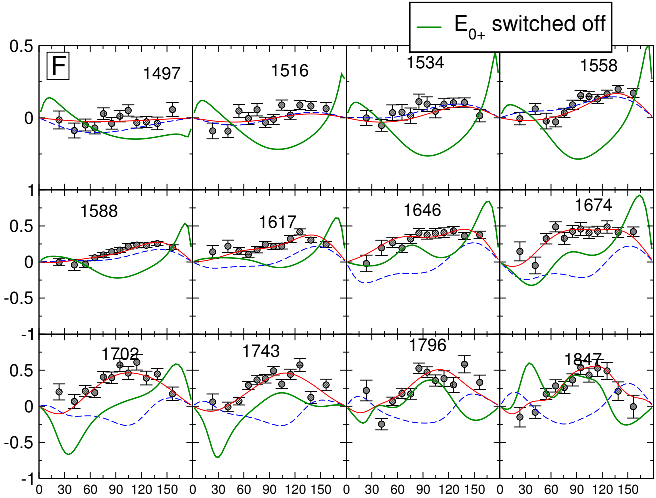
Beam	Target	Recoil
0	$+y$	0
0	$-y$	0



Beam	Target	Recoil
$+1$	$+x$	0
-1	$+x$	0

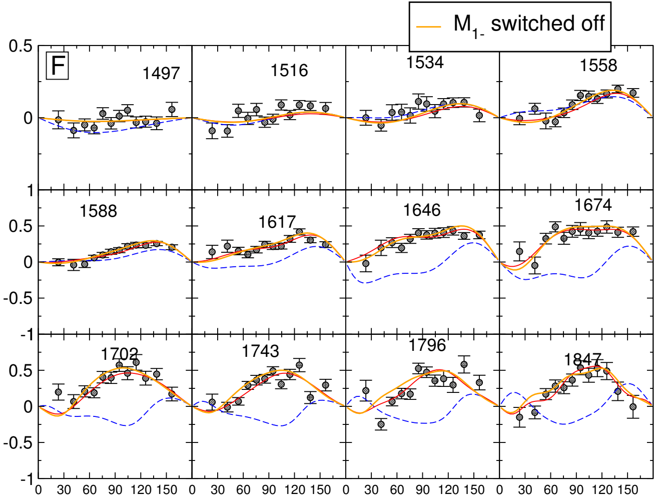
Partial wave contribution to F in $\gamma p \to \eta p$ preliminary

Switch off different PWs in **Fit B**



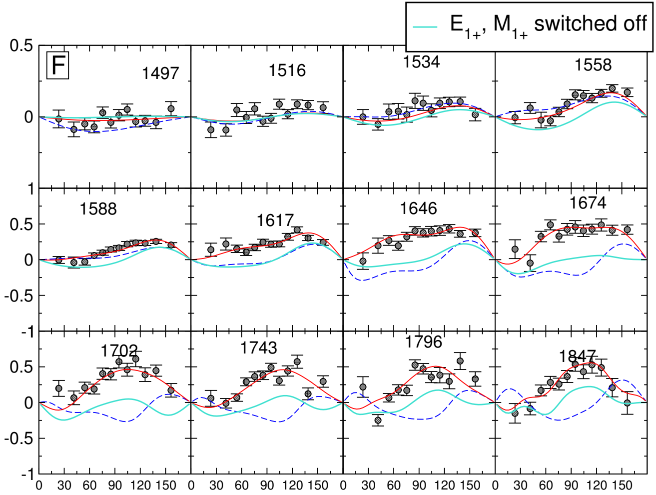
Partial wave contribution to F in $\gamma p \to \eta p$ preliminary

Switch off different PWs in **Fit B**



Partial wave contribution to F in $\gamma p \to \eta p$ preliminary

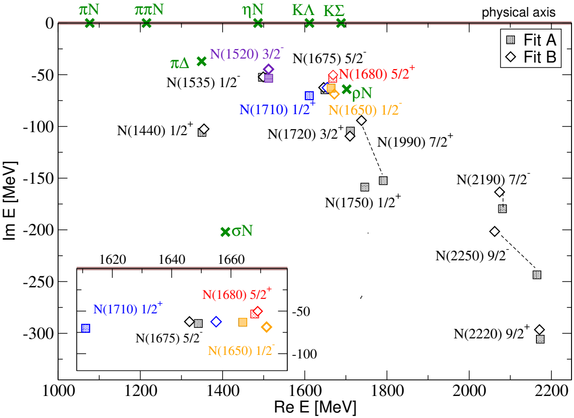
Switch off different PWs in **Fit B**



Resonance content: I=1/2

preliminary

Pole search on the 2^{nd} sheet of the scattering matrix $T_{\mu\nu}$



Resonance parameter:

- "mass" $= Re(E_0)$
- "width"$= -2Im(E_0)$
- Residues $\rightarrow$ branching ratios

E_0: pole position

x: branch points

Notation: $N(\text{"}name\text{"})\, J^{parity}$

Resonance parameters

selected results, preliminary

	ReE_0 [MeV]		-2ImE_0 [MeV]		$\Gamma_{\pi N}/\Gamma_{tot}$ [%]		$\Gamma_{\eta N}/\Gamma_{tot}$ [%]	
fit→	A	B	A	B	A	B	A	B
$N(1535)\ 1/2^-$	1497	1499	105	104	43	43	61	61
$N(1650)\ 1/2^-$	1664	1672	126	137	49	54	8	8
$N(1710)\ 1/2^+$	1611	1655	140	125	4.0	4.9	9.0	45.5
$N(1720)\ 3/2^+$	1711	1710	209	219	5.1	3.9	0.2	0.1

$\frac{\Gamma_\mu}{\Gamma_{tot}} = \frac{|r_{\pi N\to\mu}|^2}{|r_{\pi N\to\pi N}|(\Gamma_{tot}/2)}$ $r_{\pi N\to\mu}$: residue, $\Gamma_{tot} = -2\mathrm{Im}E_0$: resonance width

Photocouplings at the pole

selected results, preliminary

$$\tilde{A}^h_{pole} = A^h_{pole} e^{i\vartheta^h}$$
$$h = 1/2,\ 3/2$$

$$\tilde{A}^h_{pole} = I_F \sqrt{\frac{q_p}{k_p} \frac{2\pi\,(2J+1)\mathbf{E_0}}{m_N\ \mathbf{r_{\pi N}}}}\ \mathrm{Res}\ A^h_{L\pm}$$

I_F: isospin factor
q_p (k_p): meson (photon) momentum at the pole
$J = L \pm 1/2$ total angular momentum
E_0: pole position
$r_{\pi N}$: elastic πN residue

	$A^{1/2}_{pole}$ $[10^{-3}\ \mathrm{GeV}^{-1/2}]$		$\vartheta^{1/2}$ [deg]		$A^{3/2}_{pole}$ $[10^{-3}\ \mathrm{GeV}^{-1/2}]$		$\vartheta^{3/2}$ [deg]	
fit→	1	2	1	2	1	2	1	2
$N(1535)\ 1/2^-$	106.5	106.0	-34.7	-32.3				
$N(1650)\ 1/2^-$	60.6	59.1	-70.2	-65.3				
$N(1710)\ 1/2^+$	-6.6	21.0	-31.1	-10.1				
$N(1720)\ 3/2^+$	54.7	38.6	-12.8	-29.4	38.5	31.9	33.6	31.7

Summary

Extraction of the N^* and Δ resonance spectrum

from a simultaneous analysis of pion- and photon-induced reactions

- DCC analysis of $\pi N \to \pi N$, ηN, $K\Lambda$ and $K\Sigma$

  The Jülich model: lagrangian based, unitarity & analyticity respected

  $\to$ analysis of over 6000 data points (PWA, $d\sigma/d\Omega$, P, β)

- π and η photoproduction in a semi-phenomenological approach

  hadronic final state interaction: Jülich DCC analysis

  $\to$ analysis of more than 30 000 data points for **single** and **double polarization** observables

$\to$ extraction of resonance parameters (**poles & residues**)

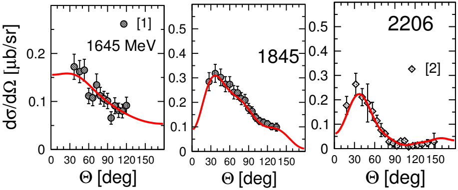
Outlook

Kaon photoproduction: preliminary results for $\gamma p \to K^+\Lambda$

simultaneous fit of $\gamma p \to \pi^0 p\,,\ \pi^+ n\,,\ \eta p,\ K^+\Lambda$

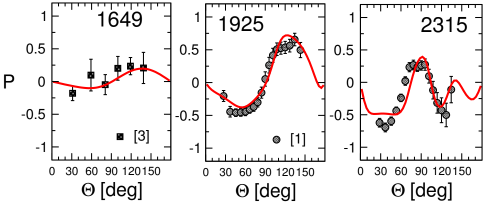
and $\pi N \to \pi N\,,\ \eta N\,,\ K\Lambda\,,\ K\Sigma$

- Differential cross section



[1] McCracken *et al.* 2010 (JLab), [2] Glander *et al.* 2004 (ELSA)

- Recoil polarization



[3] Lleres *et al.* 2007 (GRAAL)

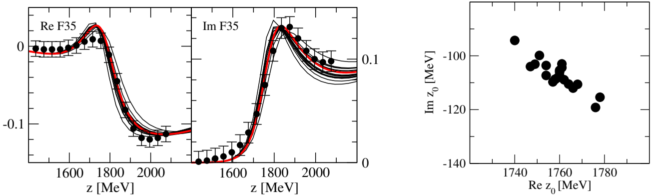
After this ...

- More double polarization observables in meson photoproduction to be published in the near future
- $\gamma N \to K\Sigma$
- Two meson photorpoduction e.g. $\gamma p \to \pi^0 \eta p$ from ELSA

Error analysis

- $\chi^2 + 1$ criterion: determination of the non-linear parameter error
  - error of parameter p_i determined by range of p_i such that χ^2_{min} rises by less than 1

$\Rightarrow$ error on pole positions and residues.



NPBA 851, 58 (2011)

BUT: numerically not possible with $\geq$ 500 free parameters

Work in progress: Developing of techniques to apply Monte-Carlo error propagation using bootstrap method (M. Döring et al.)

Matching to lattice
Prediction & analysis of lattice data

[M. Döring et al., EPJ A47, 163 (2011)]

Scattering equation:

$$T(q'',q') = V(q'',q') + \int_0^\infty dq\, q^2\, V(q'',q)\frac{1}{z-E_1(q)-E_2(q)+i\epsilon}T(q,q')$$

Discretization of momenta in the scattering equation:

$$\int \frac{\vec{d}^{\,3}q}{(2\pi)^3} f(|\vec{q}\,|^2) \quad\to\quad \frac{1}{L^3}\sum_{\vec{n}_i} f(|\vec{q}_i|^2), \quad \vec{q}_i = \frac{2\pi}{L}\vec{n}_i, \quad \vec{n}_i \in \mathbb{Z}^3$$

$$T(q'',q') = V(q'',q') + \frac{2\pi^2}{L^3}\sum_{i=0}^{\infty}\vartheta(i)V(q'',q_i)\frac{1}{z-E_1(q_i)-E_2(q_i)}T(q_i,q'),$$

$\vartheta^{(P)}(i)$ series

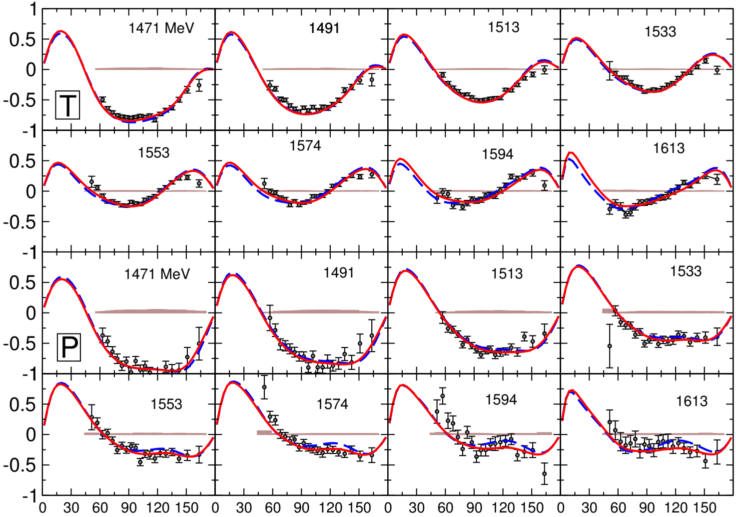
- Study finite-volume effects
- Predict lattice spectra

Thank you for your attention!

T, P in $\gamma p \to \pi^0 p$

Data NOT included in the Fit

Data: J. Hartmann *et al.* 2014 (ELSA)



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Details of the formalism

Polynomials:

$$P_i^{\mathrm{P}}(E) = \sum_{j=1}^{n} g_{i,j}^{\mathrm{P}} \left(\frac{E - E_0}{m_N}\right)^j e^{-g_{i,n+1}^{P}(E-E_0)}$$

$$P_\mu^{\mathrm{NP}}(E) = \sum_{j=0}^{n} g_{\mu,j}^{\mathrm{NP}} \left(\frac{E - E_0}{m_N}\right)^j e^{-g_{\mu,n+1}^{\mathrm{NP}}(E-E_0)}$$

- $E_0 = 1077$ MeV
- $g_{i,j}^{\mathrm{P}}$, $g_{\mu,j}^{\mathrm{NP}}$: fit parameter
- $e^{-g(E-E_0)}$: appropriate high energy behavior
- $n = 3$

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World data base on ηN, $K\Lambda$, $K\Sigma$

	PWA	σ_{tot}	$\frac{d\sigma}{d\Omega}$	P	β
$\pi N \to \pi N$	GWU/SAID 2006 up to J=9/2				
$\pi^- p \to \eta n$		62 data points	38 energy points z=1489 to 2235 MeV	12 energy points 1740 to 2235 MeV	
$\pi^- p \to K^0 \Lambda$		66 data points	46 energy points 1626 to 1405 MeV	27 energy points 1633 to 2208 MeV	7 energy points 1852 to 2262 MeV
$\pi^- p \to K^0 \Sigma^0$		16 data points	29 energy points 1694 to 2405 MeV	19 energy points 1694 to 2316 MeV	
$\pi^- p \to K^+ \Sigma^-$		14 data points	15 energy points 1739 to 2405 MeV		
$\pi^+ p \to K^+ \Sigma^+$		18 data points	32 energy points 1729 to 2318 MeV	32 energy points 1729 to 2318 MeV	2 energy pionts 2021 and 2107 MeV

$\sim$ 6000 data points

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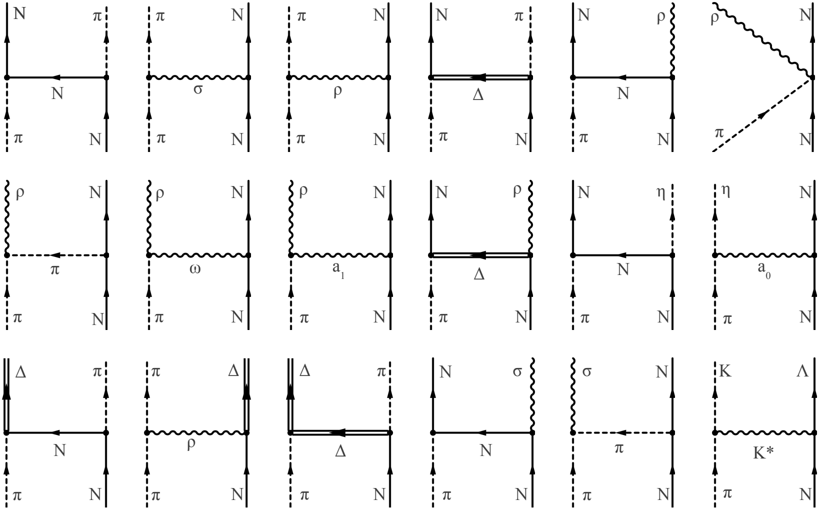
N π N π N
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π N ρ π N
N π Δ π N
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N σ N π N
σ N π π N
K Λ K* π N

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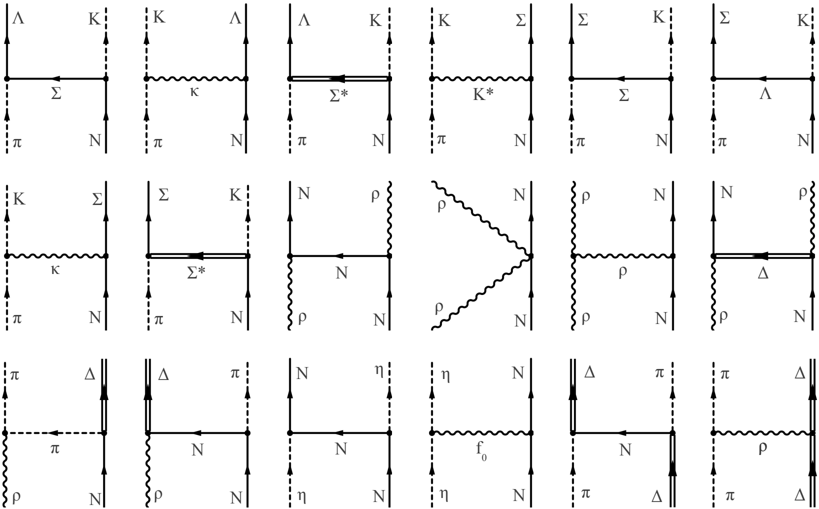
Λ K Σ π N
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Λ K Σ* π N
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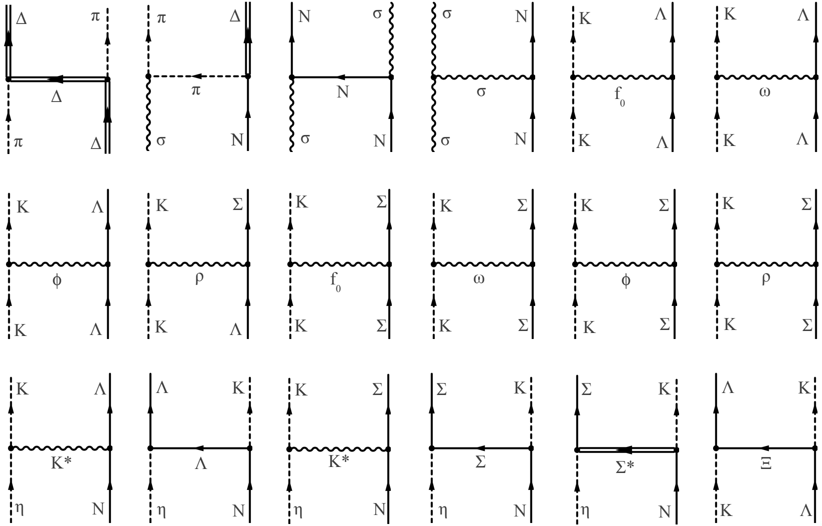

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