Gedenk-Kolloqium, Universitaet Bonn, 15. Nov. 2013

Klaus Erkelenz und das Bonn Potential

R. Machleidt
University of Idaho, USA

Outline

- Historical perspective
- The one-boson-exchange (OBE) model for the nuclear force
- Klaus Erkelenz' improvements of the OBE model
- Beyond one boson exchange
- Conclusions

Table 1. Eight Decades of Struggle: The Theory of Nuclear Forces

1935	**Yukawa: Meson Theory**
1950's	*The "Pion Theories"* One-Pion Exchange: o.k. Multi-Pion Exchange: disaster
1960's	Many pions $\equiv$ multi-pion resonances: σ, ρ, ω, ... The One-Boson-Exchange Model
1970's	Refine meson theory: More sophisticated meson-exchange models (Stony Brook, Paris, Bonn)
1980's	Nuclear physicists discover **QCD** Quark Cluster Models
1990's and beyond	Nuclear physicists discover **EFT** Weinberg, van Kolck **Back to Yukawa's Meson Theory!** *But, with Chiral Symmetry*

Table 1. Eight Decades of Struggle: The Theory of Nuclear Forces

1935	**Theory**
1950's	*ories"* ge: o.k. e: disaster
1960's	n resonances: .
1970's	xchange models , Bonn)
1980's	discover Models
1990's and beyond	over **EFT** olck **son Theory!** *But, with Chiral Symmetry*

Klaus Erkelenz 1931-1973

The one-boson-exchange (OBE) model for the nuclear force

- Pick the mesons of lowest mass.

LIGHT UNFLAVORED MESONS
$(S = C = B = 0)$

For $I = 1$ (π, b, ρ, a): $u\bar{d}$, $(u\bar{u} - d\bar{d})/\sqrt{2}$, $d\bar{u}$;
for $I = 0$ $(\eta, \eta', h, h', \omega, \phi, f, f')$: $c_1(u\bar{u} + d\bar{d}) + c_2(s\bar{s})$

$\pi^\pm$ $\quad I^G(J^P) = 1^-(0^-)$

Mass $m = 139.57018 \pm 0.00035$ MeV (S = 1.2)
Mean life $\tau = (2.6033 \pm 0.0005) \times 10^{-8}$ s (S = 1.2)
$c\tau = 7.8045$ m

π^0 $\quad I^G(J^{PC}) = 1^-(0^{-+})$

Mass $m = 134.9766 \pm 0.0006$ MeV (S = 1.1)
$m_{\pi^\pm} - m_{\pi^0} = 4.5936 \pm 0.0005$ MeV
Mean life $\tau = (8.4 \pm 0.6) \times 10^{-17}$ s (S = 3.0)
$c\tau = 25.1$ nm

η $\quad I^G(J^{PC}) = 0^+(0^{-+})$

Mass $m = 547.75 \pm 0.12$ MeV [f] (S = 2.6)
Full width $\Gamma = 1.29 \pm 0.07$ keV [g]

$f_0(600)$ [i] **or σ** $\quad I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = (400\text{–}1200)$ MeV
Full width $\Gamma = (600\text{–}1000)$ MeV

f_0(600) DECAY MODES	Fraction (Γ_i/Γ)
$\pi\pi$	dominant

$\rho(770)$ [j] $\quad I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.8 \pm 0.5$ MeV
Full width $\Gamma = 150.3 \pm 1.6$ MeV
$\Gamma_{ee} = 7.02 \pm 0.11$ keV

ρ(770) DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
$\pi\pi$	$\sim$ 100 %	

$\omega(782)$ $\quad I^G(J^{PC}) = 0^-(1^{--})$

Mass $m = 782.59 \pm 0.11$ MeV (S = 1.7)
Full width $\Gamma = 8.49 \pm 0.08$ MeV
$\Gamma_{ee} = 0.60 \pm 0.02$ keV

ω(782) DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
$\pi^+\pi^-\pi^0$	(89.1 $\pm$0.7) %	S=1.1

LIGHT UNFLAVORED MESONS
($S = C = B = 0$)

For $I = 1$ (π, b, ρ, a): $u\bar{d}$, $(u\bar{u}-d\bar{d})/\sqrt{2}$, $d\bar{u}$;
for $I = 0$ (η, η', h, h', ω, ϕ, f, f'): $c_1(u\bar{u} + d\bar{d}) + c_2(s\bar{s})$

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pseudo scalar

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scalar

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$\Gamma_{ee} = 7.02 \pm 0.11$ keV

$\rho(770)$ DECAY MODES	Fraction $(\Gamma_i/\Gamma$
$\pi\pi$	$\sim$ 100

vector

$\omega(782)$ $I^G(J^{PC}) = 0^-(1^{--})$

Mass $m = 782.$ ... 1.7)
Full width Γ
Γ_{ee}

ω ... ES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
π ...	(89.1 $\pm$0.7) %	S=1.1

Repulsive

STOP

The one-boson-exchange (OBE) model for the nuclear force

- **Pick the mesons of lowest mass.**
- **Guided by symmetries, write down appropriate Langrangians for meson-nucleon coupling**

The one-boson-exchange (OBE) model for the nuclear force

- **Pick the mesons of lowest mass.**
- **Guided by symmetries, write down appropriate Langrangians for meson-nucleon coupling**

$$\mathcal{L}_{ps} = -g_{ps}\bar{\psi} i\gamma_5 \psi \varphi^{(ps)}$$

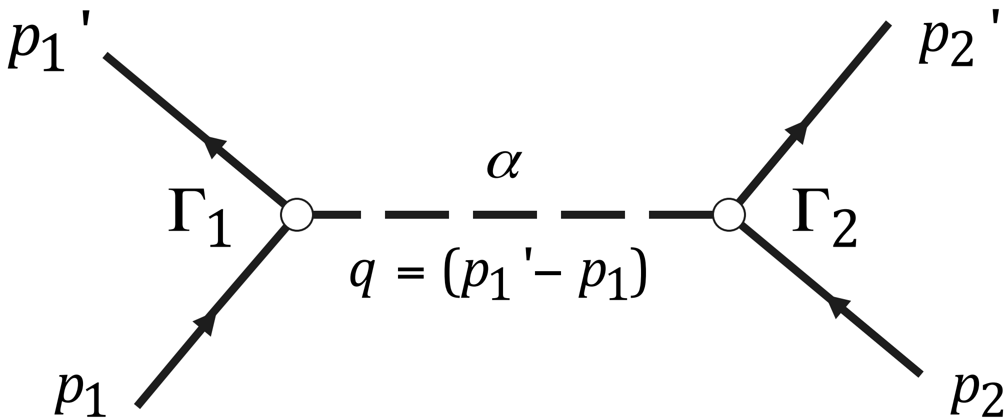
$$\mathcal{L}_{s} = +g_{s}\bar{\psi}\psi \varphi^{(s)}$$

$$\mathcal{L}_{v} = -g_{v}\bar{\psi}\gamma^{\mu}\psi\varphi_{\mu}^{(v)} - \frac{f_v}{4M}\bar{\psi}\sigma^{\mu\nu}\psi(\partial_{\mu}\varphi_{\nu}^{(v)} - \partial_{\nu}\varphi_{\mu}^{(v)})$$

The one-boson-exchange (OBE) model for the nuclear force

- Pick the mesons of lowest mass.
- Guided by symmetries, write down appropriate Langrangians for meson-nucleon coupling
- Calculate the one-meson-exchange Feynman diagrams between two nucleons and sum them up to define the NN potential ("OBEP").

Feynman diagram for NN scattering



$$\text{Amplitude: } F_\alpha(p', p) = \frac{\bar{u}_1' \Gamma_1 u_1 \; P_\alpha \; \bar{u}_2' \Gamma_2 u_2}{q^2 - m_\alpha^2}$$

$$V_{\mathrm{OBEP}} = \sum_{\alpha=\pi,\sigma,\rho,\omega,\eta,a_0,\ldots} V_\alpha$$

OBEPs in the 1960's

PHYSICAL REVIEW VOLUME 135, NUMBER 2B 27 JULY 1964

Nucleon-Nucleon Scattering from One-Boson-Exchange Potentials*

RONALD A. BRYANT†
Laboratoire de Physique Nucléaire, Orsay (Seine et Oise), France

AND

BRUCE L. SCOTT
Department of Physics, University of Southern California, Los Angeles, California

(Received 2 March 1964)

Nucleon-nucleon scattering is studied for laboratory scattering energies over the 0 to 320-MeV range for states with angular momentum $l \geq 1$. Our central hypothesis is that the interaction may be represented by a series of one-boson-exchange potentials. To this end, we attempt to fit the phenomenological models of Lassila *et al.* (Yale) and of Hamada and Johnston with the series of one-boson-exchange potentials due to the ρ, ω, π, and η, with the meson-nucleon coupling constants taken as adjustable parameters. We find that additional attraction is required in the central potentials, and we provide this by introducing two scalar mesons of isotopic spin 0 to 1, respectively. We next consider the nucleon-nucleon phase shifts that have been determined through phase-shift analysis of the N-N data by several groups. We achieve reasonable fits to the P, D, and F states with the following searched parameters: $g_\eta^2=7.0$, $g_\pi^2=11.7$, $g_\omega^2=21.5$, $g_\rho^2=0.68$, $f_\rho/g_\rho=1.8$, $m_0=560$ MeV, $g_0^2=9.4$, $m_1=770$ MeV, and $g_1^2=6.5$; the parameters of the $T=0$ and $T=1$ scalar mesons are identified by the subscripts 0 and 1, respectively, and

$$\mathfrak{L}_{\text{int}}^{(\rho)}=(4\pi)^{1/2}g_\rho\bar{\psi}\boldsymbol{\tau}\gamma^\mu\psi\boldsymbol{\varrho}_\mu+(4\pi)^{1/2}(f_\rho/2m_\rho)\bar{\psi}\boldsymbol{\tau}\sigma^{\mu\nu}\psi[\partial_\nu\boldsymbol{\varrho}_\mu-\partial_\mu\boldsymbol{\varrho}_\nu].$$

Predetermined parameters are $m_\rho=760$ MeV, $m_\omega=782$ MeV, $m_\pi=138.2$ MeV, $m_\eta=548$ MeV, and $f_\omega/g_\omega=0$. Because of the r^{-3} behavior of the potentials at the origin, all potentials are set to zero within 0.6 F. This has (surprisingly) little effect in most states but does eliminate bound ${}^3P_2$ and ${}^3F_4$ states. The effect of including the ϕ and the relation to other experiments is discussed.

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Nucleon-nucleon scattering i
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= 1 scalar

$\psi\tau\sigma^{\mu\nu}\psi[\partial_\nu\rho_\mu-\partial_\mu\rho_\nu]$.

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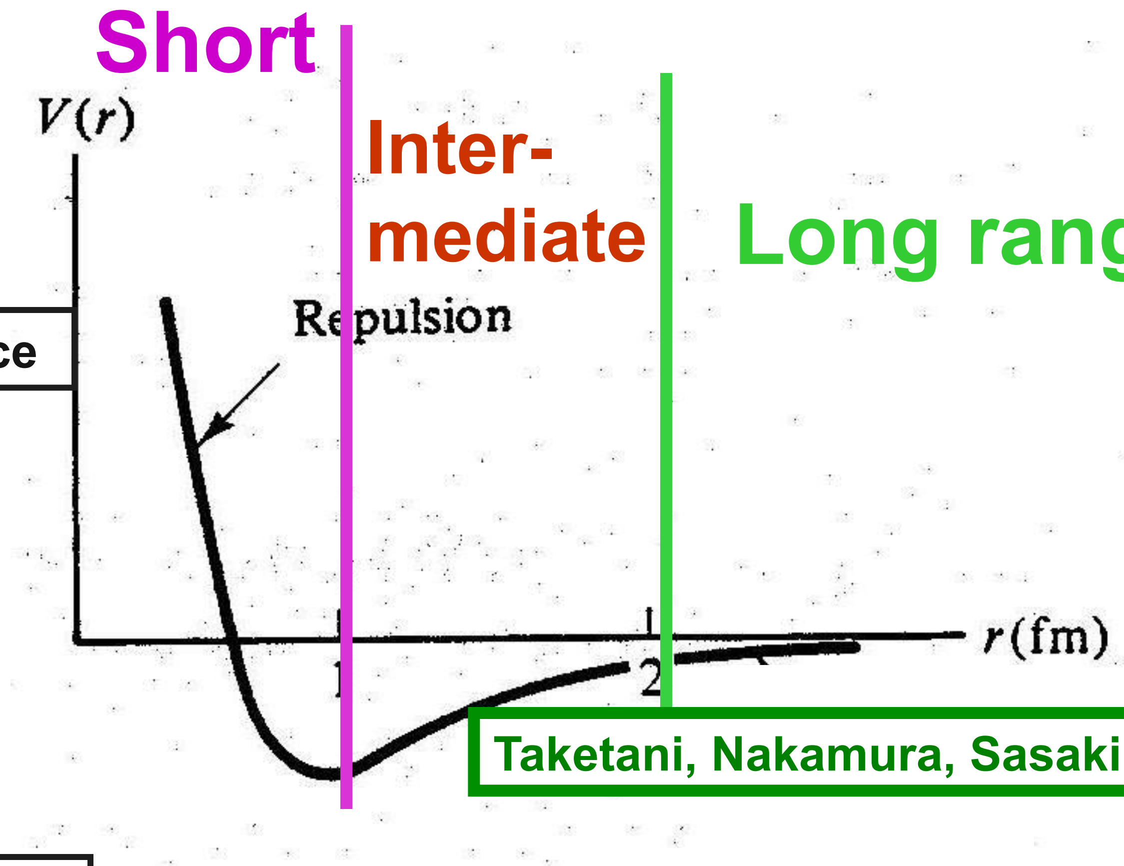
effect in most states but does eliminate bound $^3P_2$ and $^3F_4$ states. The effect of including
φ and the relation to other experiments is discussed.

Typical for 1960's OBEPs:
Q/M expansion of the momentum-space amplitudes
to allow analytic Fourier Transform to position space
to produce a local NN potential, V(r).
Why? Probably for pedagogical reasons?!

The nuclear force: three ranges



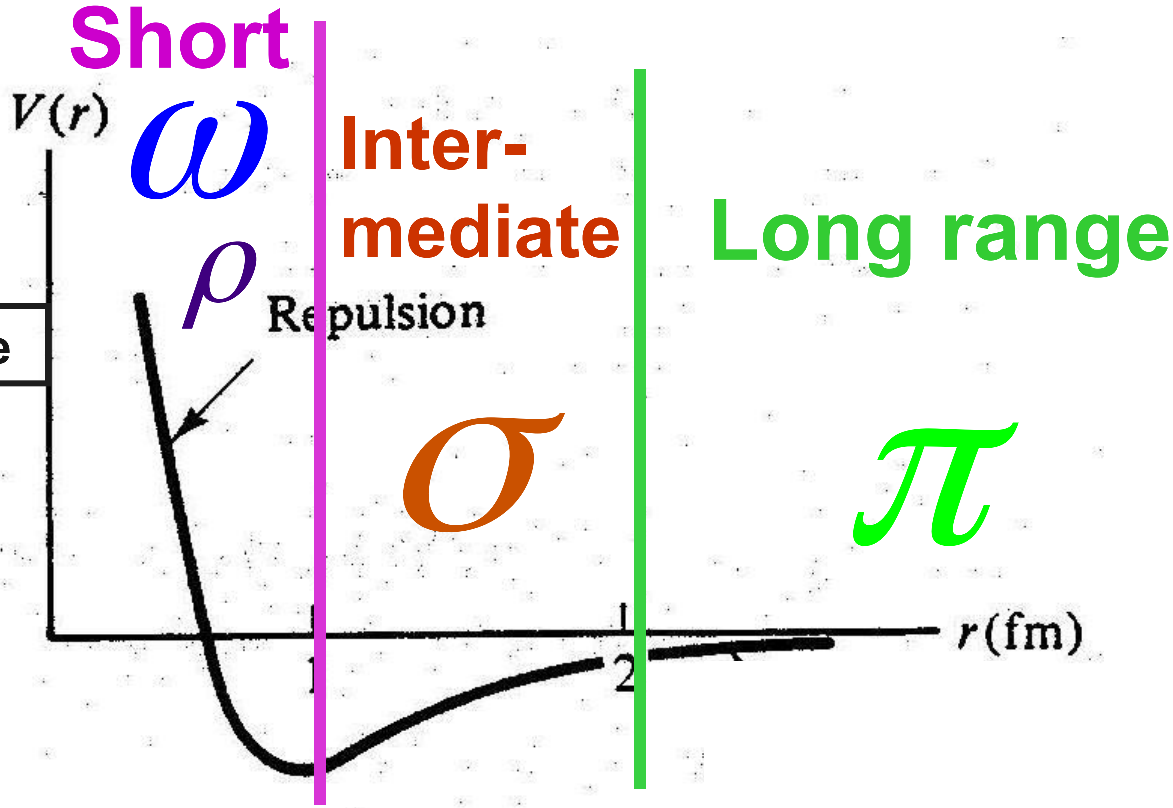
Central force

Taketani, Nakamura, Sasaki (1951): 3 ranges

Tensor force

Spin-orbit force

The nuclear force in the OBE picture



Central force

Tensor force π ρ

Spin-orbit force ω σ

Problems with those r-space OBEPs of the 1960’s

- Expressions are only approximate.
- Definition of NN potential is handwoven.
- Reproduction of the NN data not very accurate.

Problems with those r-space OBEPs of the 1960's

Klaus Erkelenz' achievements in a nutshell:
He solved all three problems!!!

uwoven

the N

curate.

How did he do that?

- Stay in momentum space (no expansions, no approximations!).
- Start from the Bethe-Salpeter equation and properly define the NN potential.

Bethe-Salpeter (BS) equation

$$\boxed{\mathcal{M} = \mathcal{V} + \mathcal{V}\mathcal{G}\mathcal{M}} \tag{1}$$

This is equivalent to two coupled equations:

$$\boxed{\begin{aligned} \mathcal{M} &= \mathcal{W} + \mathcal{W} g \mathcal{M} \\ \mathcal{W} &= \mathcal{V} + \mathcal{V}(\mathcal{G} - g)\mathcal{W} \end{aligned}} \begin{aligned} (2) \\ (3) \end{aligned}$$

where g is a **covariant three-dimensional propagator** with the same elastic unitarity cut as $\mathcal{G}$ in the physical region.

More explicit,

$$\mathcal{M}(q';q|P) = \mathcal{V}(q';q|P) + \int d^4k \mathcal{V}(q';k|P)\mathcal{G}(k|P)\mathcal{M}(k;q|P) \tag{4}$$

with

$$\mathcal{G}(k|P) = \frac{i}{(2\pi)^4} \frac{1}{(\frac{1}{2} \not{P} + \not{k} - M + i\epsilon)^{(1)}} \frac{1}{(\frac{1}{2} \not{P} - \not{k} - M + i\epsilon)^{(2)}} \tag{5}$$

$$= \frac{i}{(2\pi)^4} \left[\frac{\frac{1}{2} \not{P} + \not{k} + M}{(\frac{1}{2}P + k)^2 - M^2 + i\epsilon} \right]^{(1)} \left[\frac{\frac{1}{2} \not{P} - \not{k} + M}{(\frac{1}{2}P - k)^2 - M^2 + i\epsilon} \right]^{(2)} \tag{6}$$

with P is the total four-momentum, for which we have in the c. m. frame: $P = (\sqrt{s}, \mathbf{0})$ with $\sqrt{s}$ the total energy.

A popular choice for g, is the **Blankenbecler-Sugar** choice, which reads in manifestly covariant form,

$$\boxed{\begin{aligned} g_{\text{BbS}}(k,s) = & -\frac{1}{(2\pi)^3} \int_{4M^2}^{\infty} \frac{ds'}{s' - s - i\epsilon} \delta^{(+)}[(\frac{1}{2}P' + k)^2 - M^2] \times \delta^{(+)}[(\frac{1}{2}P' - k)^2 - M^2] \\ & \times \left[\frac{1}{2} \not{P}' + \not{k} + M \right]^{(1)} \left[\frac{1}{2} \not{P}' - \not{k} + M \right]^{(2)} \end{aligned}} \tag{7}$$

with $\delta^{(+)}$ indicating that only the positive energy root of the argument of the δ-function is to be included. By construction, g has the same singularity structure as $\mathcal{G}$.

Klaus Erkelenz proposed, as it turned out, a better g, namely,

$$g_{\rm Erk}(k,s) = -\frac{1}{(2\pi)^3}\int_{4M^2}^{\infty}\frac{ds'}{s'-s-i\epsilon}\delta^{(+)}[(\frac{1}{2}P+k)^2-M^2]\times\delta^{(+)}[(P'-\frac{1}{2}P-k)^2-M^2] \times\left[\frac{1}{2}\not{P}+\not{k}+M\right]^{(1)}\left[\not{P}'-\frac{1}{2}\not{P}-\not{k}+M\right]^{(2)} \tag{8}$$

where one nucleon is on its mass shell and, thus, the meson propagator includes a **retardation**. In the c.m. system, integration yields

$$g_{\rm Erk}(\mathbf{k},s) = \frac{1}{(2\pi)^3}\delta(k_0+\frac{1}{2}\sqrt{s}-E_k)\frac{M^2}{E_k}\frac{\Lambda_+^{(1)}(\mathbf{k})\Lambda_+^{(2)}(-\mathbf{k})}{\frac{1}{4}s-E_k^2+i\epsilon} \tag{9}$$

With this we obtain for Eq. (2) (setting $\mathcal{W}=\mathcal{V}$)

$$\mathcal{M}(\mathbf{q}',\mathbf{q}) = \mathcal{V}(\mathbf{q}',\mathbf{q}) + \int\frac{d^3k}{(2\pi)^3}\mathcal{V}(\mathbf{q}',\mathbf{k})\frac{M^2}{E_k}\frac{\Lambda_+^{(1)}(\mathbf{k})\Lambda_+^{(2)}(-\mathbf{k})}{\mathbf{q}^2-\mathbf{k}^2+i\epsilon}\mathcal{M}(\mathbf{k},\mathbf{q})\,, \tag{10}$$

and taking matrix elements between positive-energy spinors

$$\mathcal{T}(\mathbf{q}',\mathbf{q}) = V(\mathbf{q}',\mathbf{q}) + \int\frac{d^3k}{(2\pi)^3}V(\mathbf{q}',\mathbf{k})\frac{M^2}{E_k}\frac{1}{\mathbf{q}^2-\mathbf{k}^2+i\epsilon}\mathcal{T}(\mathbf{k},\mathbf{q})\,. \tag{11}$$

Defining

$$\hat{\mathcal{T}}(\mathbf{q}',\mathbf{q}) = \sqrt{\frac{M}{E_{q'}}}\mathcal{T}(\mathbf{q}',\mathbf{q})\sqrt{\frac{M}{E_q}} \tag{12}$$

and

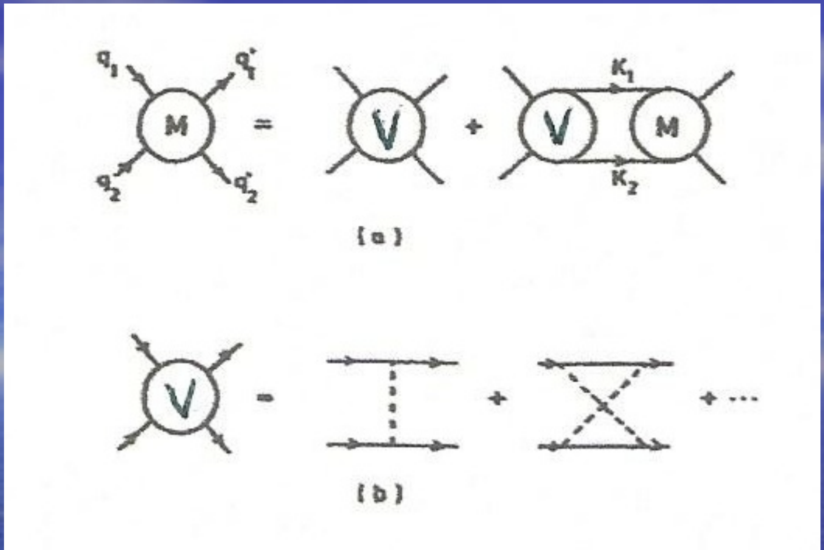
$$\hat{V}(\mathbf{q}',\mathbf{q}) = \sqrt{\frac{M}{E_{q'}}}V(\mathbf{q}',\mathbf{q})\sqrt{\frac{M}{E_q}}, \tag{13}$$

which has become known as "minimal relativity", we can rewrite Eq. (11) as

$$\hat{\mathcal{T}}(\mathbf{q}',\mathbf{q}) = \hat{V}(\mathbf{q}',\mathbf{q}) + \int\frac{d^3k}{(2\pi)^3}\hat{V}(\mathbf{q}',\mathbf{k})\frac{M}{\mathbf{q}^2-\mathbf{k}^2+i\epsilon}\hat{\mathcal{T}}(\mathbf{k},\mathbf{q}) \tag{14}$$

which is the (non-relativistic) **Lippmann-Schwinger equation**.

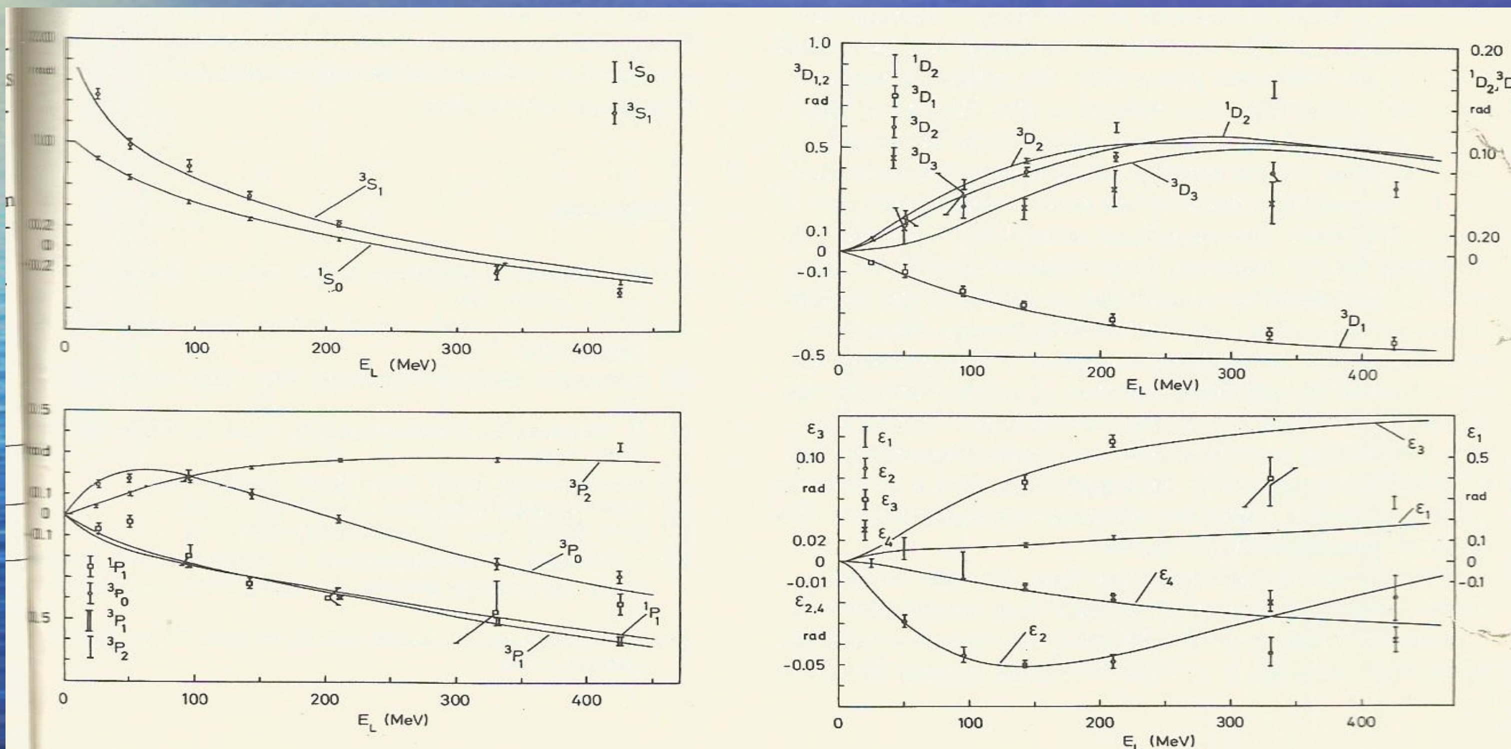
How well is BS reproduced by 3D equations?



- In the ladder approximation, BS and 3D equations disagree.
- But, ladder BS does not have the correct one-body limit, while 3D eqs. do (Gross, 1983).
- However, BS with cross-ladders has right one-body limit.
- The Erkelenz eq. in ladder approximation reproduces closely the fourth order ladder plus cross-ladder BS result (Woloshyn & Jackson, 1973).

… and the reproduction of the NN data is excellent …

Phase shift predictions of high precision by relativistic momentum-space OBEP



g. 1. The S-, P-, D- and ϵ-(bar) phases as function of the lab. energy. Error bars are from the empirical energy-independent vermore analysis [9].

Director of the Institute:
Konrad Bleuler

1.B:1.C

Nuclear Physics **A176** (1971) 413—432; © *North-Holland Publishing Co., Amsterdam*

MOMENTUM SPACE CALCULATIO
FORMALISM IN NUCLEAR

K. ERKELENZ, R. ALZETTA and

Institut für Theoretische Kernphysik der Unive

Received 10 June 1971

Abstract: Using the free reaction matrix R and the Brueckner-
nucleon scattering and nuclear matter problem is present
helicity and partial-wave state matrix elements of the m
given in the momentum space are calculated.

Volume 49B, number 3

AN IMPROVED REI
INFINIT

K. ERKELENZ*, K. HOLINDE and R. MACHLEIDT

Institut für Theoretische Kernphysik, University of Bonn, Bonn, BRD

Received 27 February 1974

A momentum-space OBEP consisting of π, η, ρ, ω, ϕ, σ and δ-meson exchange is presented. Compared to a former paper (OBEP (I)) we refine the cut-off description and try to use more realistic values of the coupling constants. The quality of the resulting phase shift fit and the deuteron data is good (χ^2/datum = 2.7) and comparable to OBEP (I). The nuclear matter data calculated in exactly the same way as in the case of OBEP(I) are improved (now 11.2 MeV at $k_F = 1.47\ \mathrm{fm}^{-1}$ compared to 12.4 MeV at $k_F = 1.55\ \mathrm{fm}^{-1}$ for OBEP(I)), since the saturation density is considerably lowered without losing too much binding.

1.B:1.C

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MOMENTUM SPACE CALCULATIONS AND HELIC FORMALISM IN NUCLEAR PHYSICS

K. ERKELENZ, R. ALZETTA and K. HOLINDE

Institut für Theoretische Kernphysik der Universität Bonn, German

Received 10 June 1971

Abstract: Using the free reaction matrix R and the Brueckner-Goldstone reaction
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PHYSICS LETTERS

TIVISTIC OBEP, TWO-NUCLEON AND NUCLEAR MATTER DATA

*, K. HOLINDE and R. MACHLEIDT

e Kernphysik, University of Bonn, Bonn, BRD

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η, ρ, ω, ϕ, σ and δ-meson exchange is presented. Compared to a former
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PHYSICS REPORTS
A review section of PHYSICS LETTERS (Section C)

Volume 13, Number 5, October 1974

CURRENT STATUS
OF THE RELATIVISTIC TWO-NUCLEON
ONE BOSON EXCHANGE POTENTIAL

K.ERKELENZ

Institut für Theoretische Kernphysik, Bonn, W.-Germany

A "Classic": 300+ citations.

PHYSICS REPORTS
A review section of PHYSICS LETTERS (Section C)

Volume 13, Number 5, October 1974

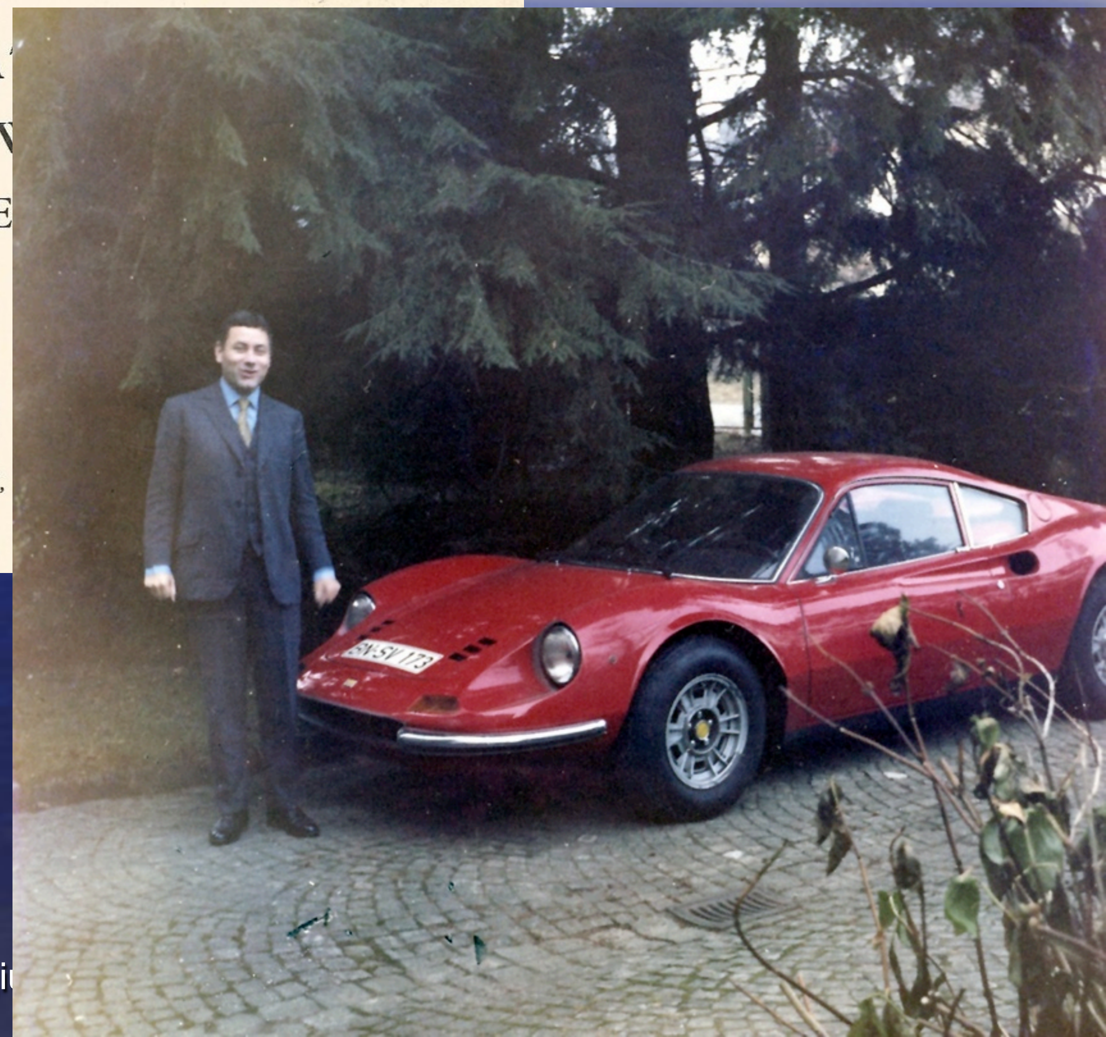
CURRENT STA

OF THE RELATIVISTIC TV

ONE BOSON EXCHANGE

K.ERKELENZ

Institut für Theoretische Kernphysik,

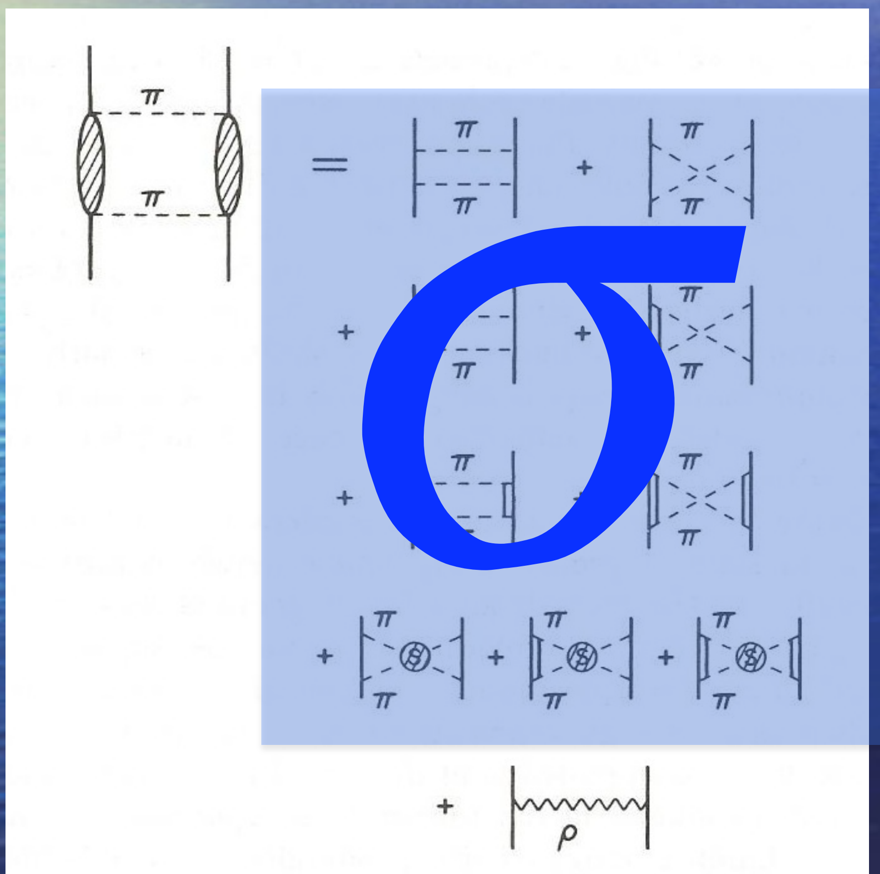


A “Classic”

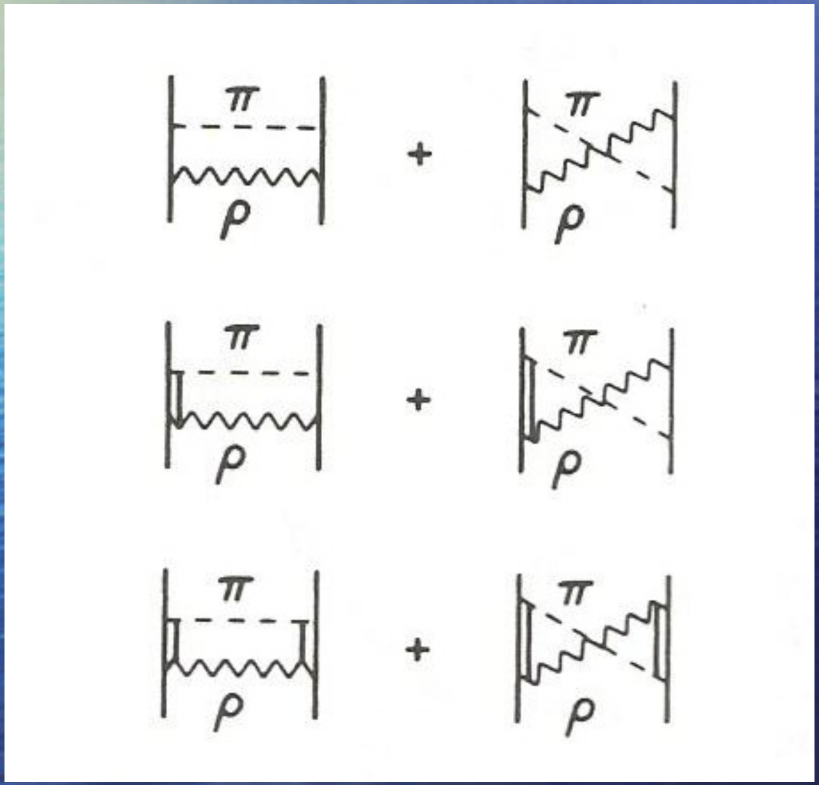
The Bonn Potential after Klaus Erkelenz: Going beyond one-boson exchange
(Karl Holinde & R.M.)

- **Non-iterative contributions**
- **Role of meson-nucleon resonances**
- **The sigma "meson" *versus* 2π exchange**

A field theoretic model for 2-pion exchange



Other important non-iterative diagrams



This resulted in the so-called Bonn Full Model (1980's)

- π + ω + 2π + π-ρ + ...
- Excellent reproduction of NN data
- Proof of principles: meson theory pursued consistently works.
- Full Model good for the investigation of specific issues, like, charge-dependence, medium effects, ...
- But: the model is complicated and energy dependent; not practical for standard nuclear structure calculations..

The last chapter in meson theory: The 1990's - Back to the beginnings

- High-precision potentials are developed.
- Among them: the "CD-Bonn potential", a very accurate relativistic momentum-space OBEP.
- It's based upon the concepts developed by Klaus Erkelenz.
- Thus, Klaus had pointed in the right direction long time ago

The last chapter in
The 1990's - Back t

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Conclusions

- Klaus Erkelenz has contributed substantially to the development of realistic and quantitative nuclear forces.
- Without him there would have never been a Bonn Potential.
- His impact is still noticeable after 40 years.
- But he was also a fun guy and great friend.