Nuclear Interactions from Chiral Effective Field Theory

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Dr. Klaus Erkelenz Prize Colloquium: HISKP Bonn, 23. Nov. 2021

- Theme: Construction of precise *NN*-potential in chiral EFT
- Crucial step: Derivation of basic two-pion exchange potential
- Efficient method to go to higher orders: Compute spectral-functions
- *NN*-scattering with coupled nucleon and $\Delta(1232)$ channels
- Three-nucleon interactions in chiral EFT:
  $\rightarrow$ density-dependent *NN*-potential for many-body applications

Many thanks to:

- Dr. Gabriele Erkelenz (donator of prize)
- Profs. Ulf Meißner, E. Epelbaum, W. Weise, ...

Encountering the work of Klaus Erkelenz

Working on *NN*-potentials, one will unavoidably meet with Dr. Klaus Erkelenz (a founding father of Bonn-potential)

MOMENTUM SPACE CALCULATIONS AND HELICITY FORMALISM IN NUCLEAR PHYSICS

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Received 10 June 1971

Abstract: Using the free reaction matrix R and the Brueckner-Goldstone reaction matrix G the two-nucleon scattering and nuclear matter problem is presented in the helicity state formalism. The helicity and partial-wave state matrix elements of the most general nucleon-nucleon potential given in the momentum space are calculated.

- *NN*-potential has central, spin-spin, tensor, and spin-orbit components

$$V_{NN} = V_C(q) + V_S(q)\vec\sigma_1\cdot\vec\sigma_2 + V_T(q)\vec\sigma_1\cdot\vec q\,\vec\sigma_2\cdot\vec q + V_{SO}(q)i(\vec\sigma_1+\vec\sigma_2)\cdot(\vec q\times\vec p) + \vec\tau_1\cdot\vec\tau_2[V\to W]$$

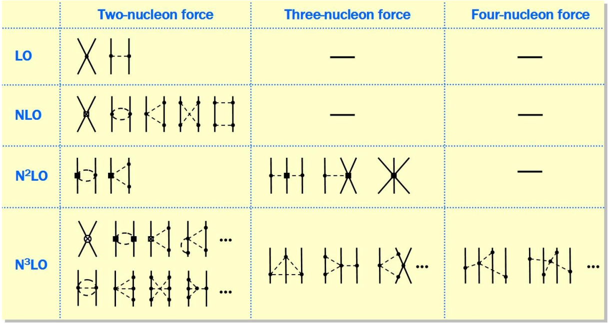
- Partial wave matrix-elements: $S=0$ spin-singlet states ${}^1L_J$ with $L=J$, $S=1$ spin-triplet states ${}^3L_J$ with $L=J, J-1, J+1$ and latter two mix
- Easy for spin-singlet states, pretend $\vec\sigma_1 = -\vec\sigma_2$

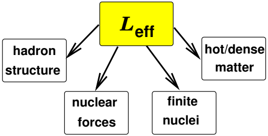
$$\langle J0J|V_{NN}|J0J\rangle = \frac{1}{2}\int_{-1}^{1} dz\,\big[V_C(q) - 3V_S(q) - q^2 V_T(q)\big]\,P_J(z), \quad q = p\sqrt{2(1-z)}$$

- Spin-triplet matrix-elements (including mixing) require helicity-formalism, first fully correct results provided by K. Erkelenz et al.

Nuclear interactions in chiral effective field theory

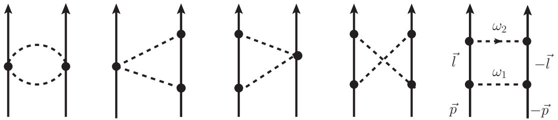
- In chiral EFT, nuclear interactions are organized hierarchically (Weinberg)





- Chiral Lagrangians $\rightarrow$ $\boxed{\text{derive } NN\text{-potentials = my contribution}}$
  $\rightarrow$ applications to *NN*-scattering, nuclear few- and many-body systems
- "High-quality nuclear forces from chiral EFT" by Patrick Reinert (2019)

- At next-to-leading order: two-pion exchange diagrams



- Ordonez, Ray, van Kolck, PRL 72, 1982 ('94) Time-ordered perturb. theory & Gaussian cutoff, not in spirit of quantum field theory: analytical loop-functions & UV-counterterms

$$W_C(q) = \frac{1}{384\pi^2 f_\pi^4}\left[4m_\pi^2(5g_A^4-4g_A^2-1)+q^2(23g_A^4-10g_A^2-1)+\frac{48g_A^4m_\pi^4}{4m_\pi^2+q^2}\right]L(q)+\dots$$

$$V_T(q) = -\frac{1}{q^2}V_S(q) = \frac{3g_A^4}{64\pi^2 f_\pi^4}L(q)+\dots, \qquad L(q) = \frac{\sqrt{4m_\pi^2+q^2}}{q}\ln\frac{q+\sqrt{4m_\pi^2+q^2}}{2m_\pi}$$

- Issue: which part of planar box goes into NN-potential V, when $T = V + V\,G\,T$?
- Three different methods lead to same answer: i) perform energy-integral & expand

$$\int\frac{dl_0}{2\pi i}[\text{planar box}] = \frac{M}{(l^2-p^2)\omega_1^2\omega_2^2} - \frac{\omega_1^2+\omega_1\omega_2+\omega_2^2}{2\omega_1^3\omega_2^3(\omega_1+\omega_2)}, \qquad \int\frac{dl_0}{2\pi i}[\text{crossed box}] = \frac{\omega_1^2+\omega_1\omega_2+\omega_2^2}{2\omega_1^3\omega_2^3(\omega_1+\omega_2)}$$

- ii) Expand sum of energy denominators from time-ordered perturbation theory

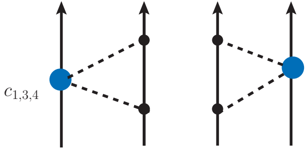
$$\frac{1}{4\omega_1\omega_2}\left[\frac{4}{2\delta T(\omega_1+\delta T)(\omega_2+\delta T)} + \frac{2}{(\omega_1+\omega_2)(\omega_1+\delta T)(\omega_2+\delta T)}\right] = \frac{1}{2\delta T\,\omega_1^2\omega_2^2} - \frac{\omega_1^2+\omega_1\omega_2+\omega_2^2}{2\omega_1^3\omega_2^3(\omega_1+\omega_2)}$$

- iii) Method of unitary transformations [Epelbaum; Okubo et al. (1954)] to project dynamics of interacting πN-system into the purely nucleonic subspace: $\tilde{H}_N = U^\dagger H_{N,\pi N}\, U$

$$V_{2\pi}^{(\mathrm{ut})}(q) = \frac{g_A^4}{16 f_\pi^4} \int \frac{d^3 l}{(2\pi)^3} \frac{\omega_1^2+\omega_1\omega_2+\omega_2^2}{\omega_1^3\omega_2^3(\omega_1+\omega_2)} \Big[2\vec{\tau}_1\cdot\vec{\tau}_2 (l^2-q^2/4)^2 + 3\vec{\sigma}_1\cdot(\vec{q}\times\vec{l})\vec{\sigma}_2\cdot(\vec{q}\times\vec{l}) \Big]$$

- Note: corresponding r-space potentials in: M. Taketani et al., Prog. Theor. Phys. 7, 45 (1952) e.g. isoscalar spin-spin potential: $V_S(r) = \frac{g_A^4 m_\pi}{32\pi^3 f_\pi^4 r^4} \Big[3x\, K_0(2x) + (3+2x^2) K_1(2x) \Big], \quad x = m_\pi r$
- NLO insufficient: no isoscalar central and isovector tensor ($\sigma + \rho$ in OBE models)
- In chiral EFT: two-pion exchange at N^{2}LO with higher-derivative πN-interactions, c_3, c_4 large due to low-lying and strongly coupled spin-$\frac{3}{2}$ $\Delta(1232)$-resonance

c_1, c_2, c_3, c_4 determined in Roy-Steiner-equation analysis of πN-scattering: Jacobo Ruiz de Elvira & Martin Hoferichter (Dr. Klaus Erkelenz prize 2015)



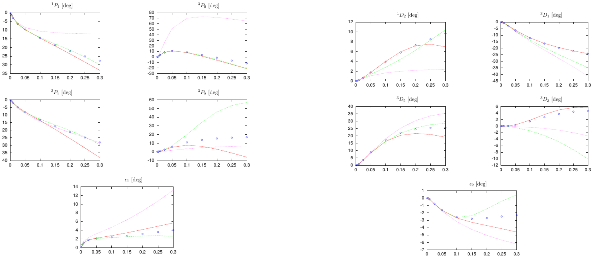
$$V_C(q) = \frac{3g_A^2}{16\pi f_\pi^4} \Big[2m_\pi^2(2c_1 - c_3) - c_3 q^2 \Big] (2m_\pi^2 + q^2) A(q) + \ldots$$

$$W_T(q) = -\frac{1}{q^2} W_S(q) = \frac{g_A^2 c_4}{32\pi f_\pi^4} (4m_\pi^2 + q^2) A(q) + \ldots, \qquad A(q) = \frac{1}{2q} \arctan\frac{q}{2m_\pi}$$

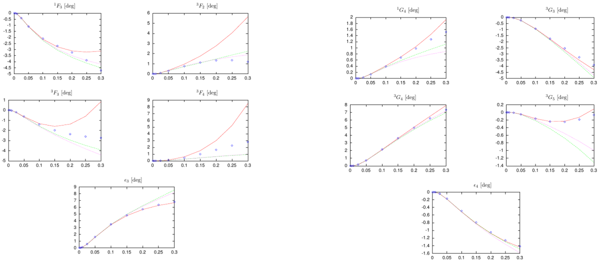
- Walter Glöckle liked this (compact form of) chiral NN-potential up to N^{2}LO

Results for *NN* phase shifts and mixing angles up to N^{2}LO

- Taken from: E. Epelbaum, W. Glöckle, U. Meißner, Nucl. Phys. A 671, 295 (2000)

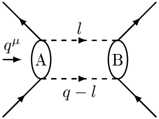


- purple lines: LO, green lines: NLO, red lines: N^{2}LO



- Substantial improvement from LO to N^{2}LO, but yet higher accuracy is needed

- Based on unitarity, loop-potentials are determined by their imaginary parts
  [similar dispersive approach used in Stony Brook and Paris NN-potentials]



$$\mathrm{Im}(2\pi - exch.) = \frac{1}{2}\int d\Phi_{2\pi} A\, B = \frac{|\vec{l}\,|}{16\pi\mu}\int_{-1}^{1} dx\, A\, B, \qquad x = \hat{l}\cdot\vec{v}$$

- For $\bar{N}N \to 2\pi \to \bar{N}N$ nucleon 4-velocity is $(0, i\vec{v})$ since $p_N = \sqrt{\mu^2/4 - M^2} = i\,M + \ldots$

$$V(q) = \frac{2}{\pi}\int_{2m_\pi}^{\infty} d\mu\, \frac{\mu\, \mathrm{Im}\, V(i\mu)}{\mu^2 + q^2}\, e^{-(q^2+\mu^2)/2\Lambda^2} \qquad (local\ \ regulator\ \ included)$$

- Short calculation reproduces NLO + N^{2}LO 2π-exchange potentials $\sim L(q), A(q)$
- Two-loop 2π-exchange from known one-loop πN-amplitudes $g^\pm(\omega,t)$, $h^\pm(\omega,t)$

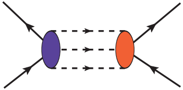
$$\mathrm{Im}\, V_C = \frac{3g_A^2}{64 f_\pi^2\,\mu}(\mu^2 - 2m_\pi^2)\mathrm{Re}[g^+(0,\mu^2)], \qquad \mathrm{Im}\, W_S = \mu^2 \mathrm{Im}\, W_T = \frac{g_A^2\mu}{128 f_\pi^2}(4m_\pi^2 - \mu^2)\mathrm{Re}[h^-(0,\mu^2)],$$

$$\mathrm{Im}\, W_C = \frac{k}{16\pi f_\pi^2\,\mu}\int_0^1 dx\left[g_A^2(2m_\pi^2 - \mu^2) + 2(g_A^2 - 1)k^2x^2\right]\mathrm{Re}\left[\frac{g^-(ikx,\mu^2)}{ikx}\right],$$

$$\mathrm{Im}\, V_S = \mu^2 \mathrm{Im}\, V_T = \frac{3g_A^2\mu k^3}{32\pi f_\pi^2}\int_0^1 dx(1-x^2)\mathrm{Re}\left[\frac{h^+(ikx,\mu^2)}{ikx}\right], \qquad k = |\vec{l}\,| = \sqrt{\mu^2/4 - m_\pi^2}$$

- Limitation: method is tailored to extreme nonrelativistic limit

- Chiral three-pion exchange NN-potentials: exploit again unitarity



$$\mathrm{Im}(3\pi - exch.) = \frac{1}{2}\int d\Phi_{3\pi}\, A\, B$$

- Integration over 3π phase-space in cm frame, $\mu > 3m_\pi$ is 3π-invariant mass

$$\int d\Phi_{3\pi} = \frac{1}{64\pi^4}\iint_{z^2<1} d\omega_1 d\omega_2 \iint_E \frac{dx\,dy}{\sqrt{1-x^2-y^2-z^2+2xyz}} \quad (E = ellipse\ with\ semiaxes\ \sqrt{1\pm z})$$

ω_1, ω_2 pion-energies, $x = \vec{v}\cdot\hat{k}_1$, $y = \vec{v}\cdot\hat{k}_2$, $z(\omega_1,\omega_2) = \hat{k}_1\cdot\hat{k}_2$ directional cosines

- Computation of Im-part analogous to total cross section $\sigma_{\rm tot}$ for $2 \to 3$ reaction

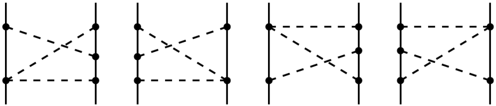


Fig.3: 3π-exchange diagrams of class VII proportional to g_A^4. The isospin factor of these diagrams is 6.

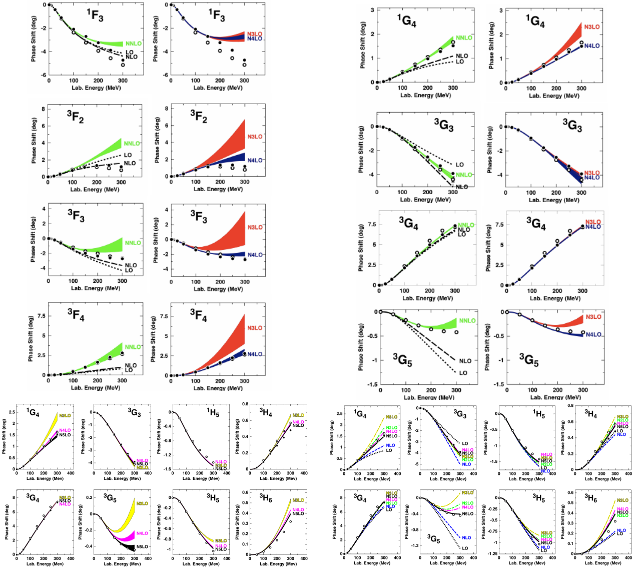
$$\mathrm{Im}\, V_S^{(VII)}(i\mu) = \frac{g_A^4(\mu-3m_\pi)^2}{35\pi(32f_\pi^3)^2}\left[2m_\pi^2 - 12\mu m_\pi - 2\mu^2 + 15\frac{m_\pi^3}{\mu} + 2\frac{m_\pi^4}{\mu^2} + 3\frac{m_\pi^5}{\mu^3}\right], \qquad (13)$$

$$\mathrm{Im}\, V_T^{(VII)}(i\mu) = \frac{g_A^4(\mu-3m_\pi)}{35\pi(32\mu f_\pi^3)^2}\left[\mu^3 + 3\mu^2 m_\pi + 2\mu m_\pi^2 + 6m_\pi^3 + 18\frac{m_\pi^4}{\mu} - 9\frac{m_\pi^5}{\mu^2} - 27\frac{m_\pi^6}{\mu^3}\right]. \qquad (14)$$

- Using spectral-function method one has derived chiral NN-potential up to N$^{4.5}$LO

Results for *NN* phase shifts and mixing angles up to N$^{4.5}$LO

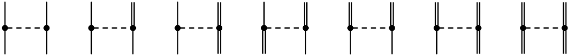
- D. Entem, N. Kaiser, R. Machleidt, Y. Nosyk, Phys. Rev. C 91, 014002 (2015); PRC 92, 064001 (2015)



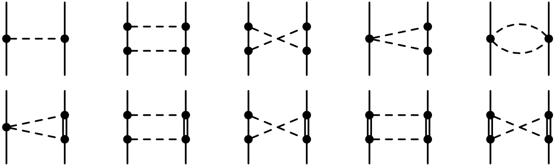
- Similar results by Bochum-group: P. Reinert, H. Krebs, E. Epelbaum, Eur. Phys. J. A 54, 86 (2018)

Elastic NN-scattering with coupled $N\Delta$-channels

- Spin-$\frac{3}{2}$ $\Delta(1232)$-isobar very important in πN-dynamics (2π-exch. NN-potential)
- Coupled $N\Delta$-channels efficient approach to deal with nuclear many-body forces



Pertinent 1π- and 2π-exchange diagrams



- Planar $N\Delta$ and $\Delta\Delta$ box include reducible part that must be subtracted $\Delta = 293$ MeV

$$\int \frac{dl_0}{2\pi i}\frac{1}{(l_0-\Delta)(-l_0+i0)(l_0^2-\omega_1^2)(l_0^2-\omega_2^2)} = \frac{1}{\Delta\omega_1^2\omega_2^2} - \frac{\omega_1^2+\omega_1\omega_2+\omega_2^2+\Delta(\omega_1+\omega_2)}{2\omega_1^2\omega_2^2(\omega_1+\omega_2)(\omega_1+\Delta)(\omega_2+\Delta)},$$

$$\int \frac{dl_0}{2\pi i}\frac{1}{(l_0-\Delta)(-l_0-\Delta)(l_0^2-\omega_1^2)(l_0^2-\omega_2^2)} = \frac{1}{2\Delta\omega_1^2\omega_2^2} - \frac{\omega_1^2+\omega_1\omega_2+\omega_2^2+\Delta(\omega_1+\omega_2)}{2\omega_1^2\omega_2^2(\omega_1+\omega_2)(\omega_1+\Delta)(\omega_2+\Delta)}$$

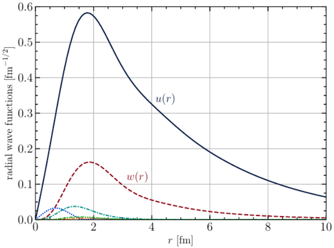
- Irreducible part of planar $N\Delta$-box and $\Delta\Delta$-box = minus crossed $N\Delta$-box

Elastic NN-scattering with coupled $N\Delta$-channels

- New feature: spin-$\frac{1}{2}$ to spin-$\frac{3}{2}$ transition operator $\vec{S}^\dagger$ and spin-$\frac{3}{2}$ operator $\vec{\Sigma}$ for Δ
- Convenient partial-wave decomposition [J. Golak et al., Eur. Phys. J. A 43, 241 (2010)]

$$H(L', S'; L, S|J) = \frac{\sqrt{\pi(2L+1)}}{2J+1} \sum_{m_J=-J}^{J} \sum_{m=-L'}^{L'} \int_{-1}^{1} dz Y_{L'm}(\arccos z, 0)\, Clebsch(L0, Sm_J, Jm_J)$$
$$\times Clebsch(L'm, S'(m_J-m), Jm_J) \langle S' m_J - m | V(\vec{q}) | S m_J \rangle$$

- Extensive coupling of $|LSJ\rangle$ states with $\Delta S = 0, 2$ and $\Delta L = 0, 2, 4, 6$
- Solve Kadyshevsky equation with coupled (NN, $N\Delta$, ΔN, $\Delta\Delta$) channels in $I = 0,1$
- Effects of $N\Delta$ and $\Delta\Delta$ contact-terms negligible for elastic NN-scattering
- Fit nine NN contact-terms up to NLO to empirical S- and P-wave phase-shifts
- Deuteron w.f. has (tiny) $\Delta\Delta$-components: $^3S_1 \to \tilde{u}$, $^3D_1 \to \tilde{w}$, $^7D_1 \to \tilde{w}_7$, $^7G_1 \to \tilde{v}$

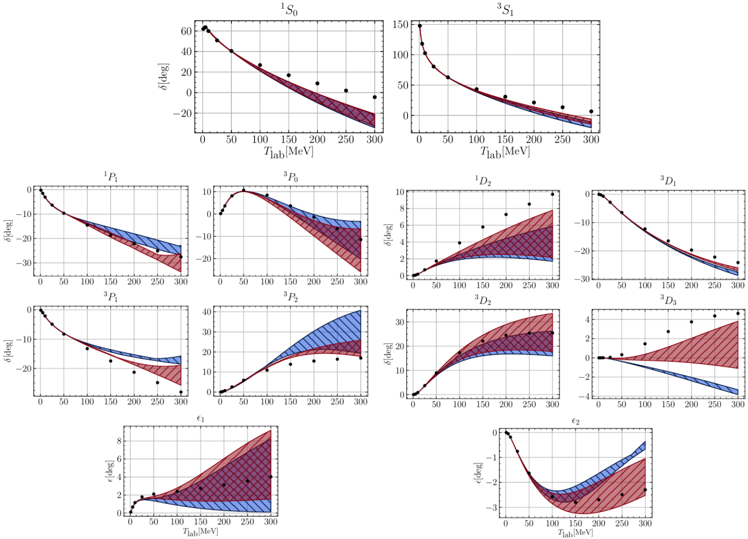


formula for computing the deuteron quadrupole moment Q_d is given by:

$$Q_d = \frac{1}{20} \int_0^\infty dr\, r^2 \Big\{ w(r)[\sqrt{8}\, u(r) - w(r)]$$
$$+ \tilde{w}(r)[\sqrt{8}\, \tilde{u}(r) - \tilde{w}(r)] + \frac{2}{7} \tilde{w}_7(r)[6\sqrt{3}\, \tilde{v}(r) - \tilde{w}_7(r)] - \frac{5}{7} \tilde{v}(r)^2 \Big\}$$

Results for NN phase shifts using coupled $N\Delta$-channels

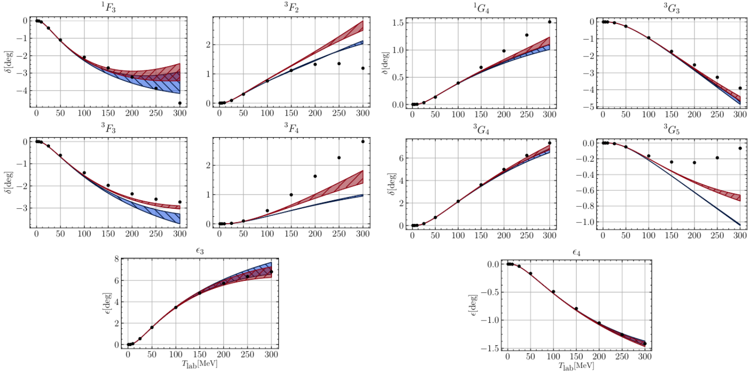
- blue bands NN only, red bands coupled (NN, $N\Delta$, ΔN, $\Delta\Delta$) channels up to N^{2}LO



- Partial improvement by including coupled channels, bands from $\Lambda = 450 - 700$ MeV

Results for NN phase shifts using coupled $N\Delta$-channels

- Taken from: S. Strohmeier, N. Kaiser, Nucl. Phys. A 1002, 121980 (2020)
- blue bands NN only, red bands coupled (NN, $N\Delta$, ΔN, $\Delta\Delta$) channels up to N^{2}LO

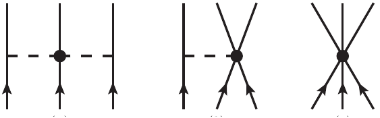


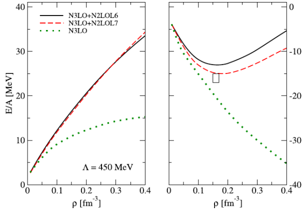
- Improvement in some F- and G-waves, but yet no perfect description
- Rather involved N^{2}LO calculation with coupled $N\Delta$ channels cannot provide a substitute for full chiral NN-potential constructed up to N^{4}LO

Chiral three-nucleon interactions

- Three-body forces are indispensable for accurate description of nuclear few- and many-body systems
- Examples: neutron-deuteron scattering differential cross section, spectra of light nuclei, saturation properties of nuclear matter, neutron matter EoS, . . .
- At leading order in chiral EFT $3N$-force consists of: long-range 2π-exchange ($c_{1,3,4}$), mid-range 1π-exchange (c_D), short-range contact (c_E) components

J. Fujita, H. Miyazawa, Prog. Theor. Phys. 17, 360 (1957)



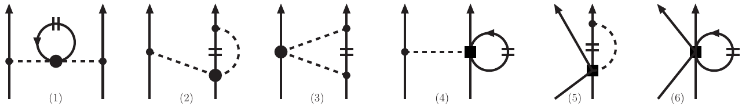


- For existing many-body methods to calculate ($A > 12$) nuclei and nuclear matter it is technically very challenging to include chiral $3N$-forces directly
- Alternative and simpler approach: density-depend. in-medium NN-potential V_{med}
- Consider $N_1(\vec{p}) + N_2(-\vec{p}) \to N_1(\vec{p}\,') + N_2(-\vec{p}\,')$ onshell in nuclear matter rest frame: same spin/isospin structures as in vacuum although Galilei invariance is broken

In-medium NN-potential V_{med}

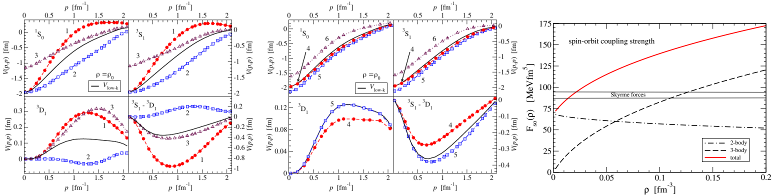
- Evaluate one-loop diagrams with in-medium nucleon or particle-hole propagator

$$\frac{i}{l_0 + i\epsilon} - 2\pi\delta(l_0)\theta(k_f - |\vec{l}\,|), \qquad \textit{density}\ \rho = \frac{2k_f^3}{3\pi^2}$$



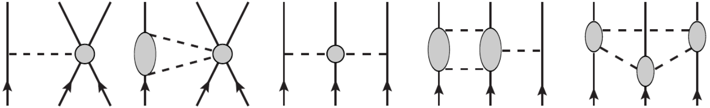
$$V_{\text{med}}^{(1)} = \frac{g_A^2 k_f^3}{3\pi^2 f_\pi^4}(c_3 q^2 + 2c_1 m_\pi^2)\vec{\tau}_1\cdot\vec{\tau}_2 \frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{(m_\pi^2+q^2)^2}, \qquad V_{\text{med}}^{(6)} \sim c_E k_f^3$$

$$V_{med}^{(2,4)} = \frac{g_A^2}{4f_\pi^2}\vec{\tau}_1\cdot\vec{\tau}_2 \frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{m_\pi^2+q^2}\Gamma_{vert}(p,q,k_f), \qquad V_{\text{med}}^{(3)} = \left\{ i(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}\times\vec{p})\,[c_{1,3,4}\dots] + \dots \right\}$$



- V_{med} derived from chiral 3N-force brings repulsion that increases with density

- $V_{\rm med} \sim c_{1,3,4}, c_{D,E}$ derived in [J. Holt et al., PRC 81, 024002 ('10)] has found many applications in calculations of nuclear matter, its thermodynamics, finite nuclei
- Subleading chiral 3N-forces parameterfree: Long-range terms built up by many pion-loop diagrams [V. Bernard, E. Epelbaum, H. Krebs, UGM, PRC 77, 064004 ('08)], short-range terms and relativistic $1/M$-corrections [BEKM, PRC 84, 054001 ('10)]
- Extension to subsubleading order: Long and intermediate range components in [H. Krebs, A. Gasparyan, E. Epelbaum, PRC 85, 054006 ('12); PRC 87, 054007 ('13)]



$$V_{3N}^{\rm ring} = \frac{g_A^4}{32 f_\pi^6} \int \frac{d^3 l_2}{(2\pi)^3} \frac{1}{(m_\pi^2 + l_1^2)(m_\pi^2 + l_2^2)(m_\pi^2 + l_3^2)} \Big\{ 2\vec\tau_1 \cdot \vec\tau_2 \Big[\vec l_1 \cdot \vec l_2 \, \vec l_2 \cdot \vec l_3 - \vec\sigma_1 \cdot (\vec l_2 \times \vec l_3) \vec\sigma_3 \cdot (\vec l_1 \times \vec l_2) \Big]$$

$$+ \vec\tau_1 \cdot (\vec\tau_2 \times \vec\tau_3) \vec\sigma_1 \cdot (\vec l_2 \times \vec l_3) \, \vec l_1 \cdot \vec l_2 + \frac{g_A^2}{m_\pi^2 + l_2^2} \Big[\dots \Big] \Big\}, \qquad \vec l_1 = \vec l_2 - \vec q_3, \quad \vec l_3 = \vec l_2 + \vec q_1$$

- Selfclosing of one nucleon-line gives isovector central potential linear in density ρ

$$V_{\rm med}^{(0)} = -\frac{g_A^4 k_f^3}{96 \pi^3 f_\pi^6} \vec\tau_1 \cdot \vec\tau_2 \Big\{ 2m_\pi + \frac{m_\pi^3}{4m_\pi^2 + q^2} + \frac{3m_\pi^2 + q^2}{q} \arctan \frac{q}{2m_\pi} \Big\} + \dots$$

- More difficult to evaluate: Concatenations of two nucleon lines $\to$ an example

$$V_{\rm med}^{(cc)} = \frac{3g_A^4}{(4\pi)^4 f_\pi^6} \int_0^\infty dl \bigg\{ l\, \Gamma_1(l,k_f) \bigg[\big[m_\pi^2(8p^2-q^2) + (4p^2+q^2)(p^2-l^2)\big] \frac{\Lambda(l)}{p^2} + \frac{l}{p^2}(q^2-4p^2)$$
$$+2(2m_\pi^2+q^2)(l^2-p^2-m_\pi^2)\Omega(l)\bigg] + \frac{16k_f^3}{3}\bigg\}, \qquad \Gamma_1(l,k_f),\, \Lambda(l),\, \Omega(l) \;\; \textit{logarithmic functions}$$

- $V_{\rm med}$ given either in analytical form or requires at most one numerical integration
  [N. Kaiser et al., PRC 98, 054002 ('18); PRC 100, 014002 ('19); PRC 101, 014001 ('20); nucl-th/2010.02739]
- Suitable form to implement chiral $3N$-forces into nuclear many-body calculations

Summary

- Derivation of basic two-pion exchange potential: difference to traditional separation of irreducible part from planar box, but it makes $V_{2\pi}$ simpler
- Spectral-function method to go beyond one-loop order: clue is nucleon four-velocity $v^\mu = (1, \vec{0})$ for NN-potential and $v^\mu = (0, i\vec{v})$ for its Im-part
- Elastic NN-scattering with coupled $(NN, N\Delta, \Delta N, \Delta\Delta)$-channels
- Construction of ρ-dep. in-medium NN-potential from chiral $3N$-forces

Thank you for your attention!