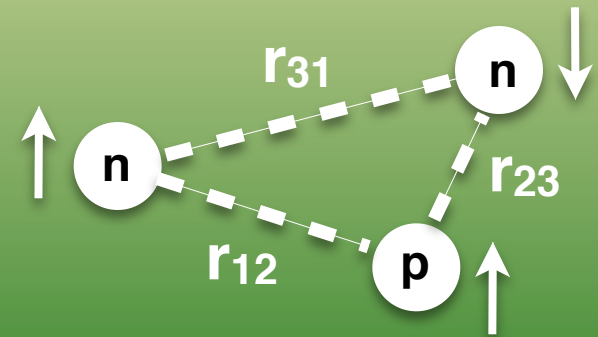


Status of three-nucleon forces

Evgeny Epelbaum, RUB

μ -Workshop, CRC 110, TU München, October 25, 2012



Outline

- Introduction
- 3NF: Where do we stand
- 3NF beyond leading order
- Summary and outlook

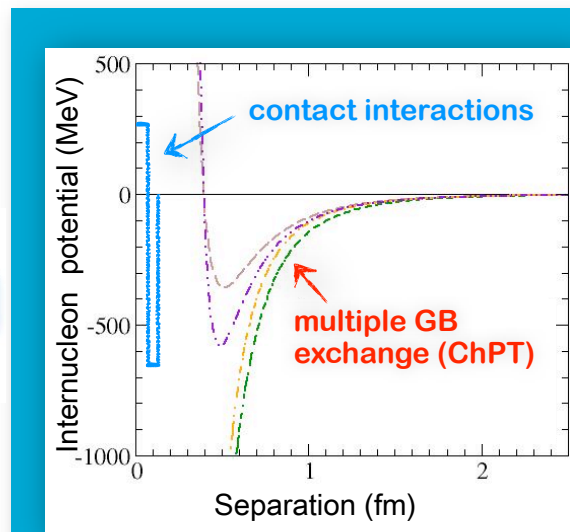
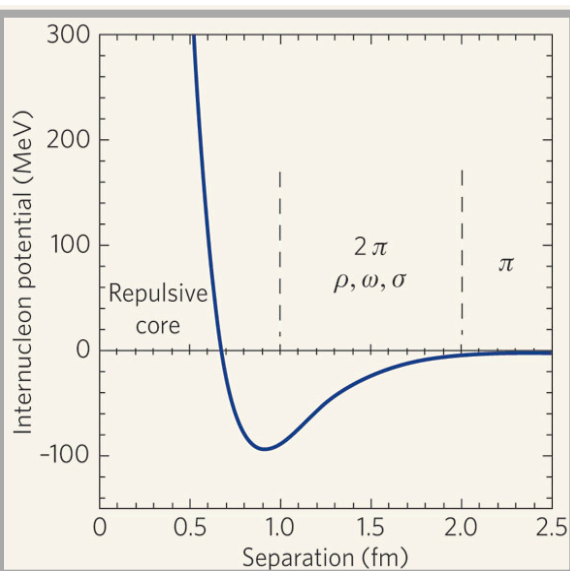


From QCD to nuclear physics

The roadmap: QCD \rightarrow Chiral Perturbation Theory \rightarrow hadron dynamics




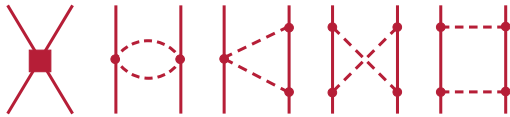



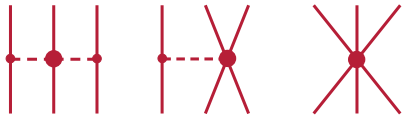


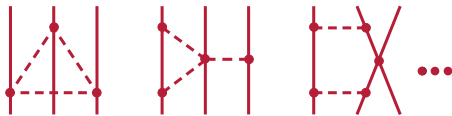
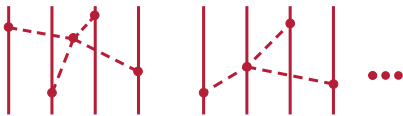
- Pions and up to 1 nucleon: **ChPT for the scattering amplitude**
- 2 and more nucleons: **ChPT for nuclear forces/currents** Weinberg '91

$$\left[\left(\sum_{i=1}^A \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3}) \right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \dots}_{\text{derived within ChPT}} \right] |\Psi\rangle = E|\Psi\rangle$$



- unified description of $\pi\pi$, πN and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, π -prod., ...)
- precision physics with/from light nuclei

Chiral expansion of nuclear forces

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)			
NLO (Q^2)			
N ² LO (Q^3)			
N ³ LO (Q^4)			

$\langle V_{2N} \rangle \sim 20$ MeV/pair

$\langle V_{3N} \rangle \sim 1$ MeV/triplet

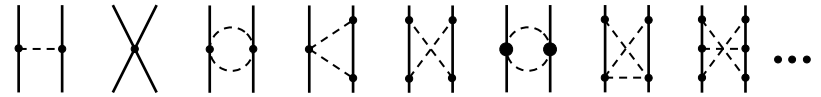
$\langle V_{4N} \rangle \sim 0.1$ MeV/quartet

(numbers from Pudliner et al. PRL 74 (95) 4396)

Nucleon-nucleon potential at N³LO

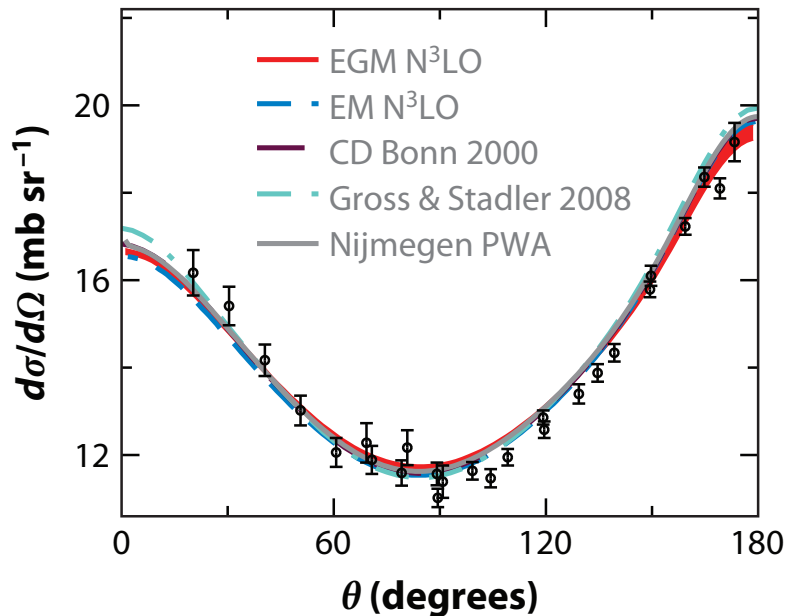
van Kolck et al.'94; Friar & Coon '94; Kaiser et al. '97; E.E. et al. '98,'03; Kaiser '99-'01; Higa, Robilotta '03; ...

- Long-range: parameter-free (all LECs from πN)
- Short-range part: 24 LECs tuned to NN data
- **Accurate description of NN data up to ~ 200 MeV**

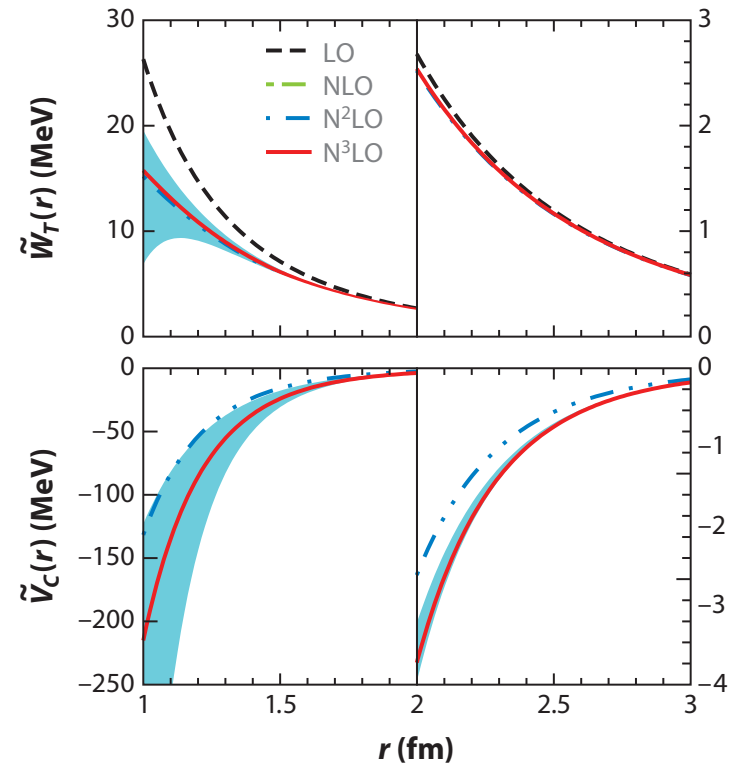


Entem-Machleidt, EE-Glöckle-Meißner

np cross section @ 50 MeV



χ expansion of the long-range force

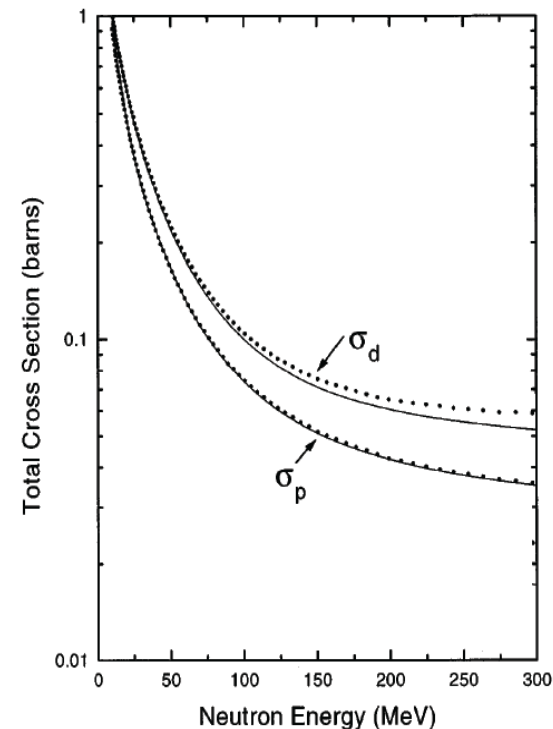


Recent reviews:

- EE, Prog. Part Nucl. Phys. 57 (06) 654;
- EE, Hammer, Meißner, Rev. Mod. Phys. 81 (09) 1773;
- Entem, Machleidt, Phys. Rept. 503 (11) 1;
- EE, Meißner, Ann. Rev. Nucl. Part. Sci. 62 (2012) 159.

The challenge: Understanding the 3N force

- Today's few- and many-body calculations have reached the level of accuracy at which it is necessary to include 3NFs
- In spite of decades of efforts, the structure of the 3NF is still poorly understood
Kalantar-Nayestanaki, EE, Messchendorp, Nogga, Rev. Mod. Phys. 75 (2012) 016301
- All the necessary ingredients for a precision description of the 3NF are available:
Chiral EFT + authom. PWD + FY equations + few-body data
→ first results already coming...



Most general structure of a local 3NF

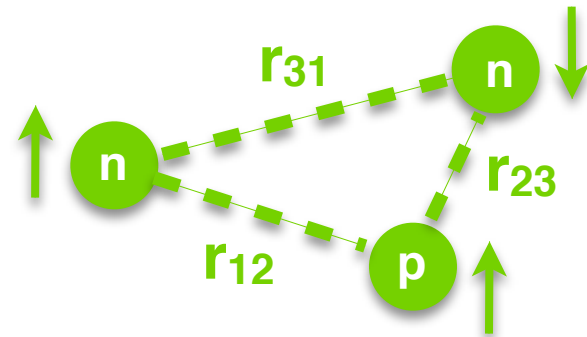
Krebs, Gasparyan, EE, in preparation

A meaningful comparison of 3NF terms requires a **complete set of independent operators**.
 Most general local 3NF involves **89 operators**, can be generated (by permutations) from **22 structures**:

$$V_{3N} = \sum_{i=1}^{22} G_i F_i(r_{12}, r_{23}, r_{31}) + \text{perm.}$$

calculated in ChPT; long-range terms parameter free...

Generators \mathcal{G} of 89 independent operators	S	A	G_1	G_2	$G_1(12)$	$G_2(12)$
1	O_1	0	0	0	0	0
$\tau_1 \cdot \tau_2$	O_2	0	O_3	O_4	O_3	O_4
$\vec{\sigma}_1 \cdot \vec{\sigma}_3$	O_5	0	O_6	O_7	$-\frac{1}{2}O_6$	$-\frac{1}{2}O_7$
$\tau_1 \cdot \tau_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3$	O_8	0	O_9	O_{10}	$-\frac{1}{2}O_9$	$-\frac{1}{2}O_{10}$
$\tau_2 \cdot \tau_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2$	O_{11}	O_{12}	O_{13}	O_{14}	O_{15}	O_{16}
$\tau_1 \cdot (\tau_2 \times \tau_3) \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$	O_{17}	0	0	0	0	0
$\tau_1 \cdot (\tau_2 \times \tau_3) \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{18}	0	O_{19}	O_{20}	$-\frac{1}{2}O_{19}$	$-\frac{1}{2}O_{20}$
$\vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_3$	O_{21}	O_{22}	O_{23}	O_{24}	O_{25}	O_{26}
$\vec{q}_1 \cdot \vec{\sigma}_3 \vec{q}_3 \cdot \vec{\sigma}_1$	O_{27}	0	O_{28}	O_{29}	$-\frac{1}{2}O_{28}$	$-\frac{1}{2}O_{29}$
$\vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	O_{30}	0	O_{31}	O_{32}	$-\frac{1}{2}O_{31}$	$-\frac{1}{2}O_{32}$
$\tau_2 \cdot \tau_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	O_{33}	O_{34}	O_{35}	O_{36}	O_{37}	O_{38}
$\tau_2 \cdot \tau_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	O_{39}	O_{40}	O_{41}	O_{42}	O_{43}	O_{44}
$\tau_2 \cdot \tau_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	O_{45}	O_{46}	O_{47}	O_{48}	O_{49}	O_{50}
$\tau_2 \cdot \tau_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	O_{51}	O_{52}	O_{53}	O_{54}	O_{55}	O_{56}
$\tau_2 \cdot \tau_3 \vec{q}_1 \cdot \vec{\sigma}_2 \vec{q}_1 \cdot \vec{\sigma}_3$	O_{57}	0	O_{58}	O_{59}	$-2O_{58}$	$-2O_{59}$
$\tau_2 \cdot \tau_3 \vec{q}_3 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3$	O_{60}	O_{61}	O_{62}	O_{63}	O_{64}	O_{65}
$\tau_2 \cdot \tau_3 \vec{q}_1 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3$	O_{66}	$-O_{61}$	O_{67}	O_{68}	$-2O_{62} - O_{64} - 2O_{67}$	$-2O_{63} - O_{65} - 2O_{68}$
$\tau_1 \cdot (\tau_2 \times \tau_3) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\sigma}_3 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{69}	0	O_{70}	O_{71}	O_{70}	O_{71}
$\tau_1 \cdot (\tau_2 \times \tau_3) \vec{\sigma}_3 \cdot \vec{q}_1 \vec{q}_1 \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$	O_{72}	O_{73}	O_{74}	O_{75}	O_{76}	O_{77}
$\tau_1 \cdot (\tau_2 \times \tau_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_2 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{78}	O_{79}	O_{80}	O_{81}	O_{82}	O_{83}
$\tau_1 \cdot (\tau_2 \times \tau_3) \vec{\sigma}_1 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot \vec{q}_3 \vec{\sigma}_3 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{84}	0	O_{85}	O_{86}	O_{85}	O_{86}
$\tau_1 \cdot (\tau_2 \times \tau_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_2 \cdot \vec{q}_3 \vec{\sigma}_3 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{87}	$-O_{79}$	O_{88}	O_{89}	$O_{80} - O_{82} + O_{88}$	$O_{81} - O_{83} + O_{89}$



The generators are defined as:

$$S(\mathcal{O}) := \frac{1}{6} \sum_{P \in S_3} P\mathcal{O}$$

$$A(\mathcal{O}) := \frac{1}{6} \sum_{P \in S_3} (-1)^P P\mathcal{O}$$

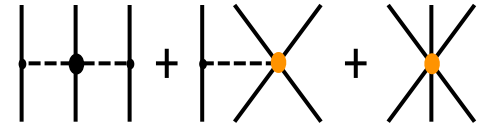
$$G_2(\mathcal{O}) := \frac{\sqrt{3}}{2} [S_{23}S_{13} - S_{12}S_{13}](\mathcal{O}).$$

$$G_1(\mathcal{O}) := \left[S_{13} - \frac{1}{2} (S_{23}S_{13} + S_{12}S_{13}) \right](\mathcal{O})$$

3N force: Where do we stand

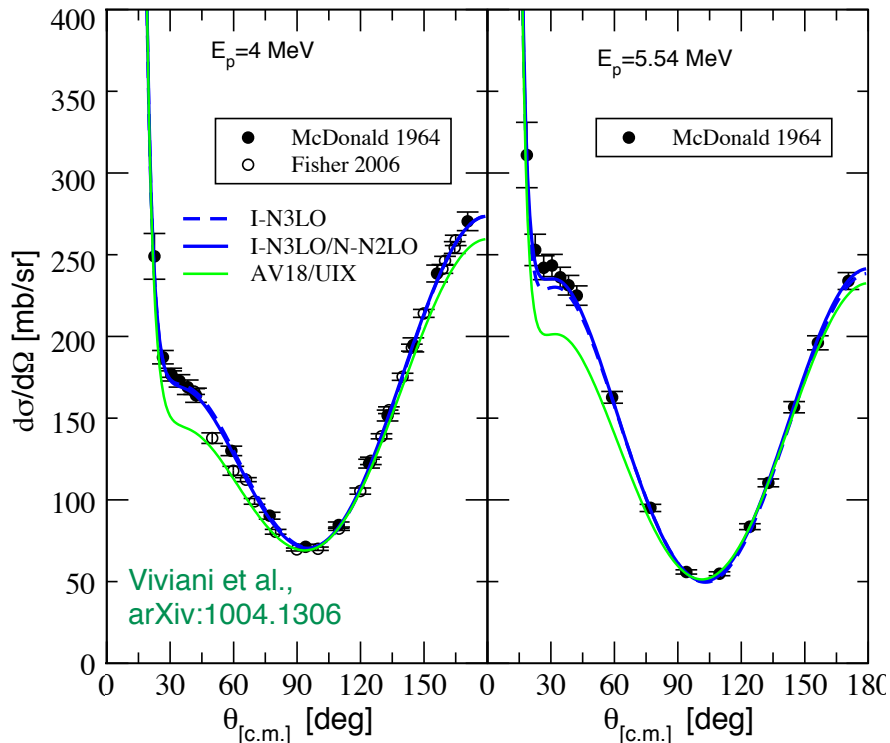
Leading chiral 3NF and 3N/4N continuum

- Nd scattering: accurate description at low energy except for **A_y -puzzle** (fine tuned) and some breakup configurations
- Uncertainty grows rapidly with energy (**higher orders ?**)
- **4N continuum**: an emerging field...

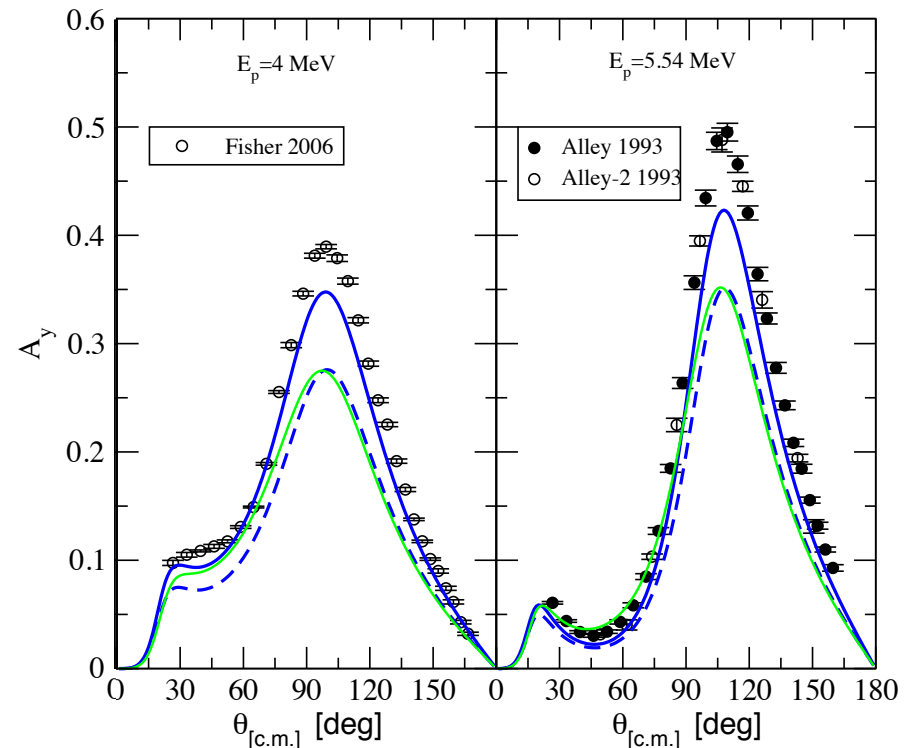


2 LECs tuned to few-N data
(e.g. ^3H , ^4He BEs)

p - ^3He differential cross section



A_y -puzzle in p - ^3He elastic scattering



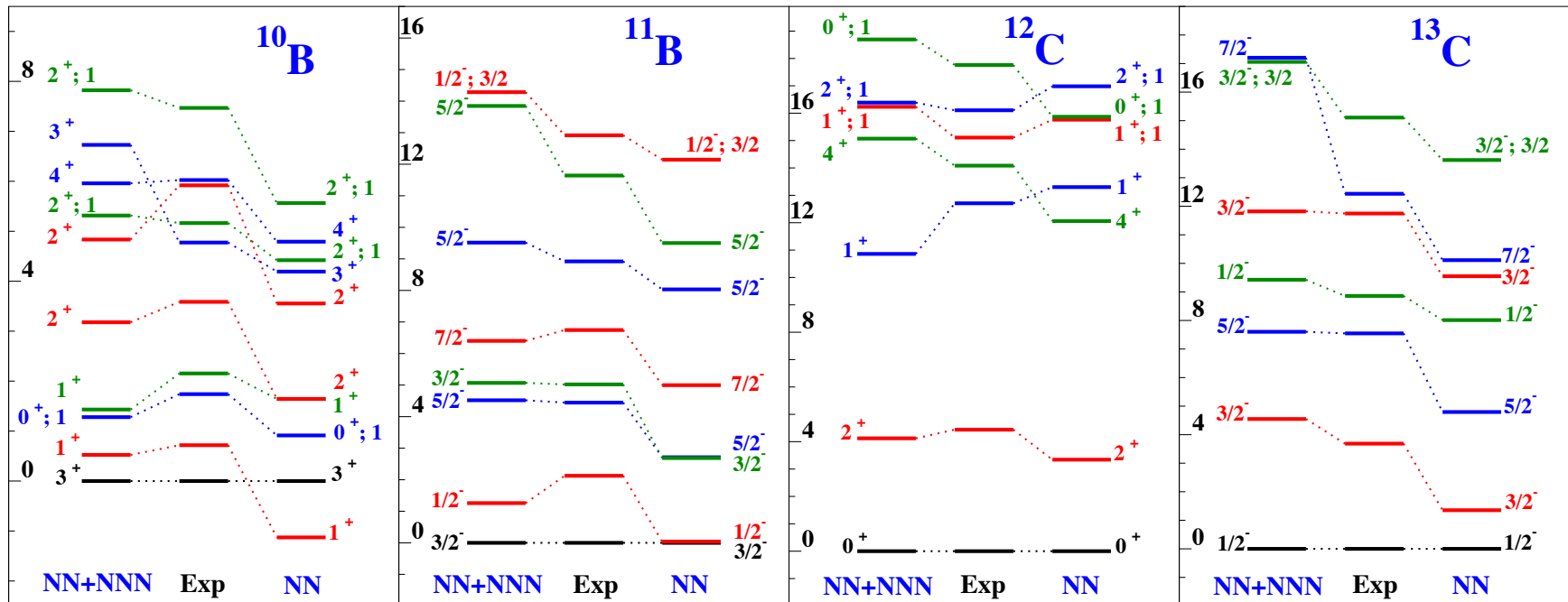
Leading chiral 3NF and nuclear structure

Ab initio methods (NCSM, GFMC, CCM, Lattice, ...) + renormalization ideas (SRG, $V_{\text{low-k}}$, UCOM)

+ computational resources \longrightarrow precision ab initio nuclear structure calculations

Barrett, Navratil, Nogga, Roth, Schwenk, Hebeler, Furnstahl, Vary, Ormand, ...

NCSM calculation of p-shell nuclei with chiral 2NF+3NF Navratil et al. '07

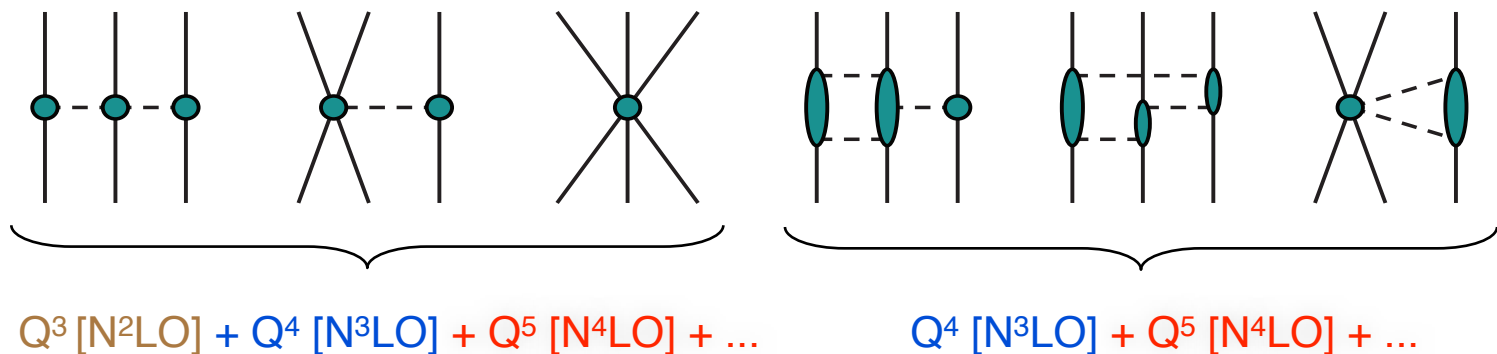


- sensitive to details of the 3NF
- many promising results (neutron-rich nuclei, long lifetime of ^{14}C , neutron star radii, ...)
- still room for improvement and some open questions \longrightarrow higher-order 3NFs...

3N force beyond leading order

3N force: corrections beyond LO

3NF topologies up to N⁴LO (subleading one-loop order)

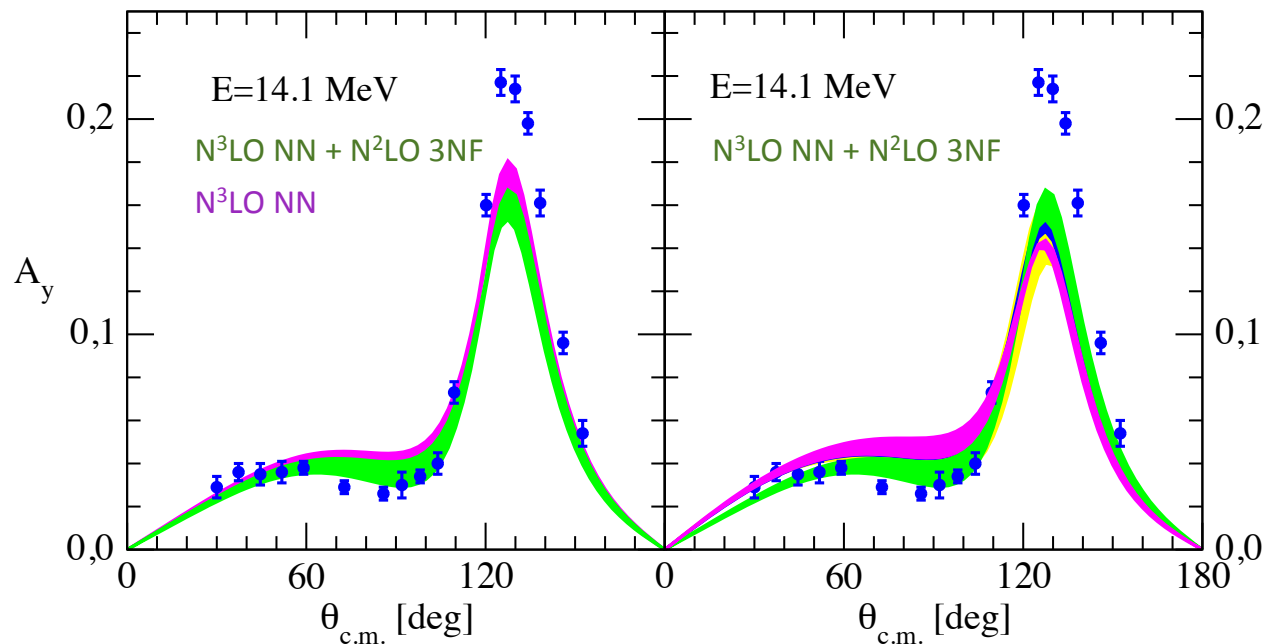


- N²LO contributions (leading 3NF) nowadays included in most few-/many-body calculations
- The leading corrections (N³LO) have been worked out recently
 - Ishikawa, Robilotta, PRC76 (07); Bernard, EE, Krebs, Meißner, PRC77 (08); PRC84 (11)
 - parameter-free!
 - rich spin-momentum structure, especially from the ring diagrams
 - partial wave decomposition is still in progress
 - the technology for numerical PWD has been developed Skibinski et al., PRC84 (11)
 - calculations presently running at JUROPA@FZJ, OSC@OhioState,...
 - estimated need: ~ 10.000.000 CPU hours

3N force: corrections beyond LO

Impact of some of the $N^3\text{LO}$ 3NF terms on nd A_y (incomplete)

Witala et al., in proceedings of FB20



Right panel: $X = N^3\text{LO NN} + N^2\text{LO 3NF} + N^3\text{LO 3NF (1}\pi\text{-cont.)} + N^3\text{LO 3NF (cont.)}$

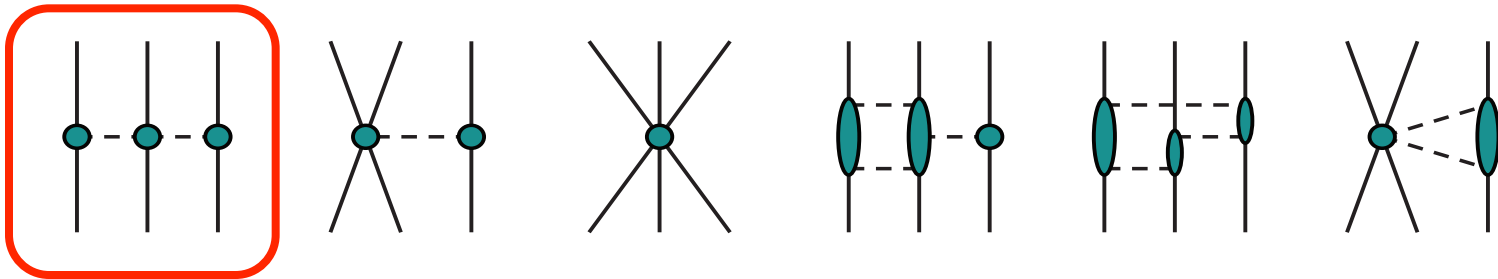
■ = $X + N^3\text{LO 3NF (2}\pi\text{-exch.)}$

■ = $X + N^3\text{LO 3NF (2}\pi\text{-exch. \& 2}\pi\text{-1}\pi\text{-exch.)}$

■ = $X + N^3\text{LO 3NF (2}\pi\text{-exch. \& 2}\pi\text{-1}\pi\text{-exch. \& ring)}$

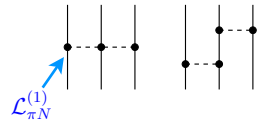
- $N^3\text{LO}$ corrections seem to be rather small,
- chiral expansion of the 3NF is NOT yet converged → need to go to higher orders...

3NF: chiral expansion of the longest-range piece (2π)



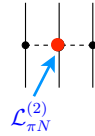
Two-pion exchange 3NF up to N^4LO

Next-to-leading order



← yield vanishing 3NF contributions

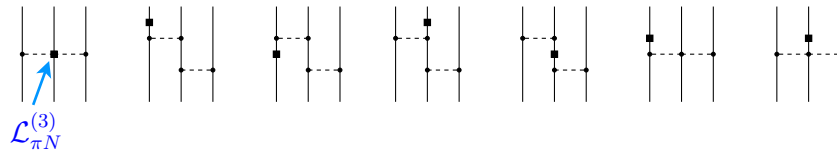
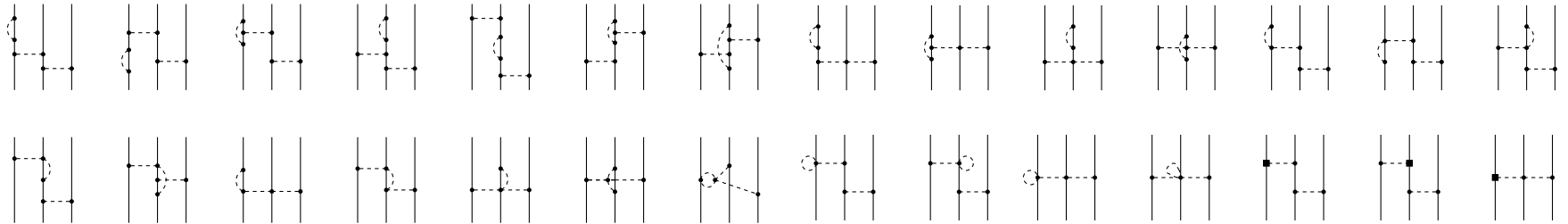
Next-to-next-to-leading order



← first nonvanishing 3NF, encodes information about the Δ :

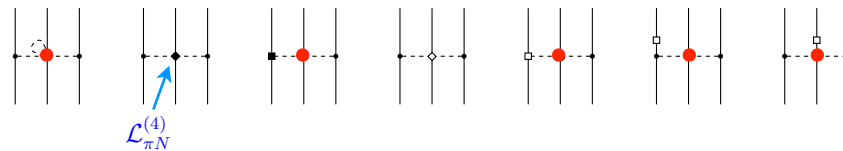
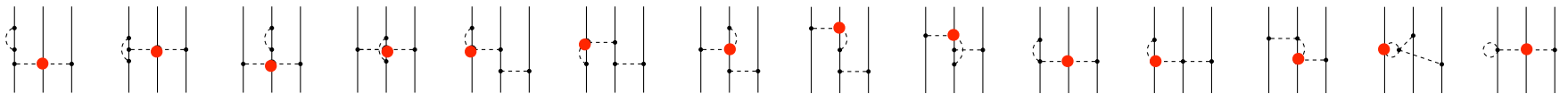


Next-to-next-to-next-to-leading order (leading 1 loop)



renormalization nontrivial !

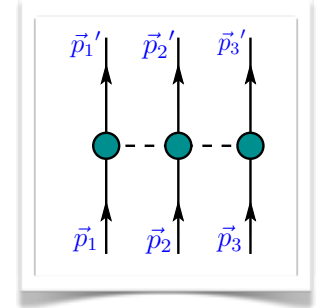
Next-to-next-to-next-to-next-to-leading order (subleading 1 loop)



Two-pion exchange 3NF up to N⁴LO

The TPE 3NF has the form (modulo 1/m-terms):

$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2][q_3^2 + M_\pi^2]} \left(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \mathcal{B}(q_2) \right)$$



- **leading-order:** $\mathcal{A}^{(3)}(q_2) = \frac{g_A^2}{8F_\pi^4} \left((2c_3 - 4c_1)M_\pi^2 + c_3q_2^2 \right), \quad \mathcal{B}^{(3)}(q_2) = \frac{g_A^2 c_4}{8F_\pi^4}$
van Kolck '94

- **subleading:** $\mathcal{A}^{(4)}(q_2) = \frac{g_A^4}{256\pi F_\pi^6} \left[A(q_2) \left(2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4 \right) + (4g_A^2 + 1) M_\pi^3 + 2(g_A^2 + 1) M_\pi q_2^2 \right],$
 $\mathcal{B}^{(4)}(q_2) = -\frac{g_A^4}{256\pi F_\pi^6} \left[A(q_2) \left(4M_\pi^2 + q_2^2 \right) + (2g_A^2 + 1)M_\pi \right]$
Ishikawa, Robilotta '07
Bernard, EE, Krebs, Meißner '08

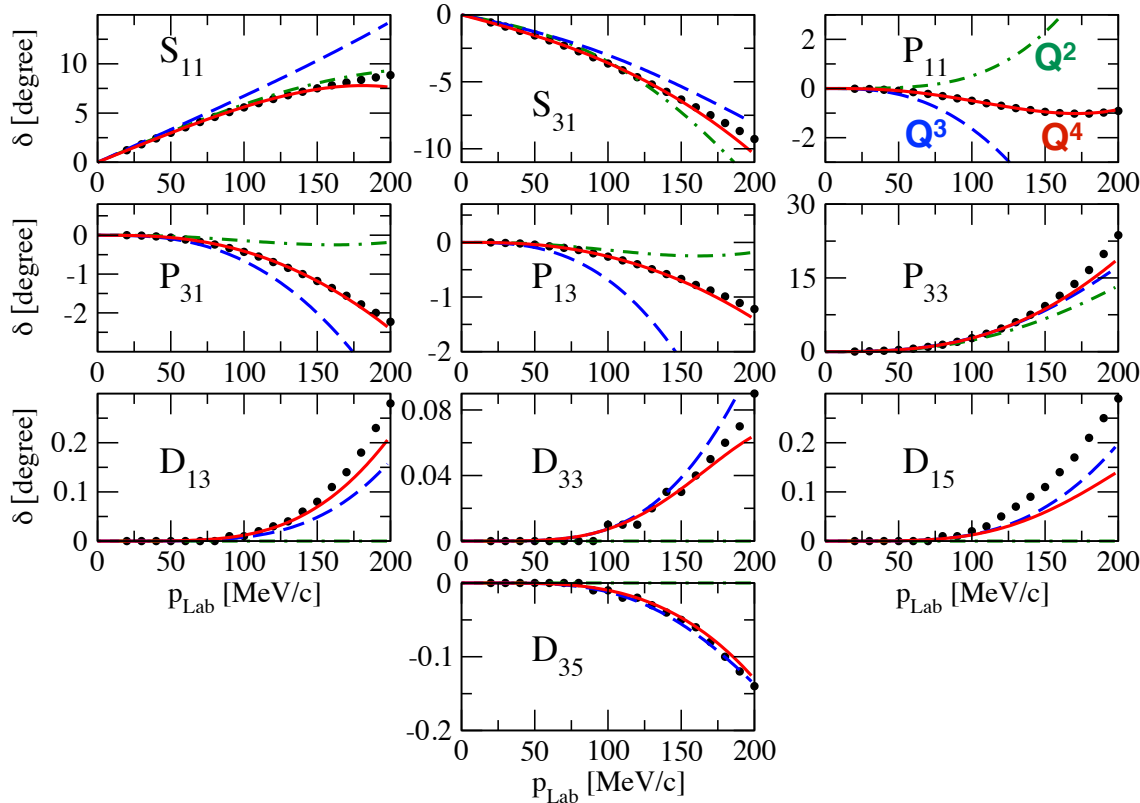
- **sub-subleading:**
Krebs, Gasparyan, EE '12

$$\begin{aligned} \mathcal{A}^{(5)}(q_2) &= \frac{g_A}{4608\pi^2 F_\pi^6} \left[M_\pi^2 q_2^2 (F_\pi^2 (2304\pi^2 g_A (4\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36}) - 2304\pi^2 \bar{d}_{18} c_3) \right. \\ &+ g_A (144c_1 - 53c_2 - 90c_3)) + M_\pi^4 (F_\pi^2 (4608\pi^2 \bar{d}_{18} (2c_1 - c_3) + 4608\pi^2 g_A (2\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{36} - 4\bar{e}_{38})) \\ &+ g_A (72 (64\pi^2 \bar{l}_3 + 1) c_1 - 24c_2 - 36c_3)) + q_2^4 (2304\pi^2 \bar{e}_{14} F_\pi^2 g_A - 2g_A (5c_2 + 18c_3)) \left. \right] \\ &- \frac{g_A^2}{768\pi^2 F_\pi^6} L(q_2) \left(M_\pi^2 + 2q_2^2 \right) \left(4M_\pi^2 (6c_1 - c_2 - 3c_3) + q_2^2 (-c_2 - 6c_3) \right), \\ \mathcal{B}^{(5)}(q_2) &= -\frac{g_A}{2304\pi^2 F_\pi^6} \left[M_\pi^2 (F_\pi^2 (1152\pi^2 \bar{d}_{18} c_4 - 1152\pi^2 g_A (2\bar{e}_{17} + 2\bar{e}_{21} - \bar{e}_{37})) + 108g_A^3 c_4 + 24g_A c_4) \right. \\ &+ q_2^2 (5g_A c_4 - 1152\pi^2 \bar{e}_{17} F_\pi^2 g_A) \left. \right] + \frac{g_A^2 c_4}{384\pi^2 F_\pi^6} L(q_2) \left(4M_\pi^2 + q_2^2 \right) \end{aligned}$$

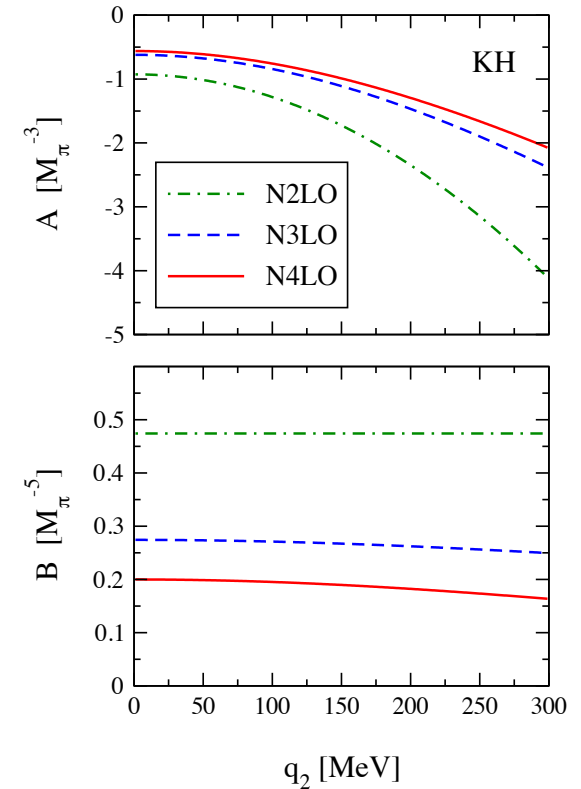
Two-pion exchange 3NF up to N⁴LO

Krebs, Gasparyan, EE '12

πN phase shifts in HB ChPT up to Q⁴ (KH PWA)



3NF „structure functions“



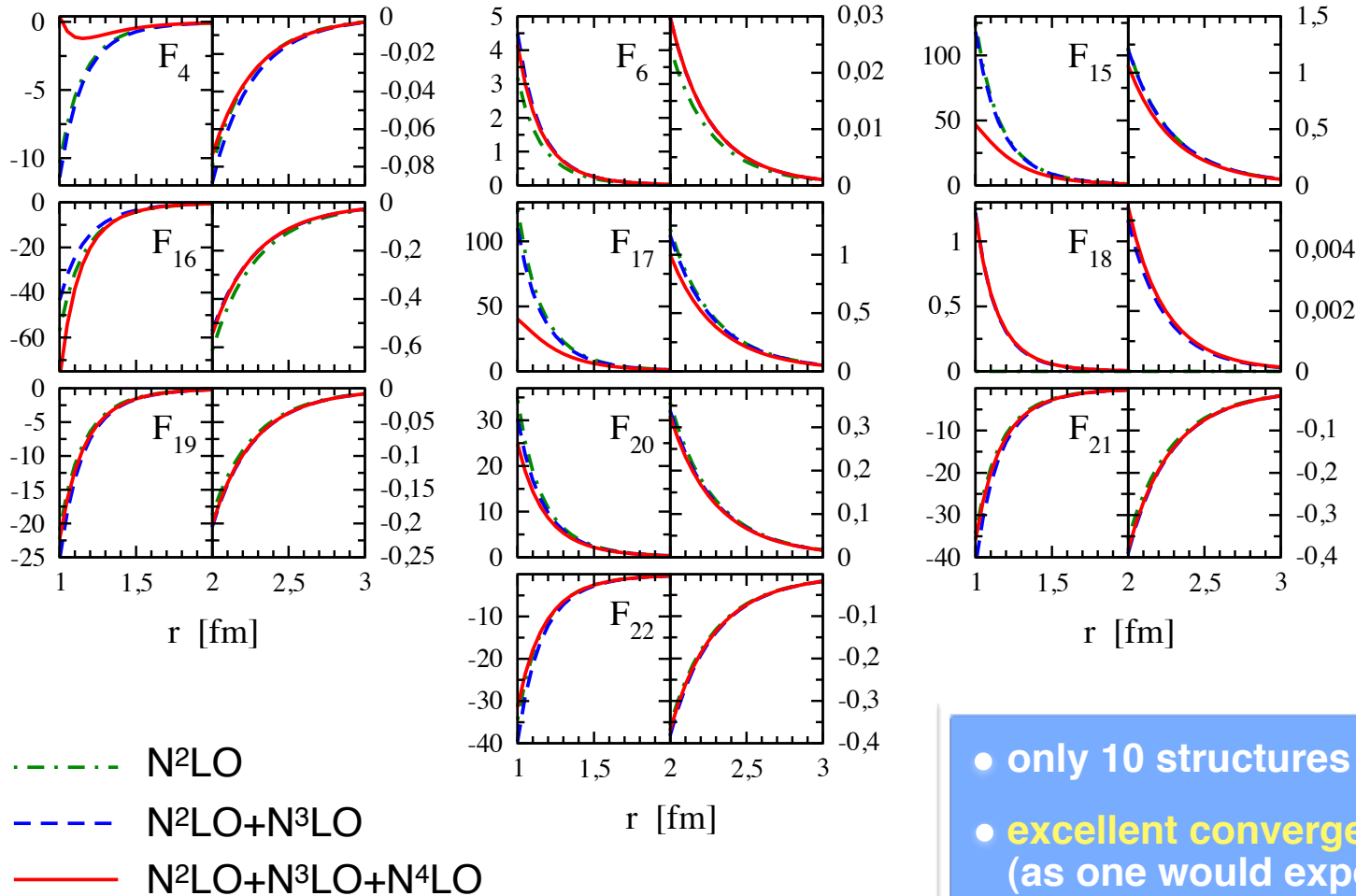
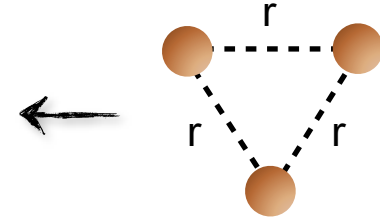
The determined values of LECs

	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	\bar{d}_3	\bar{d}_5	$\bar{d}_{14} - \bar{d}_{15}$	\bar{e}_{14}	\bar{e}_{15}	\bar{e}_{16}	\bar{e}_{17}	\bar{e}_{18}
Q ⁴ fit to GW	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-5.80	1.76	-0.58	0.96
Q ⁴ fit to KH	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26

Two-pion exchange 3NF up to N⁴LO

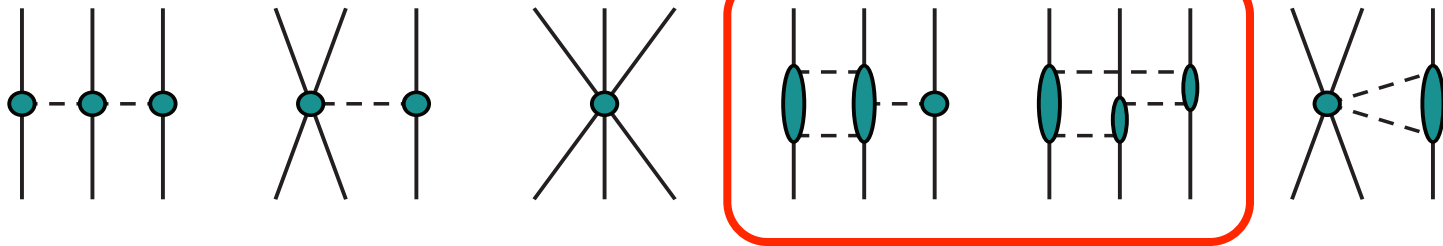
Krebs, Gasparyan, EE '12

Chiral expansion of TPE „structure functions“ F_i (in MeV) in the equilateral-triangle configuration



- only 10 structures out of 22...
- excellent convergence for $r > 1.5$ fm (as one would expect)

3NF: chiral expansion of the intermediate-range terms



$2\pi-1\pi$ & ring graphs up to N⁴LO

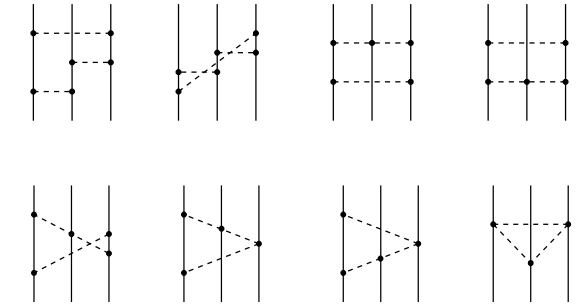
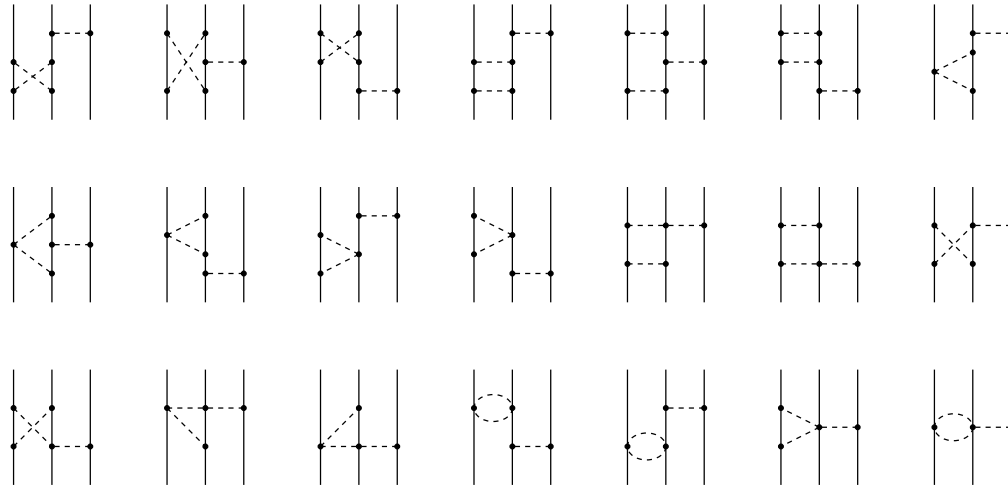
The $2\pi-1\pi$ -exchange 3NF topology



Ring 3NF diagrams

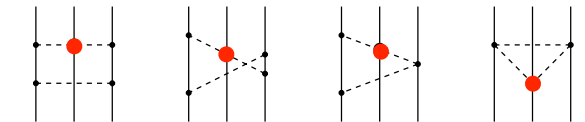
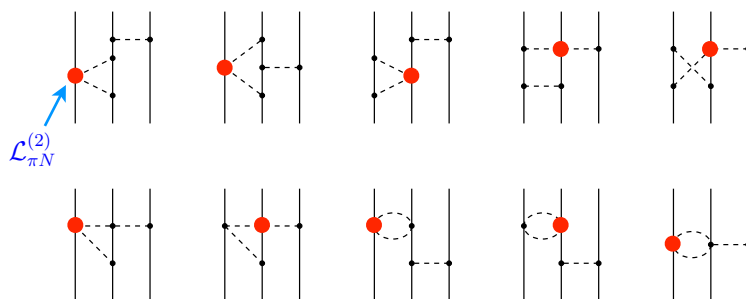


N³LO



Bernard, EE, Krebs, Meißner PRC 77 (2008) 064004

N⁴LO

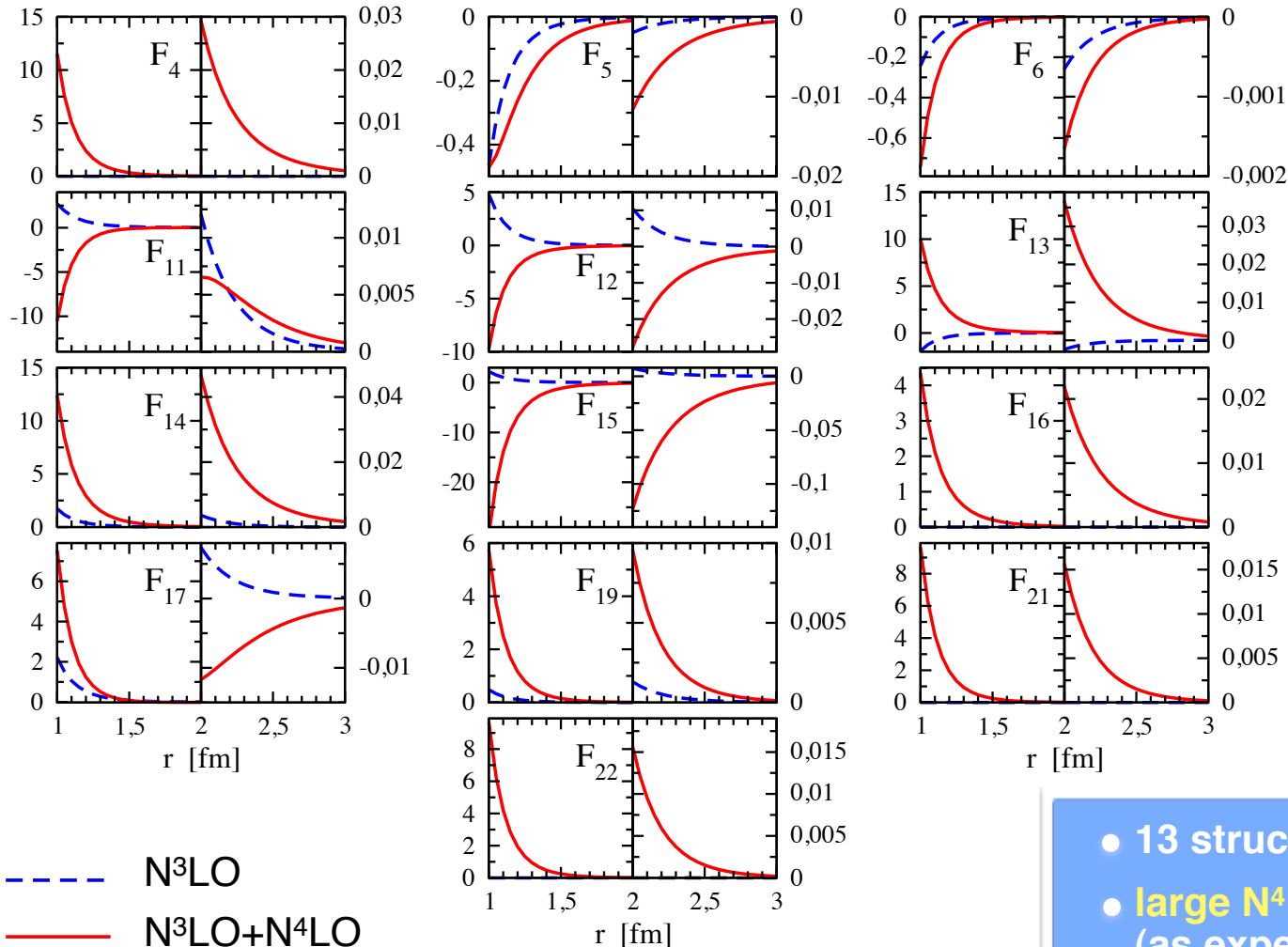
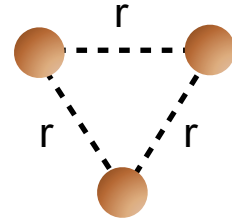


Krebs, Gasparyan, EE, in preparation

2π - 1π -exchange 3NF at N⁴LO

Krebs, Gasparyan, EE, in preparation

Chiral expansion of 2π - 1π „structure functions“ F_i (in MeV) in the equilateral-triangle configuration

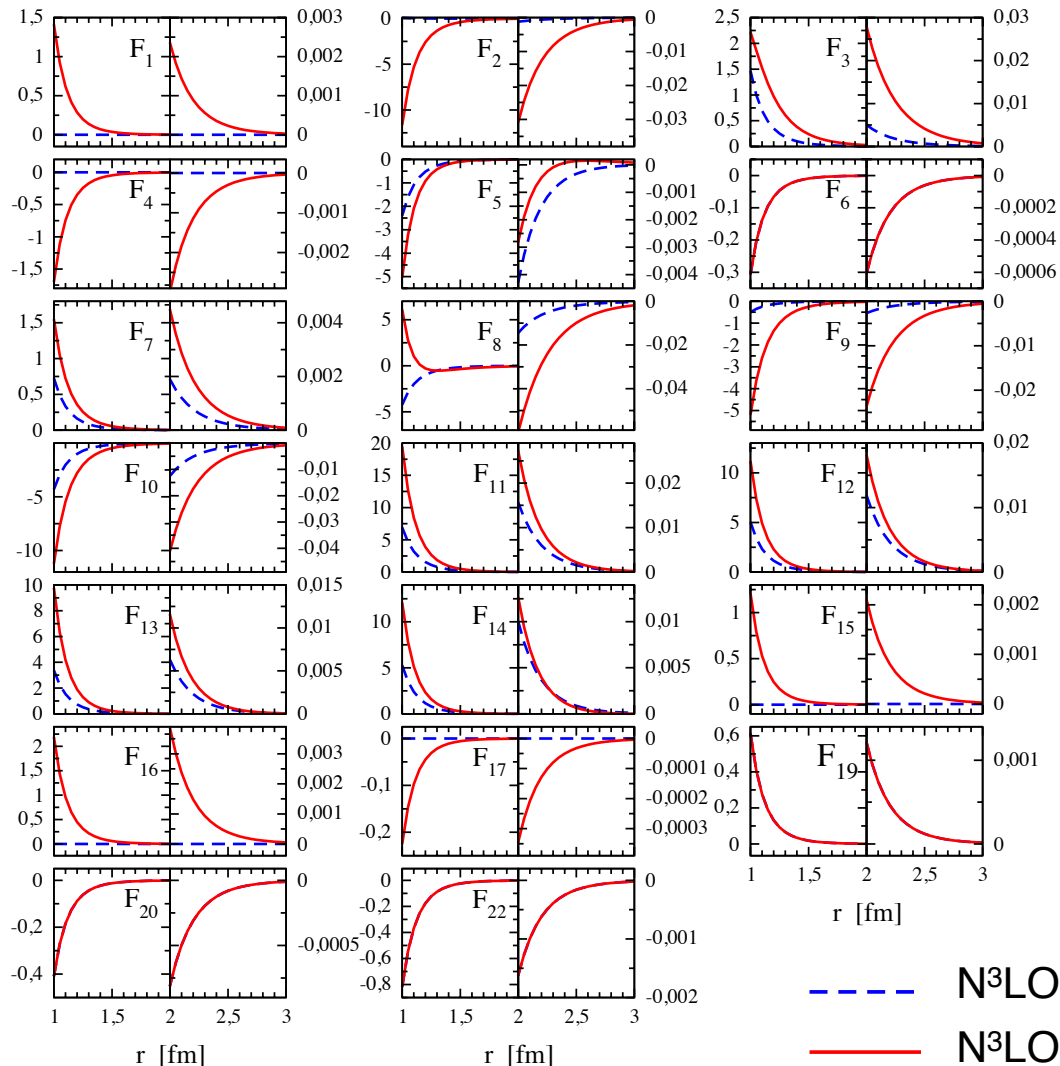


- 13 structures out of 22...
- large N⁴LO contributions (as expected)

Ring 3NF topology at N⁴LO

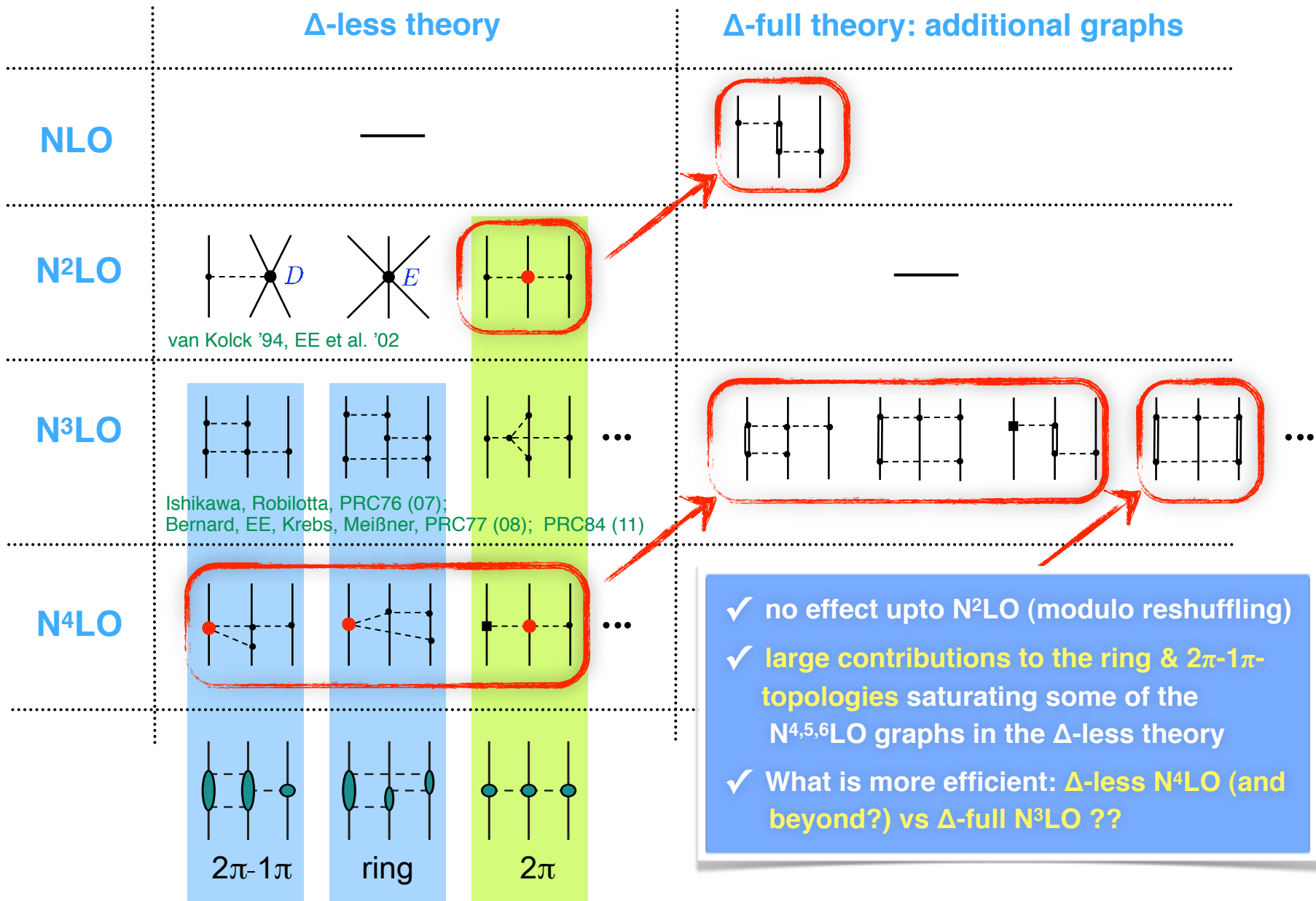
Krebs, Gasparyan, EE, in preparation

Ring „structure functions“ F_i (in MeV) in the equilateral-triangle configuration



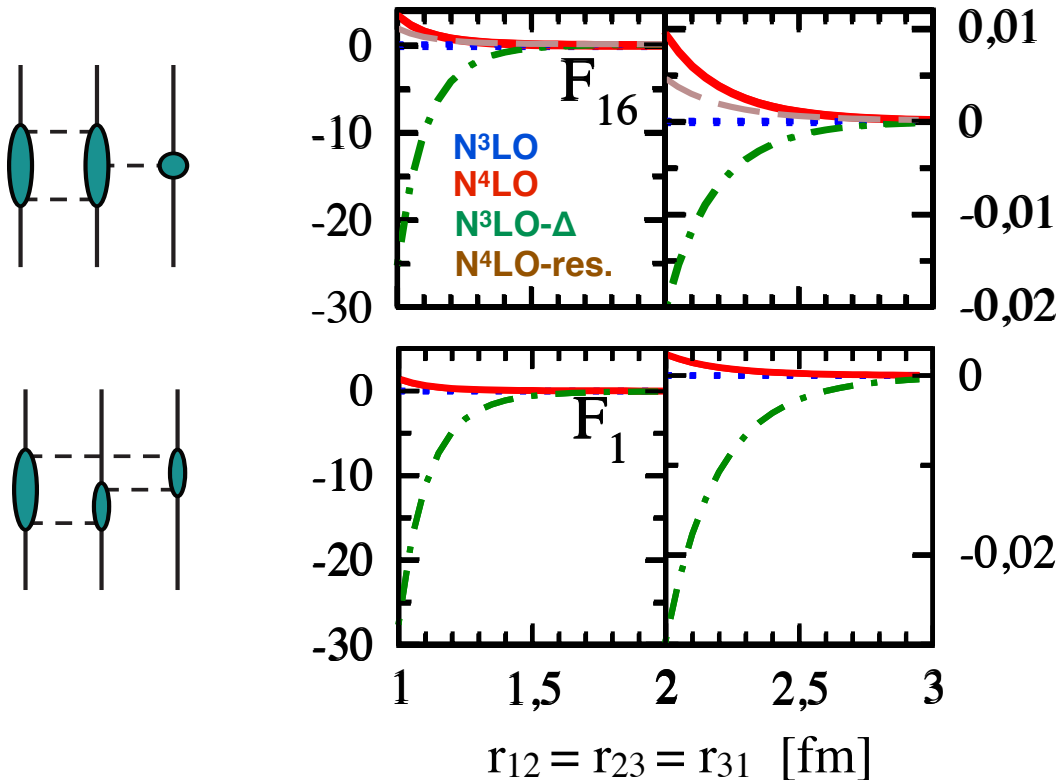
- 20 structures out of 22...
- large N⁴LO contributions (as expected)
- N⁴LO converged ??

Chiral expansion of the 3NF



Chiral 3NF: Δ -less vs. Δ -full

Krebs, Gasparyan, EE, to appear



include terms $\sim \frac{1}{m_\Delta - m_N}$
 (Δ -saturation of c_3, c_4)
 but **NOT** $\sim \frac{1}{(m_\Delta - m_N)^2}, \dots$

Chiral EFT with $\Delta(1232)$:
 (partial) resummation of terms:

$$\sim \frac{1}{m_\Delta - m_N}, \frac{1}{(m_\Delta - m_N)^2}, \dots$$

$N^4\text{LO}$, resonance saturation
 (only effects of the Δ)

- $N^4\text{LO}$ contributions still miss important physics of double and triple Δ -excitations
 → it is more efficient to include Δ as an explicit DOF
- there are indications that $N^3\text{LO}-\Delta$ results for F_i already provide a good approximation
- intermediate and short-range topologies in progress...

Summary and outlook

Towards high-precision chiral three-nucleon forces

- **technology to (numerically) carry out PWD for ANY 3NF** has been developed (but requires huge computational resources...)
- **3NF at N²LO**: nowadays standard, promising results, room for improvement
- **3NF at N³LO**: work in progress (PWD, determination of D, E). First results indicate that N³LO corrections might be small; **evidence for significant higher-order contributions (2π - 1π , ring)**
- **3NF beyond N³LO**: long-range terms (parameter-free) worked out completely at N⁴LO and **N³LO- Δ (seems to be the most efficient approach)**; shorter-range contributions in progress

Future: determination of LECs in the short-range parts of the 3NF, effects of the novel structures in the 3N and 4N continuum and light nuclei, lots of interesting physics...