Nuclear Thermodynamics with Chiral Low-Momentum Interactions

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Motivation (I): Applications and Constraints of the Nuclear EoS

Zero-Temperature Equation of State

- Bulk properties of (heavy) nuclei (e.g. saturation point, symmetry energy, . . .)
- Two-solar-mass neutron stars: PSR J0348+0432 (2013), PSR J1614-2230 (2010)

Finite-Temperature Equation of State

- Multifragmentation and fission experiments ($\rightarrow$ critical temperature)
- Core-collapse supernovae simulations
- Heavy-ion collisions (not ultra-relativistic)
- Thermodynamics of the in-medium chiral condensate

Motivation (I): Applications and Constraints of the Nuclear EoS

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Motivation (II): Parameter-Dependence of the Nuclear EoS from χEFT?

In χEFT, the parameters (LECs) of nuclear potentials are fitted to few-body properties (NN phase-shifts, binding energies of light nuclei, ...)

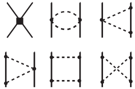
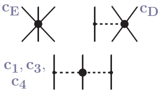
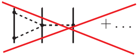
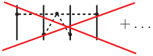
Problem: fitting procedure not unique, there exist several different sets of LECs in the literature

$\rightarrow$ **how do these different parameter-sets compare in the many-body sector?**

Nuclear Forces in Chiral Effective Field Theory

- Hierarchy controlled by **chiral expansion parameter** $\frac{Q}{\Lambda_\chi}$

  $Q \sim m_\pi \simeq 140$ MeV, $\Lambda_\chi \sim 4\pi f_\pi \simeq 1.2$ GeV
- Two types of vertices: pion-exchange- & contact-vertices
- Nuclear forces parametrized by Low-Energy Constants (**LECs**)

	NN Force	3N Force	4N Force
LO $(Q/\Lambda_\chi)^0$		—	—
NLO $(Q/\Lambda_\chi)^2$		—	—
NNLO $(Q/\Lambda_\chi)^3$	 $+ \dots$		—
N^{3}LO $(Q/\Lambda_\chi)^4$	 $+ \dots$	 not considered here	 not considered here

- Introduce regulator function (with "cutoff"-parameter Λ) in order to restrict resolution in momentum space

  $\Rightarrow$ Nuclear potentials $\mathbf{V_{NN}} = V_{NN}(\Lambda; c_i, d_i)$ and $\mathbf{V_{3N}} = V_{3N}(\Lambda; c_i, c_E, c_D)$
- $c_i(\Lambda)$, $d_i(\Lambda)$ from NN-scattering phase shifts, $c_D(\Lambda)$, $c_E(\Lambda)$ from 3N & 4N observables

Chiral-Low Momentum Interactions

- $V_{\mathbf{NN}}(\Lambda_{\boldsymbol{R}})$: low-momentum potential with smooth regulator

$$f(k', k) = \exp\left[-\left(\frac{k}{\Lambda_{\boldsymbol{R}}}\right)^{2n} - \left(\frac{k'}{\Lambda_{\boldsymbol{R}}}\right)^{2n} \right]$$

Here: n3lo-potentials constructed by Coraggio, Entem, Machleidt, Gazit et al.

arXiv:nucl-th/0701065

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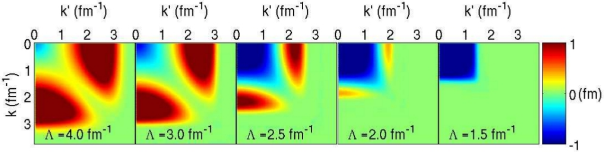
- $V_{\text{low-}k}(\Lambda)$: RG-evolution to low (half-)relative-momenta ($\Lambda \sim 2.0\,\text{fm}^{-1} \Rightarrow$ universality)



Bogner, Furnstahl, Schwenk; arXiv:0912.3688v3

Advantage: do not have to worry about refitting LECs for different Λ
Disadvantage: don't know how LECs change with Λ, but need c_1, c_3 & c_4 for 3N forces

Here: Nijmegen LECs + $cE(\Lambda)$ and $cD(\Lambda)$ refitted to 3N & 4N binding energies

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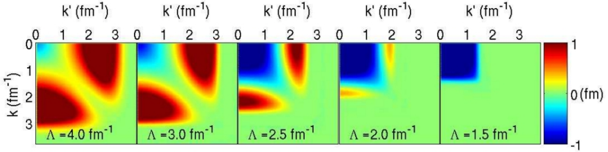

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Summary: sets of low-momentum NN and 3N potentials used in this work

identifier	$\Lambda_{(R)}$	n	c_E	c_D	c_1 [GeV^{-1}]	c_3 [GeV^{-1}]	c_4 [GeV^{-1}]
n3lo414	2.1 fm^{-1}	10	-0.072	-0.40	-0.81	-3.0	3.4
n3lo450	2.3 fm^{-1}	3	-0.106	-0.24	-0.81	-3.4	3.4
n3lo500	2.5 fm^{-1}	2	-0.205	-0.20	-0.81	-3.2	5.4
VLK21	2.1 fm^{-1}	∞	-0.625	-2.062	-0.76	-4.78	3.96
VLK23	2.3 fm^{-1}	∞	-0.822	-2.785	-0.76	-4.78	3.96

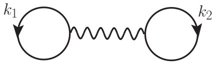
PART II:
Many-Body Perturbation Theory for Nuclear Matter

Many-Body Perturbation Theory: T=0 vs. T=finite

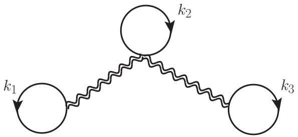
- **T = 0:** $E(\kappa_F) = E_0 + E_{1,\mathrm{NN}} + E_{1,3\mathrm{N}} + E_{2,\mathrm{normal}} + \ldots$ (BG-formula)

  κ_F = **Fermi-momentum of non-interacting system,** $\rho = \frac{2}{3\pi^2}\kappa_F^3$

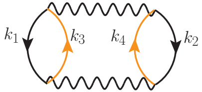
  $\rightsquigarrow$ **calculation in the canonical ensemble, i.e.** $F(\rho, T = 0) = E(\kappa_F)$



(1,NN)



(1,3N)



(2,normal)

Antisymmetrized Goldstone-diagrams (T = 0): hole-lines $|\vec{k}| \leq \kappa_F$
particle-lines $|\vec{k}| \geq \kappa_F$

Many-Body Perturbation Theory: T=0 vs. T=finite

- **T = 0:** $E(\kappa_F) = E_0 + E_{1,NN} + E_{1,3N} + E_{2,\text{normal}} + \dots$ (BG-formula)
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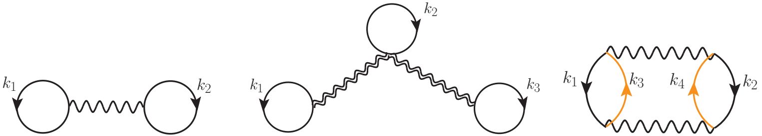
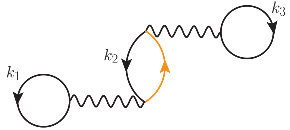
  $\rightsquigarrow$ **calculation in the canonical ensemble, i.e.** $F(\rho, T=0) = E(\kappa_F)$
- **T ≠ 0:** $\Omega(\mu, T) = \Omega_0 + \Omega_{1,NN} + \Omega_{1,3N} + \Omega_{2,\text{normal}} + \Omega_{2,\text{anomalous}} + \dots$
  grand-canonical, $\mu =$ **chemical potential of the interacting system**



(1,NN) (1,3N) (2,normal)



(2,anomalous)

Antisymmetrized Goldstone-diagrams T ≠ 0: hole-lines $\sim f_k = \frac{1}{1+\exp[-\beta(\epsilon_{\boldsymbol{k}}-\mu)]}$
particle-lines $\sim \bar{f}_k = 1 - f_k$

Kohn-Luttinger-Ward Method

Idea: Expand about non-interacting system (Free Fermi Gas)

1. Formally postulate equality of densities

$$\rho(\mu_0, T) = -\frac{\partial \Omega_0(\mu_0, T)}{\partial \mu_0} \stackrel{!}{=} -\frac{\partial \Omega(\mu, T)}{\partial \mu} = \rho(\mu, T)$$

2. Formally expand chemical potential in terms of interaction strength λ

$$\mu = \mu_0 + \lambda \mu_1 + \lambda^2 \mu_2 + \mathcal{O}(\lambda^3)$$

3. Expand the right-hand side of the density equation around μ_0 and solve iteratively for increasing powers of λ: gives $\mu_i(\mu_o)\,,\ i \geq 1$

4. Expand every term in the grand-canonical perturbation series around μ_0

$$\boxed{\begin{aligned} F(\mu_0) =& F_0(\mu_0) + \Omega_{1,\mathrm{NN}}(\mu_0) + \Omega_{1,3\mathrm{N}}(\mu_0) + \Omega_{2,\mathrm{normal}}(\mu_0) \\ &+ \left[\Omega_{2,\mathrm{anomalous}}(\mu_0) - \frac{1}{2}\frac{(\partial \Omega_{1,\mathrm{NN}}/\partial \mu_0)^2}{\partial^2 \Omega_0/\partial \mu_0^2}\right] \\ \equiv& F_0(\mu_0) + F_{1,\mathrm{NN}}(\mu_0) + F_{1,3\mathrm{N}}(\mu_0) + F_{2,\mathrm{normal}}(\mu_0) + [F_{2,\mathrm{anomalous}}(\mu_0) + F_{\mathrm{ADT}}(\mu_0)] \end{aligned}}$$

The additional ADT-terms cancel the anomalous contributions in the $T \to 0$ limit

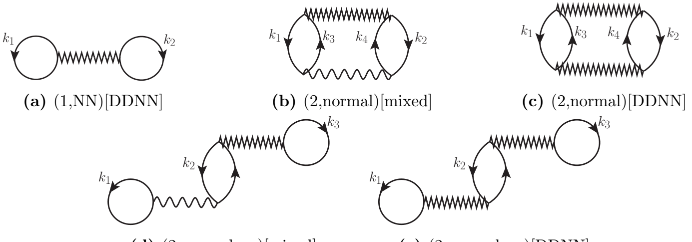
$$\Rightarrow \quad \boxed{F(\mu_0) \xrightarrow{T \to 0} E(\kappa_F), \quad \mu_0 \xrightarrow{T \to 0} \frac{\kappa_F^2}{2M_N}}$$

Summary: KLW-method gives the consistent continuation of the BG-formula to T=finite

Density-Dependent Two-Nucleon Interaction (DDNN)

- Effective NN potential $\tilde{V}_{3N}$ constructed from genuine 3N forces by integrating out (i.e. closing) one nucleon-line in the 3N scattering-diagram

  Heavyside step-function $\Rightarrow$ $\tilde{V}_{3N}$ is density-dependent
- At T=finite $\tilde{V}_{3N}$ becomes also temperature-dependent: $\tilde{V}_{3N}(\rho, T)$
- **Regularization of $\tilde{V}_{3N}$:** in n3lo-potential-sets $\tilde{V}_{3N}$ has a smooth regulator, in VLK-potential-sets a sharp cutoff



(a) (1,NN)[DDNN] **(b)** (2,normal)[mixed] **(c)** (2,normal)[DDNN]

(d) (2,anomalous)[mixed] **(e)** (2,anomalous)[DDNN]

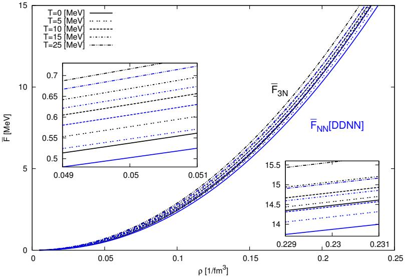
Antisymmetrized Goldstone diagrams representing the 1st-order and 2nd-order contributions associated with the DDNN potential (represented by zigzag lines)

Diagrams (a) and (d) have a topological factor of $\frac{1}{3}$, and diagram (e) one of $\frac{1}{9}$

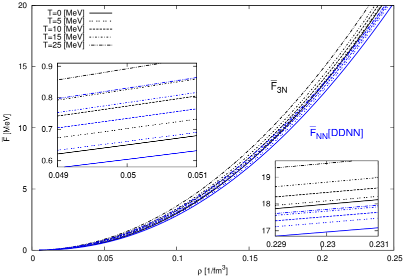
Diagrams (b) and (d) have a symmetry factor 2 due to exchange of interaction-lines

How Good is this Approximation?

Compare first-order DDNN contribution with genuine 3N contribution:



(a) (1,3N) vs. (1,NN)[DDNN] for n3lo414

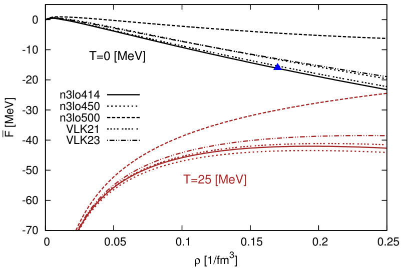


(b) (1,3N) vs. (1,NN)[DDNN] for n3lo450

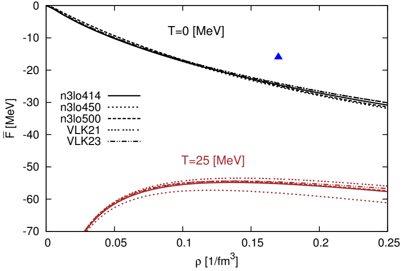
$\Rightarrow$ Using $\tilde{V}_{3N}(\rho, T)$ instead of V_{3N} at second order is justified

PART V:
Nuclear Equation of State

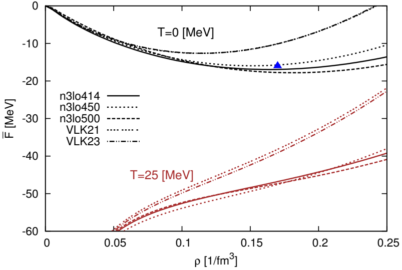
Convergence of the Perturbation Series



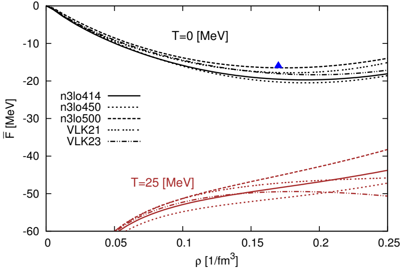
(a) NN first order, no 3N



(b) NN second order, no 3N

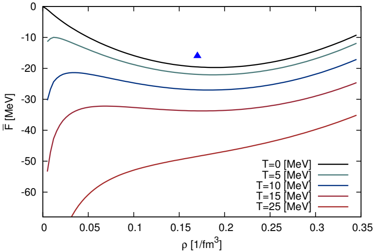


(c) NN second order, 3N first order

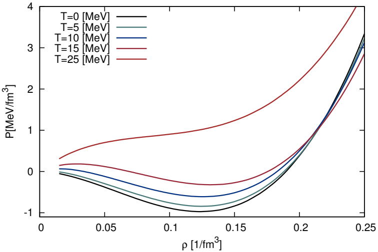


(d) NN second order, 3N second order

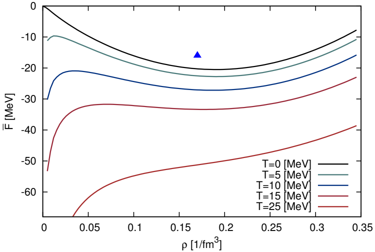
Free Energy per Nucleon and Pressure Isotherms (I)



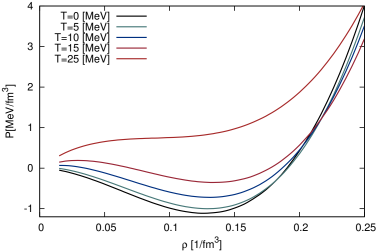
(a) Free energy per particle for n3lo414



(b) Pressure isotherms for n3lo414

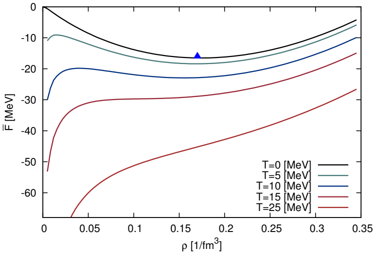


(c) Free energy per particle for n3lo450

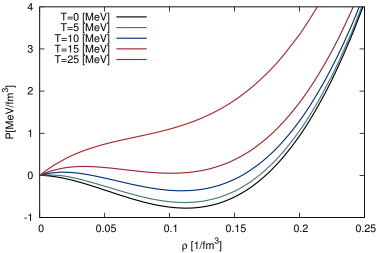


(d) Pressure isotherms for n3lo450

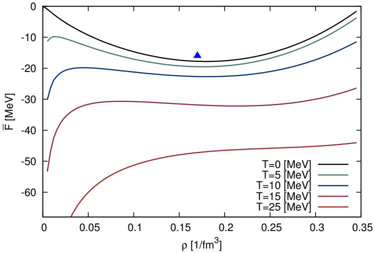
Free Energy per Nucleon and Pressure Isotherms (II)



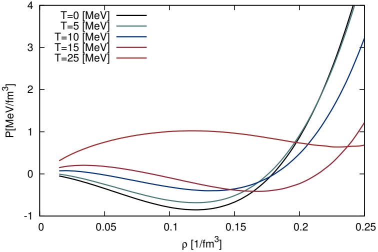
(e) Free energy per particle for n3lo500



(f) Pressure isotherms for n3lo500

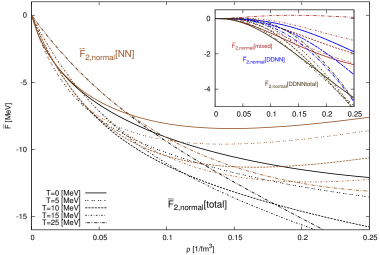


(g) Free energy per particle for VLK21

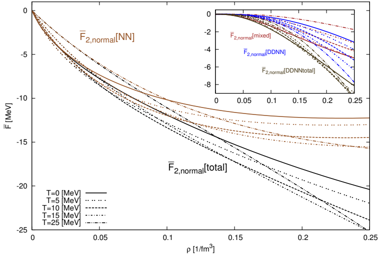


(h) Pressure isotherms for VLK21

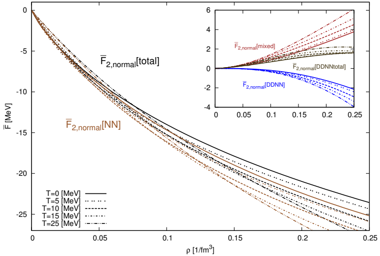
Second-Order Normal Contributions



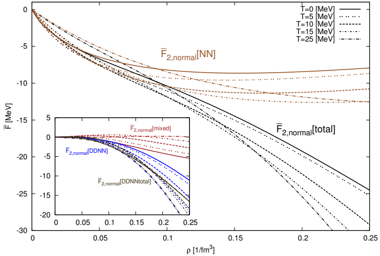
(a) (2,normal)[NN+DDNN] for n3lo414



(b) (2,normal)[NN+DDNN] for n3lo450



(c) (2,normal)[NN+DDNN] for n3lo500



(d) (2,normal)[NN+DDNN] for VLK21

Summary

Summary

- Zero-Temperature EoS:

  Relatively good model-independence, for n3lo500 best agreement with empirical saturation point , $\bar{E}_0 = -16.50$ MeV, $\rho_0 = 0.174\ \mathrm{fm}^{-3}$ and compression modulus, $K = 250$ MeV

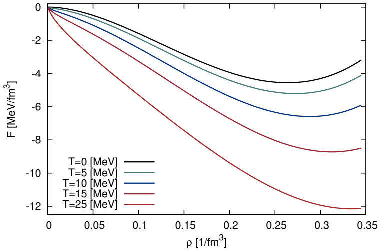
- Thermodynamical EoS:

  Reasonable results only for n3lo-potentials, for VLK21 (and VLK23) pressure isotherm crossing caused by large 2nd-order normal DDNN contributions

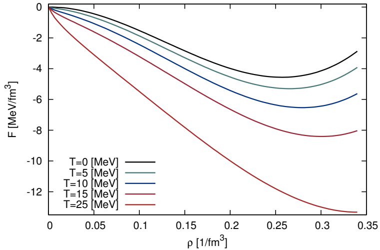
  $\Rightarrow$ Nijmegen LECs do not work for many-nucleon system in 2nd-order calculation

Appendix

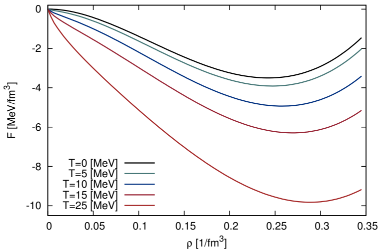
Free Energy Density (Non-Convex ⇒ Phase Coexistence)



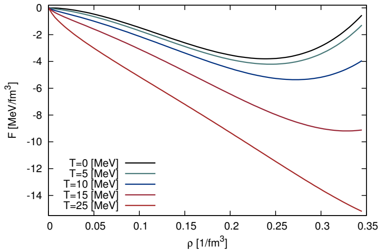
(a) Free energy density for n3lo414



(b) Free energy density for n3lo450

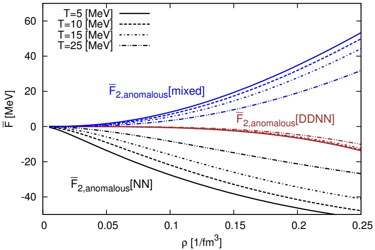


(c) Free energy density for n3lo500

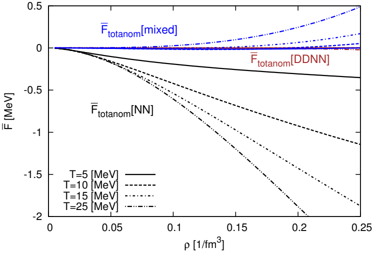


(d) Free energy density for VLK21

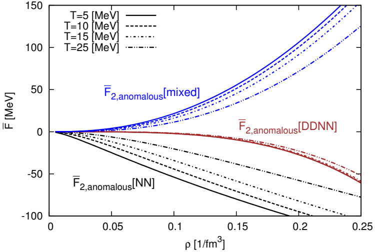
Second-Order Anomalous Contributions



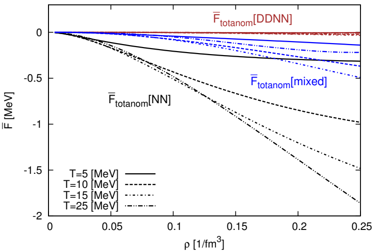
(a) (2,anomalous) for n3lo500



(b) (2,anomalous)+ADT for n3lo500



(c) (2,anomalous) for VLK21



(d) (2,anomalous)+ADT for VLK21