Results for YN at NLO

Johann Haidenbauer

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Outline

1. Introduction
2. *YN* in chiral effective field theory
3. *YN* Results

Chiral Effective Field Theory

- *NN* interaction
  - S. Weinberg, PLB 251 (1990) 288; NPB 363 (1991) 3
  - C. Ordoñez, L. Ray, and U. van Kolck, PRC 53 (1996) 2086
  - J.L. Friar, S.A. Coon
  - C.A. da Rocha, R. Higa, M.R. Robilotta
  - N. Kaiser, R. Brockmann, S. Gerstendörfer, W. Weise
  - D.B. Kaplan, M.J. Savage, M.B. Wise
  - .
  - D.R. Entem and R. Machleidt, PRC 68 (2003) 041001 (N^3LO)
  - E. Epelbaum, W. Glöckle, U.-G. Meißner, NPA 747 (2005) 362 (N^3LO)
- *YN* interaction
  - C.K. Korpa et al., PRC 65 (2001) 015208
  - S.R. Beane et al., NPA 747 (2005) 55
  - H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 24
- *YN* interaction at NLO
  - J.H., N. Kaiser, U.-G. Meißner, A. Nogga, S. Petschauer, W. Weise, in preparation

YN in chiral effective field theory

We follow the scheme of S. Weinberg
in complete analogy to the work of E. Epelbaum et al.

Lowest order (LO): $\nu = 0$

a) non-derivative four-baryon contact terms

b) one-meson (Goldstone boson) exchange diagrams

NLO: $\nu = 2$

a) four-baryon contact terms with two derivates

b) two-meson (Goldstone boson) exchange diagrams

YN in chiral effective field theory

Advantages:

- Power counting
  systematic improvement by going to higher order
- Possibility to derive two- and three baryon forces and external current operators in a consistent way

Obstacle: YN data base is rather poor
($\rightarrow$ impose $SU(3)_f$ constraints)

YN data

- about 35 data points, all from the 1960s
- 10 new data points, from the KEK-PS E251 collaboration (from $\gtrsim$ 2000)
- constraints from hypernuclei

Contact terms for BB

e.g., LO contact terms for BB:

$$\mathcal{L} = C_i \left(\bar{N}\Gamma_i N\right)\left(\bar{N}\Gamma_i N\right) \quad \Rightarrow \quad \begin{aligned} \mathcal{L}^1 &= \tilde{C}^1_i \left\langle \bar{B}_a \bar{B}_b \left(\Gamma_i B\right)_b \left(\Gamma_i B\right)_a \right\rangle , \\ \mathcal{L}^2 &= \tilde{C}^2_i \left\langle \bar{B}_a \left(\Gamma_i B\right)_a \bar{B}_b \left(\Gamma_i B\right)_b \right\rangle , \\ \mathcal{L}^3 &= \tilde{C}^3_i \left\langle \bar{B}_a \left(\Gamma_i B\right)_a \right\rangle \left\langle \bar{B}_b \left(\Gamma_i B\right)_b \right\rangle \end{aligned}$$

$$B = \left(\begin{array}{ccc} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{array} \right)$$

a, b ... Dirac indices of the particles

$$\Gamma_1 = 1\,,\ \Gamma_2 = \gamma^\mu\,,\ \Gamma_3 = \sigma^{\mu\nu}\,,\ \Gamma_4 = \gamma^\mu\gamma_5\,,\ \Gamma_5 = \gamma_5$$

C_i, $\tilde{C}_i$ ... low-energy coefficients spin-momentum structure of the contact term potential:

BB contact terms without derivatives (LO):

$$V^{(0)}_{BB\to BB} = C_{S,BB\to BB} + C_{T,BB\to BB}\,\vec\sigma_1\cdot\vec\sigma_2$$

BB contact terms with two derivatives (NLO):

$$\begin{aligned} V^{(2)}_{BB\to BB} &= C_1\vec q^{\,2} + C_2\vec k^{\,2} + (C_3\vec q^{\,2} + C_4\vec k^{\,2})(\vec\sigma_1\cdot\vec\sigma_2) \\ &+ iC_5(\vec\sigma_1+\vec\sigma_2)\cdot(\vec q\times\vec k) + C_6(\vec q\cdot\vec\sigma_1)(\vec q\cdot\vec\sigma_2) \\ &+ C_7(\vec k\cdot\vec\sigma_1)(\vec k\cdot\vec\sigma_2) + iC_8(\vec\sigma_1-\vec\sigma_2)\cdot(\vec q\times\vec k) \end{aligned}$$

note: $C_i \to C_{i,BB\to BB}$

$\vec q = \vec p\,' - \vec p$; $\vec k = (\vec p\,' + \vec p)/2$

$SU(3)$ symmetry

10 independent spin-isospin channels in NN and YN (for L=0)
(NN (I=0), NN (I=1), ΛN, ΣN (I=1/2), ΣN (I=3/2), $\Lambda N \leftrightarrow \Sigma N$)

$\Rightarrow$ in principle (at LO), 10 low-energy constants

$SU(3)$ symmetry $\Rightarrow$ only 5 independent low-energy constants

$SU(3)$ structure for scattering of two octet baryons:
direct product:

$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$

$C_{S,i}$, $C_{T,i}$, $C_{1,i}$, etc., can be expressed by the coefficients corresponding to the $SU(3)_f$ irreducible representations:
C^1, C^{8_a}, C^{8_s}, C^{10^*}, C^{10}, C^{27}

$SU(3)$ structure of contact terms for BB

	Channel	I	V_{1S0}, $V_{3P0,3P1,3P2}$	V_{3S1}, $V_{3S1-3D1}$, V_{1P1}	$V_{1P1-3P1}$
$S = 0$	$NN \to NN$	0	–	C^{10^*}	–
	$NN \to NN$	1	C^{27}	–	–
$S = -1$	$\Lambda N \to \Lambda N$	$\frac{1}{2}$	$\frac{1}{10}\left(9C^{27} + C^{8_s}\right)$	$\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$	$\frac{-1}{\sqrt{20}} C^{8_s 8_a}$
	$\Lambda N \to \Sigma N$	$\frac{1}{2}$	$\frac{3}{10}\left(-C^{27} + C^{8_s}\right)$	$\frac{1}{2}\left(-C^{8_a} + C^{10^*}\right)$	$\frac{3}{\sqrt{20}} C^{8_s 8_a}$
					$\frac{-1}{\sqrt{20}} C^{8_s 8_a}$
	$\Sigma N \to \Sigma N$	$\frac{1}{2}$	$\frac{1}{10}\left(C^{27} + 9C^{8_s}\right)$	$\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$	$\frac{3}{\sqrt{20}} C^{8_s 8_a}$
	$\Sigma N \to \Sigma N$	$\frac{3}{2}$	C^{27}	C^{10}	–

Number of contact terms:

NN: 2 (LO) 7 (NLO)
YN: +3 (LO) +11 (NLO)
YY: +1 (LO) + 4 (NLO)

$\Rightarrow C^1$ contributes only to $I = 0$, $S = -2$ channels!!

Pseudoscalar-meson exchange

SU(3)$_f$-invariant pseudoscalar-meson-baryon interaction Lagrangian:

$$\mathcal{L} = -\left\langle \frac{g_A(1-\alpha)}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5\{\partial_\mu P, B\} + \frac{g_A\alpha}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5[\partial_\mu P, B]\right\rangle$$

$$f = g_A/(2F_\pi); \quad g_A \simeq 1.26, \quad F_\pi = 92.4 \text{ MeV}$$

$$\alpha = F/(F+D) \text{ with } g_A = F + D$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

$$\begin{aligned}
f_{NN\pi} &= f & f_{NN\eta_8} &= \frac{1}{\sqrt{3}}(4\alpha-1)f & f_{\Lambda NK} &= -\frac{1}{\sqrt{3}}(1+2\alpha)f \\
f_{\Xi\Xi\pi} &= -(1-2\alpha)f & f_{\Xi\Xi\eta_8} &= -\frac{1}{\sqrt{3}}(1+2\alpha)f & f_{\Xi\Lambda K} &= \frac{1}{\sqrt{3}}(4\alpha-1)f \\
f_{\Lambda\Sigma\pi} &= \frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Sigma\Sigma\eta_8} &= \frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Sigma NK} &= (1-2\alpha)f \\
f_{\Sigma\Sigma\pi} &= 2\alpha f & f_{\Lambda\Lambda\eta_8} &= -\frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Xi\Sigma K} &= -f
\end{aligned}$$

Pseudoscalar-meson (boson) exchange

- One-pseudoscalar-meson exchange (V^{OBE}) [LO]

$$V^{OBE}_{B_1 B_2 \to B'_1 B'_2} = -f_{B_1 B'_1 P} f_{B_2 B'_2 P} \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^{\,2} + m_P^2}$$

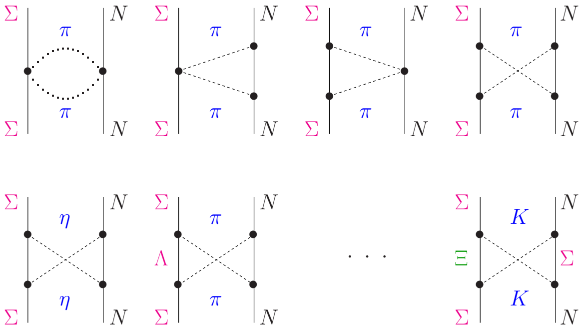
$f_{B_1 B'_1 P}$ ... coupling constants
m_P ... mass of the exchanged pseudoscalar meson

- dynamical breaking of $SU(3)$ symmetry due to the mass splitting of the ps mesons
 (m_π = 138.0 MeV, m_K = 495.7 MeV, m_η = 547.3 MeV)
 taken into account already at LO!

Details:

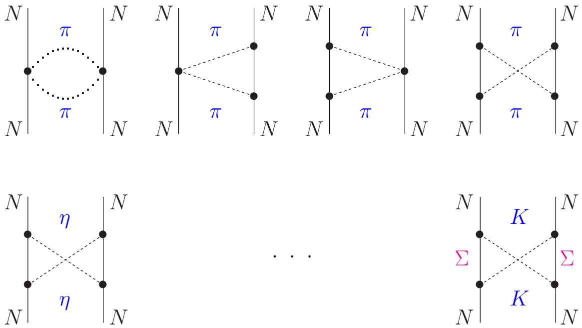
(H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 244; PLB 653 (2007) 29)

- Two-pseudoscalar-meson exchange diagrams (V^{TBE}) [NLO]



Two-pseudoscalar-meson exchange diagrams

- Two-pseudoscalar-meson exchange diagrams (V^{TBE}) [NLO]



Coupled channels Lippmann-Schwinger Equation

$$
\begin{aligned}
T_{\rho'\rho}^{\nu'\nu,J}(p',p) &= V_{\rho'\rho}^{\nu'\nu,J}(p',p) \\
&+ \sum_{\rho'',\nu''} \int_0^\infty \frac{dp'' p''^2}{(2\pi)^3} V_{\rho'\rho''}^{\nu'\nu'',J}(p',p'') \frac{2\mu_{\nu''}}{p^2 - p''^2 + i\eta} T_{\rho''\rho}^{\nu''\nu,J}(p'',p)
\end{aligned}
$$

$\rho', \ \rho = \Lambda N, \ \Sigma N$

LS equation is solved for particle channels (in momentum space)

Coulomb interaction is included via the Vincent-Phatak method

The potential in the LS equation is cut off with the regulator function:

$$
V_{\rho'\rho}^{\nu'\nu,J}(p',p) \to f^\Lambda(p') V_{\rho'\rho}^{\nu'\nu,J}(p',p) f^\Lambda(p); \quad f^\Lambda(p) = e^{-(p/\Lambda)^4}
$$

consider values Λ = 500 - 700 MeV

Pseudoscalar-meson exchange

- All one- and two-pseudoscalar-meson exchange diagrams are included
- $SU(3)$ symmetry is broken by using the physical masses of the pion, kaon, and eta
- $SU(3)$ breaking in the coupling constants is ignored
  $F_\pi = F_K = F_\eta = F_0 = 92.4$ MeV
- assume that $\eta \equiv \eta_8$ (i.e. $\theta_P = 0^o$ and $f_{BB\eta_1} \equiv 0$)
- assume that $\alpha = F/(F+D) = 2/5$
  (semi-leptonic decays $\Rightarrow \alpha \approx 0.364$)
- Correction to V^{OBE} due to baryon mass differences are ignored
- (A fit of comparable quality with two-pion-meson exchange diagrams is possible!)

Correction to V^{OBE} due to baryon mass differences:

$m_P^2 \Rightarrow m_P^2 - \Delta M^2$ in

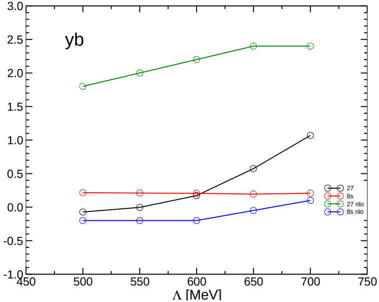
$$\begin{aligned}
\pi-\text{exchange in } \Sigma N \to \Lambda N: &\quad \Delta M^2 = ((M_\Lambda - M_\Sigma)/2)^2 \\
K-\text{exchange in } \Sigma N \to \Lambda N: &\quad \Delta M^2 = ((M_\Lambda + M_\Sigma)/2 - M_N)^2 \\
K-\text{exchange in } \Sigma N \to \Sigma N: &\quad \Delta M^2 = (M_\Sigma - M_N)^2 \\
K-\text{exchange in } \Lambda N \to \Lambda N: &\quad \Delta M^2 = (M_\Lambda - M_N)^2
\end{aligned}$$

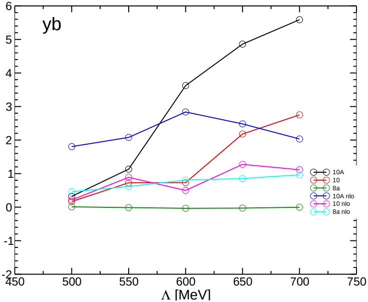
Contact terms

- $SU(3)$ symmetry is assumed
- (at NLO $SU(3)$ breaking corrections to the LO contact terms arise!)
- no SU(3) constraints from the NN sector are imposed
- 10 contact terms in S-waves
- 12 contact terms in P-waves and in ${}^3S_1 - {}^3D_1$
- 1 contact term in ${}^1P_1 - {}^3P_1$ (singlet-triplet mixing) is set to zero

- contact terms in S-waves: can be fairly well fixed from data
- some correlations between NLO and LO LECs

Contact terms





Contact terms in P-waves

- contact terms in P-waves are much less constrained
- at present they are fixed from “bulk” properties:

  (1) $\sigma_{\Lambda p} \approx 10$ mb at $p_{lab} \approx 700 - 900$ MeV/c

  (2) $d\sigma/d\Omega_{\Sigma^- p \to \Lambda n}$ at $p_{lab} \approx 135 - 160$ MeV/c

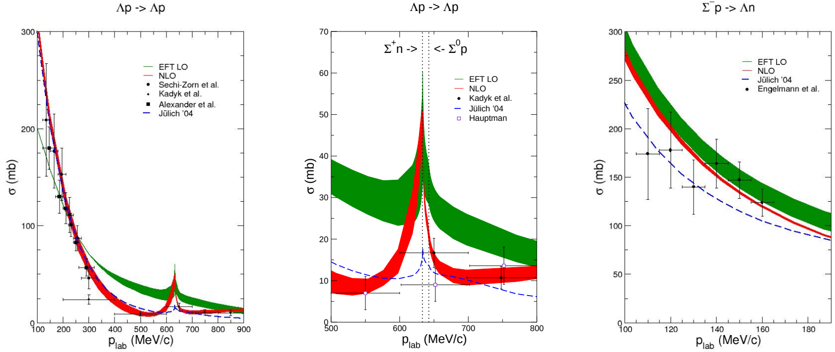
Other (further) options:

- use SU(3) and fix at least some LECs from NN
- consider matter properties:

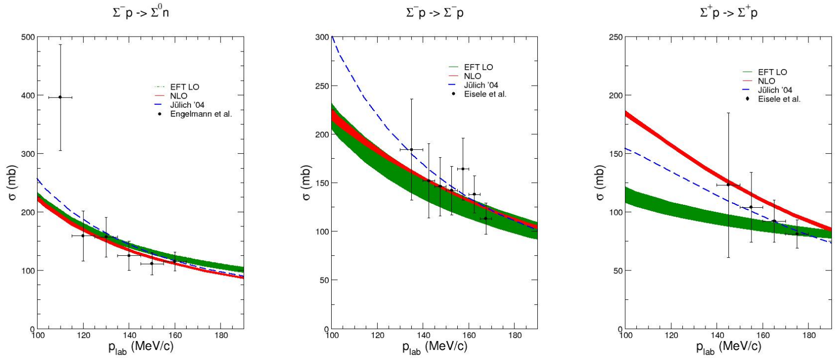
  use spin-orbit splitting of the Λ single particle levels in nuclei

  Consider the Scheerbaum factor S_Λ calculated in nuclear matter to relate the strength of the Λ-nucleus spin-orbit potential to the two body ΛN interaction
  (R.R. Scheerbaum, NPA 257 (1976) 77)

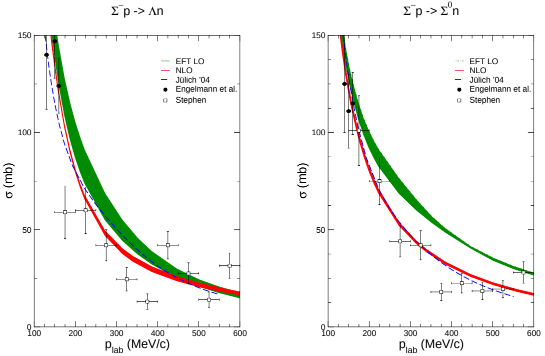
YN integrated cross sections



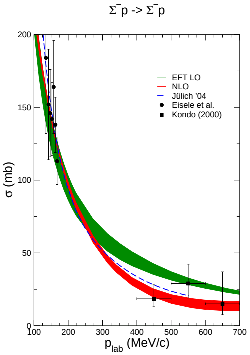
YN integrated cross sections

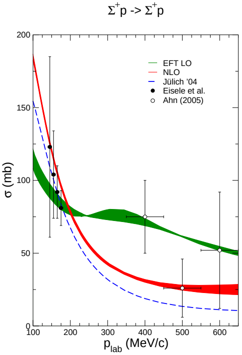


YN integrated cross sections

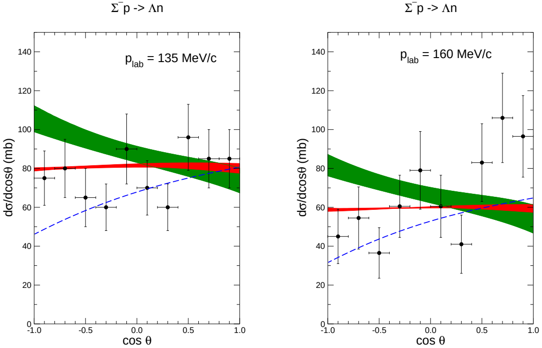


YN integrated cross sections

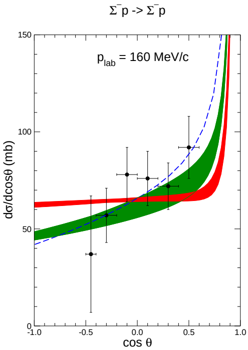


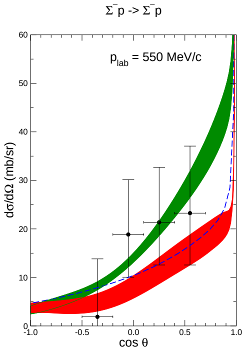


YN differential cross sections

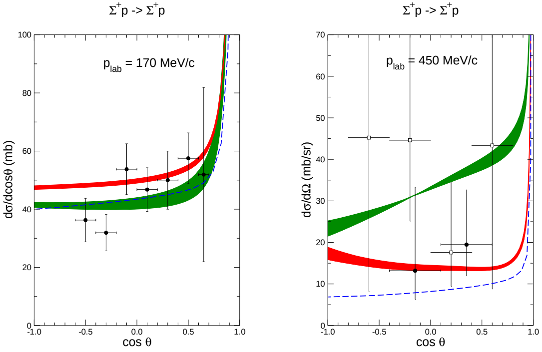


YN differential cross sections





YN differential cross sections



YN scattering lengths [fm]

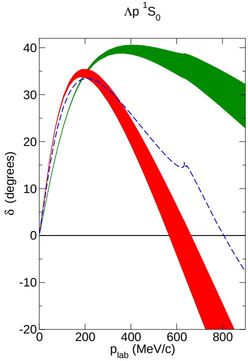
	EFT LO	EFT NLO	Jülich '04	NSC97f	experiment*
Λ [MeV]	$550 \cdots 700$	$500 \cdots 700$			
$a_s^{\Lambda p}$	$-1.90 \cdots -1.91$	$-2.88 \cdots -2.89$	-2.56	-2.51	$-1.8^{+2.3}_{-4.2}$
$a_t^{\Lambda p}$	$-1.22 \cdots -1.23$	$-1.59 \cdots -1.61$	-1.66	-1.75	$-1.6^{+1.1}_{-0.8}$
$a_s^{\Sigma^+ p}$	$-2.24 \cdots -2.36$	$-3.90 \cdots -3.83$	-4.71	-4.35	
$a_t^{\Sigma^+ p}$	$0.70 \cdots 0.60$	$0.51 \cdots 0.47$	0.29	-0.25	
χ^2	≈ 30	$18.0 \cdots 24.0$	≈ 25	16.7	
$({}^3_\Lambda\mathrm{H})\ E_B$	$-2.34 \cdots -2.36$	$-2.31 \cdots -2.34$	-2.27	-2.30	-2.354(50)

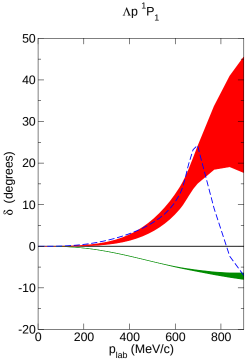
* A. Gasparyan et al., PRC 69 (2004) 034006 $\Rightarrow$ extract from final-state interaction:

$pp \rightarrow K^+\Lambda p$ (COSY, Jülich)

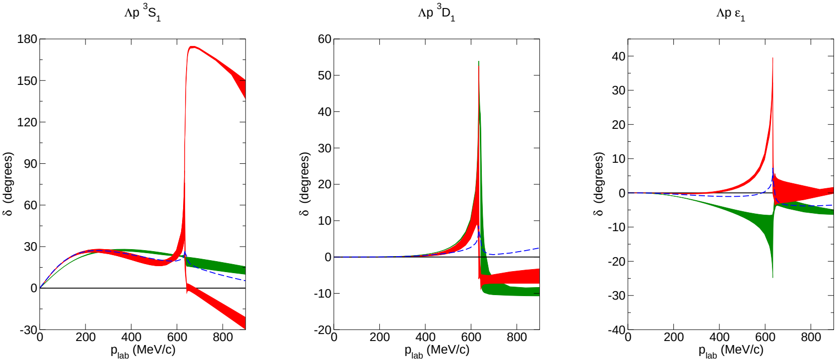
$\gamma d \rightarrow K^+\Lambda n$ (SPring-8)

Λp phase shifts

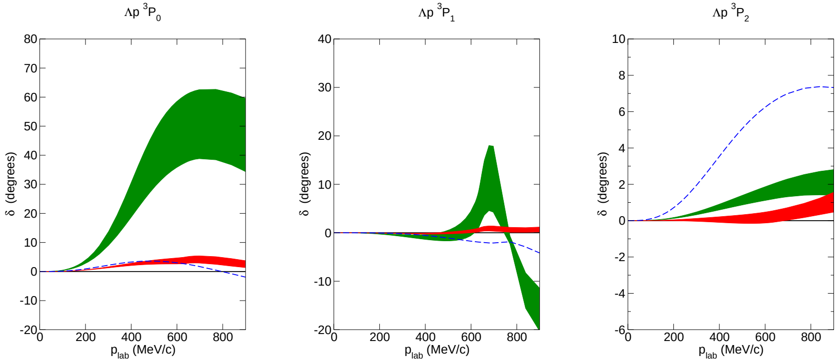




Λp phase shifts



Λp phase shifts



ΣN (I=3/2)

ΣN with maximal isospin ($\Sigma^+ p$, $\Sigma^- n$)

- no coupling to the ΛN system
- $^1S_0$: test for $SU(3)$ symmetry $\longrightarrow V_{np} \equiv V_{\Sigma^- n}$
- $^3S_1 - {}^3D_1$: properties closely linked with the Σ properties in nuclear matter (attractive or repulsive)
- advantageous for lattice simulations
  $\longrightarrow$ S.R. Beane et al. [NPLQCD], arXiv:1204.3606 [hep-lat]

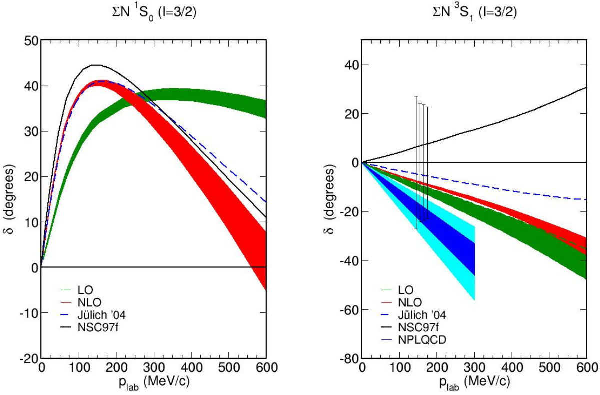
$^1S_0$

- description of pp phase shifts and $\sigma_{\Sigma^+ p}$ with LEC's that fulfill $SU(3)$ symmetry is not possible
- LEC's that are fitted to the pp $^1S_0$ phase shift produce a bound state in $\Sigma^+ p$
  $\rightarrow \sigma_{^1S_0} \approx 4 \times \sigma_{\Sigma^+ p}$
- simultaneous fit is possible if we assume that there is $SU(3)$ breaking in the LO contact term only.

$^3S_1 - {}^3D_1$:

- A description of all YN data is possible with an attractive as well as a repulsive $^3S_1 - {}^3D_1$ interaction
- However, the χ^2 is found to be slightly larger for a repulsive $^3S_1 - {}^3D_1$ interaction

ΣN (I=3/2) phase shifts



Nuclear matter properties

conventional first-order Brueckner calculation:

Partial wave contributions to $-U_\Lambda(p_\Lambda = 0)$ (in MeV) at $k_F = 1.35\ \mathrm{fm}^{-1}$

	$^1S_0$	$^3S_1 + ^3D_1$	$^3P_0$	$^1P_1 + ^3P_1$	$^3P_2 + ^3F_2$	Total
EFT LO	12.0	25.5	1.7	-3.3	0.4	36.5
EFT NLO	12.5	12.4	0.2	-0.7	-0.2	24.4
Jülich '04	9.9	35.0	0.7	0.2	3.3	49.7
Jülich '94	3.6	27.2	-0.6	-2.0	0.8	29.8
NSC97f	14.4	22.9	-0.5	-6.4	0.7	31.1

"Empirical" value for the Λ binding energy in nuclear matter:
$\approx$ 30 MeV

Nuclear matter properties

	EFT LO	EFT NLO	Jülich '04	Jülich '94	NSC97f
Λ [MeV]	$550 \cdots 700$	$500 \cdots 700$			
$-U_\Lambda(0)$	$38.0 \cdots 34.4$	$33.1 \cdots 24.4$	49.7	29.8	31.1
$-U_\Sigma(0)$	$-28.0 \cdots -11.1$	$-16.2 \cdots -7.4$	22.2	71.45	16.1

Summary

YN interaction based on chiral EFT

- approach is based on a modified Weinberg power counting, analogous to the NN case
- The potential (contact terms, pseudoscalar-meson exchanges) is derived imposing $SU(3)_f$ constraints
- Good description of the empirical YN data was achieved already at LO
  (with only 5 free parameters!)

- (Preliminary) Results at next-to-leading order (NLO) look very promising
- YN data are reproduced with a quality comparable to phenomenological models
- $SU(3)$ symmetry for the LEC's can be maintained in the YN system (ΛN, ΣN) but not between YN and NN