# Hadronic electroweak processes in a finite volume

Andria Agadjanov, University of Bonn

Dr. Klaus Erkelenz Kolloquium

November 29, 2016

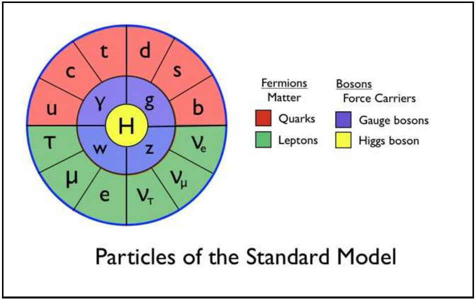
Collaborators: V. Bernard, Ulf-G. Meißner and A. Rusetsky

Bonn-Cologne Graduate School
of Physics and Astronomy

# Outline

- Introduction
- Radiative decays of resonances:
  - the $\Delta N\gamma^*$ transition — Nucl. Phys. B **886** (2014) 1199
  - the $B \to K^*(892)l^+l^-$ decay — Nucl. Phys. B **910** (2016) 387
- Compton scattering at low energies — arXiv:1610.05545
- Summary and Outlook

# Standard Model

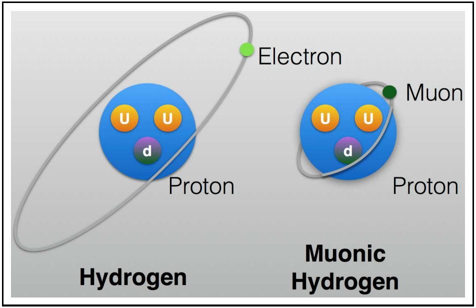



- Standard Model is a triumph of theory
- Higgs particle - a great discovery of the $21^{st}$ century Nobel Prize 2013

↪along with gravitational waves by LIGO and Virgo

Electroweak sector × Strong sector

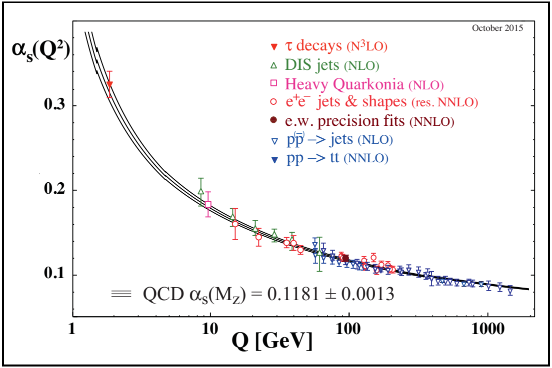
# Experimental tensions




- ▷ Proton radius puzzle
- ▷ Anomalous magnetic moment of muon ("muon g−2")
- ▷ Anomalies in $B$ meson decays
- Strong sector of SM has to be better understood!

# Strong interaction

- Quantum chromodynamics (QCD): non-abelian gauge theory

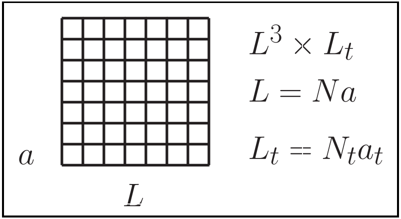



The measurements of $\alpha_s$ as a function of the energy scale $Q$

- High energies → Asymptotic Freedom — Nobel Prize 2004
- Low energies ($\lesssim$ 1 GeV) → Confinement
- Perturbation theory breaks down → lattice QCD, disp. relations,. . .

# Lattice QCD

- Path integral formulation in Euclidean space-time




- Space-time is discretized and finite $\Rightarrow$ natural UV cut-off $\sim 1/a$

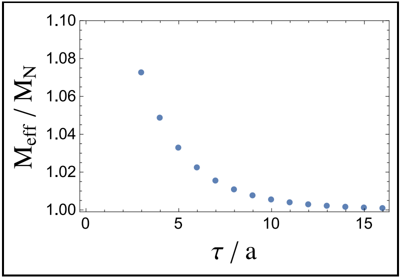
  K. G. Wilson, Phys. Rev. D **10** (1974) 2445

- Integration $\rightarrow$ Monte Carlo methods

- Correlation functions $\rightarrow$ energy levels, current matrix elements

FLAG: S. Aoki *et al.*, arXiv:1607.00299 (2016)

# Stable hadrons




The nucleon two-point function

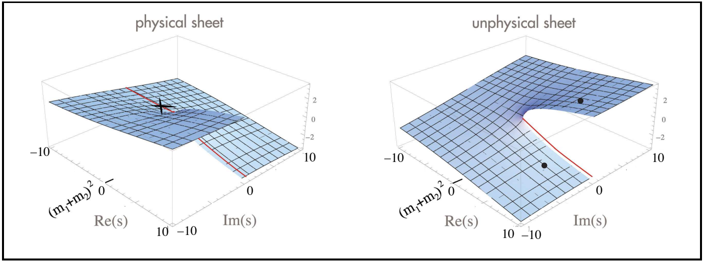
- Two-point correlation function:

$$C(\tau) = \sum_{\mathbf{x}} \langle 0 | O_N(\mathbf{x}, \tau) O_N^\dagger(\mathbf{x}, 0) | 0 \rangle = |Z_O|^2 e^{-M_N \tau} \left[ 1 + \sum_n |Z_n|^2 e^{-\Delta E_n \tau} \right]$$

  ▷ $O_N(\mathbf{x}, \tau)$ - nucleon field, $Z_O$, $Z_n$ - overlap factors, $\Delta E_n > 0$

- Effective mass: $M_{\mathrm{eff}}(\tau) = -\frac{1}{a} \log \frac{C(\tau+a)}{C(\tau)}$, $\quad M_{\mathrm{eff}}(\tau \to \infty) = M_N$

# Resonances




Riemann sheets on the complex $s$-plane

- Energy levels → do not correspond to any resonance
- Resonances → complex poles of the scattering amplitude:

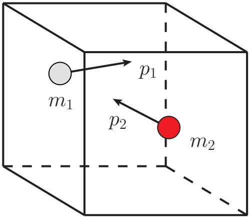
$$E_R = m_R - i\frac{\Gamma_R}{2}, \quad s_R = E_R^2$$

▷ $m_R$ - mass, $\Gamma_R$ - width PDG: Chin. Phys. C **40**, 100001 (2016)

# Lüscher method

- A standard tool to study resonances on the lattice
- scattering phase shift $\leftrightarrow$ finite volume energy spectrum

M. Lüscher, Nucl. Phys. B **354** (1991) 531.




$$\cot\delta(p_n) + \cot\phi(q_n) = 0 \quad \text{(Lüscher equation)}$$

no partial − wave mixing

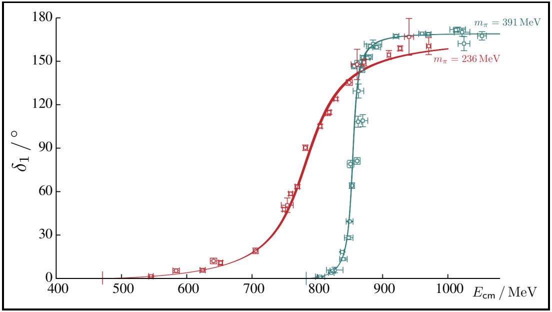
$$p^2 = \lambda(s, m_1^2, m_2^2)/4s, \quad q = \frac{pL}{2\pi}$$

$$\cot\phi(q) = -\frac{1}{2\pi^2 q}\sum_{\mathbf{n}\in\mathbf{Z^3}}\frac{1}{\mathbf{n}^2 - q^2} \quad (l = 0,\ m = 0)$$

▷ energy levels → Lüscher equation → scattering phase

▷ effective-range expansion (ERE) → resonance pole position $E_R$

# An example: $\rho$ resonance



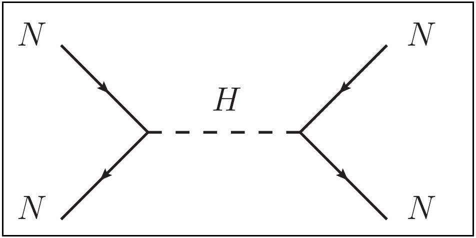

Source: Hadron Spectrum Collaboration, Phys. Rev. D **92** (2015) 094502

| $m_\pi$ [MeV] | $m_R$ [MeV] | $\Gamma_R$ [MeV] |
|---|---|---|
| 236 | 783(2) | 85(2) |
| 391 | 853(2) | 12.4(6) |

▷ I=1, P-wave $\pi\pi$ scattering phase shift

▷ rapid change of phase shift → resonance

# Work of K. Erkelenz



The nucleon-nucleon interaction

- $M = \pi,\ \sigma,\ \rho,\ \omega, \ldots \ \rightarrow$ one-boson exchange models

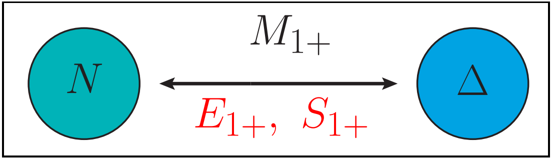
K. Erkelenz, Phys. Rept. **13** (1974) 191

- Bonn Model: R. Machleidt, *et al.*, Phys. Rept. **149** (1987) 1

- Resonances play a crucial role!

# Radiative decays of resonances

# $\Delta N \gamma^*$ transition




- Amplitudes $E_{1+},\ S_{1+}$ are small: $E_{1+},\ S_{1+} \ll M_{1+}$

R. Beck et al., Phys. Rev. Lett. **78** (1997) 606

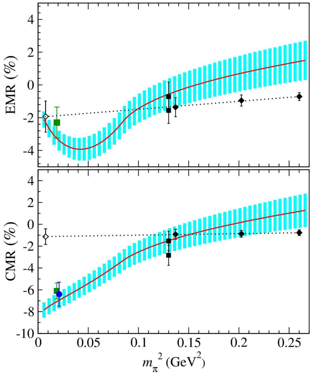
- Form factors $\rightarrow$ the transition probability (transverse densities)

- The recent lattice measurement (with a stable $\Delta$) :

C. Alexandrou et al., Phys. Rev. D **83** (2011) 014501

$$\mathrm{EMR} \equiv \frac{\mathrm{Im} E_{1+}}{\mathrm{Im} M_{1+}}, \quad \mathrm{CMR} \equiv \frac{\mathrm{Im} S_{1+}}{\mathrm{Im} M_{1+}}$$

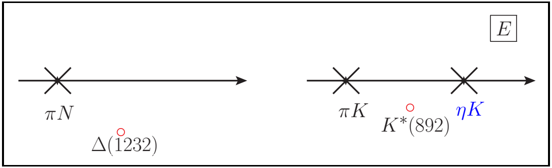
# Experiment vs. Lattice




Source: V. Pascalutsa, M. Vanderhaeghen, Phys. Rev. D **73** (2006) 034003

▷ the chiral extrapolation is not reliable for $m_\pi \gtrsim 300$ MeV

# Theoretical issues

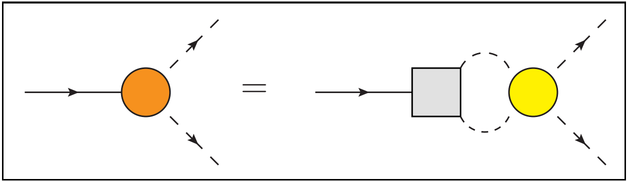



- $\Delta$ and $K^*$ are resonances $\rightarrow$ Lüscher method
- Lattice data might be affected by another threshold ($\eta K$)
- Resonance matrix elements should be properly defined
- Form factors: real axis vs. complex resonance pole

# Electroweak processes

- Seminal work on $K \to \pi\pi$ by

  L. Lellouch and M. Lüscher, Commun.Math.Phys. **219**, 31 (2001)



Lellouch-Lüscher formula

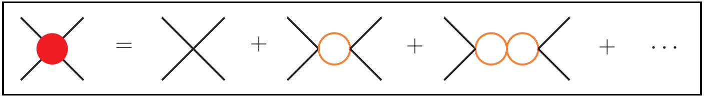
$$|A(K \to \pi\pi)| \propto |\langle \pi\pi | H_{\mathrm{weak}} | K \rangle| \times \left( \frac{p^2}{\frac{d\delta(p)}{dp} + \frac{d\phi(q)}{dp}} \right)^{-1/2}$$

- The range of applicability:

  large volumes ($m_\pi L \gtrsim 4$), below 3(4)-particle threshold

# The framework: non-relativistic EFT

- Ideally suited for the problems we study



Bubble-chain diagrams

$$T \propto 1 + cG + c^2G^2 + \cdots = \frac{1}{1 - cG}$$

J. Gasser, B. Kubis and A. Rusetsky, Nucl. Phys. B **850** (2011) 96

- In a finite volume: $G \to G_L$ (the loop function)
- A bridge: finite volume spectrum $\leftrightarrow$ scattering sector

# Form factors

- on the real energy axis → model- and process-dependent

$$|\mathrm{Im}\,\mathcal{A}(E_{BW},|\mathbf{q}|)| = \sqrt{\frac{8\pi}{p_{BW}\Gamma}}|F_A(E_{BW},|\mathbf{q}|)|$$

▷ $F_A(E_{BW},|\mathbf{q}|)$ → current matrix elements, $\Gamma$ – resonance width

- at the resonance pole → process-independent ⇒ favourable!

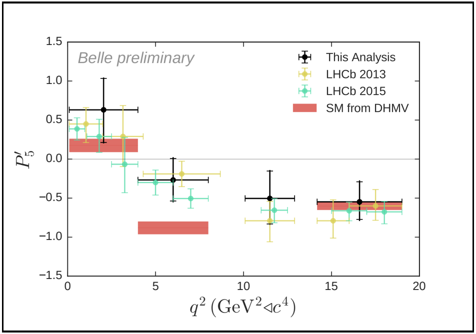
$$\langle P,\,\mathrm{resonance}|J(0)|Q,\,\mathrm{stable}\rangle = \lim_{P^2\to s_R,\,Q^2\to M^2} Z_R^{-1/2} Z^{-1/2}(s_R-P^2)(M^2-Q^2)F(P,Q)$$

▷ $F(P,Q)$ – 3-point Green's function

▷ $Z$, $Z_R$ – wave-function renormalization constants

generalization of: S. Mandelstam, Proc. Roy. Soc. Lond. A **233** (1955) 248

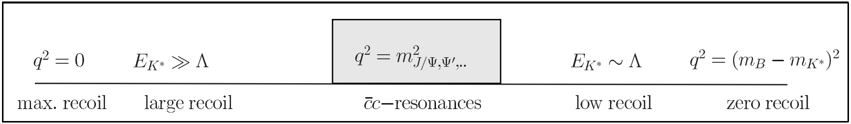
# $B \to K^*(892) l^+ l^-$




LHCb and Belle data on the observable $P'_5$

A. Abdesselam et al., arXiv:1604.04042 [hep-ex]

- Example: observable $P'_5 \to 3\sigma$ deviation from Standard Model
- Observed tensions $\to$ in same Wilson coefficients ($C_7, C_9, C_{10}$)




Source: C. Hambrock et al., Phys. Rev. D 89 (2014), 074014

- Pattern of deviations observed by LHCb and Belle Collaboration

BSM or QCD?

- Hadronic uncertainties: form factors, long-distance effects
- In the low recoil region, lattice QCD simulations are reliable

# Low recoil

- For $q^2 \gg \Lambda_{QCD}$, the decay amplitude $A$ is approximately

$$A \approx \sum_{M=1}^{7} c_M(C_7, ...) f^M(q^2)$$

$\hookrightarrow$ see, however, J. Lyon and R. Zwicky, arXiv:1406.0566 [hep-ph]

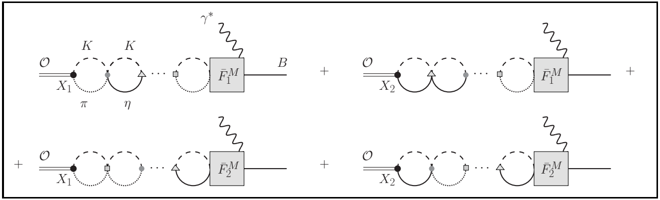
- Seven semileptonic form factors $f^M(q^2), M = 1, ...7$,

$$\langle K^*(k,\lambda)|\bar{s}\gamma^\mu b|B(p)\rangle = \frac{2iV(q^2)}{m_B + m_V}\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu k_\rho p_\sigma, \quad \text{etc.}$$

- The first unquenched lattice calculation (with a stable $K^*$) :

R. R. Horgan et al., Phys. Rev. D **89**, 094501 (2014)

# Lellouch-Lüscher formula for $B \to K^*(892) l^+ l^-$



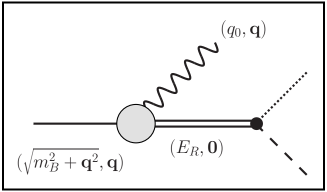

$$|F^M(E_n, |\mathbf{q}|)| = \frac{\mathcal{V}^{-1}}{8\pi E} \left| v_1 \, \bar{F}_1^M + v_2 \, \bar{F}_2^M \right| \Big|_{E=E_n}$$

- ▷ $\mathcal{V}$ – (asymmetric) volume
- ▷ $v_1 = v_1(E, L)$, $v_2 = v_2(E, L)$ – known functions (gener. of LL factor)
- ▷ $F^M(E_n, |\mathbf{q}|) \to$ current matrix elements, measured on the lattice

- Two-particle irreducible vertices $\bar{F}_1^M(E_n, |\mathbf{q}|)$, $\bar{F}_2^M(E_n, |\mathbf{q}|) \to$

$\to$ decay amplitudes $\mathcal{A}^M(B \to \pi K l^+ l^-)$ | Watson's theorem, agrees with S. Sharpe and M. Hansen, Phys. Rev. D **86** (2012) 016007

# Form factors at the $K^*$ pole




- Suppose, the $K^*$ pole is on the Riemann sheet *II*
- Analytic continuation: vary $E$ with $|\mathbf{q}|$ fixed (analog of the ERE)

$$F_R^M(E_R, |\mathbf{q}|) = -\frac{i}{8\pi E}\left(w_1 \bar{F}_1^M - w_2 \bar{F}_2^M\right)\Big|_{E=E_R}$$

▷ $w_1 = w_1(E)$, $w_2 = w_2(E)$ – volume-independent quantities

- Form factors at the pole → complex

# Infinitely narrow width

- Results are simplified in the limit $\Gamma \to 0$ ($K^*$ is above $\eta K$)
- Assume the Breit-Wigner form in the vicinity of $E = E_{BW}$
- Real axis:

$$\left|F^M(E_n, |\mathbf{q}|)\right| = \frac{\mathcal{V}^{-1}}{\sqrt{2E_n}} \left|F_A^M(E_n, |\mathbf{q}|)\right| + O(\Gamma^{1/2}), \quad E_n = E_{BW} + O(\Gamma)$$

- Complex plane ($E_R \to E_{BW}$):

$$F_R^M(E_R, |\mathbf{q}|)\big|_{\Gamma \to 0} = F_A^M(E_{BW}, |\mathbf{q}|) + O(\Gamma^{1/2})$$

$\hookrightarrow$ similar to the one-channel case of the $\Delta N \gamma^*$ transition:

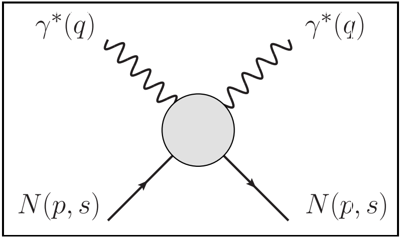
AA, V. Bernard, U.-G. Meißner and A. Rusetsky, Nucl. Phys. B **886** (2014) 1199

# Summary

- Extraction of the $B \to K^*$ form factors on the lattice is studied
- Possible admixture of the $\eta K$ to $\pi K$ channels is taken into account
- Equation for the $B \to K^* l^+ l^-$ amplitude at low recoil is derived
- Form factors at the $K^*$ pole are determined
- Infinitely-narrow width approximation of the results is considered
- Similar work on the $\Delta N \gamma^*$ transition: Nucl. Phys. B **886** (2014) 1199
- An open issue: long-distance effects $\Leftrightarrow$ non-local matrix elements

$$T^\mu \propto i \int d^4 x e^{iqx} \langle K^* | T j^\mu_{\rm em}(x) H_{4q}(0) | B \rangle$$

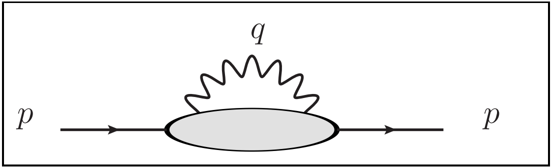
# External fields on the lattice: Compton scattering




Compton scattering

- Two invariant amplitudes: $T_1(\nu, q^2)$ and $T_2(\nu, q^2)$, $\quad \nu \equiv p \cdot q/m$
- Why important?
  - ▷ the proton-neutron mass difference
  - ▷ Lamb shift in the muonic hydrogen (low $q^2$)
  - ▷ the existence of a fixed pole in Regge theory

# Cottingham formula



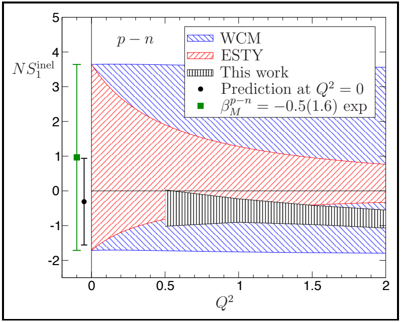

▷ Electromagnetic contribution to the proton-neutron mass difference:

$$(m_p - m_n)_{\rm em} = \frac{ie^2}{2m(2\pi)^4} \int^{\Lambda} d^4q D(q^2)\{3q^2 T_1 + (2\nu^2 + q^2) T_2\} + \text{counter terms}$$

▷ $D(q^2)$ - photon propagator

- Electroproduction cross sections $\rightarrow T_1, T_2 \quad (q^2 < 0)$
- A problem: $S_1(q^2) \equiv T_1(0, q^2)$ is not fixed by experiment

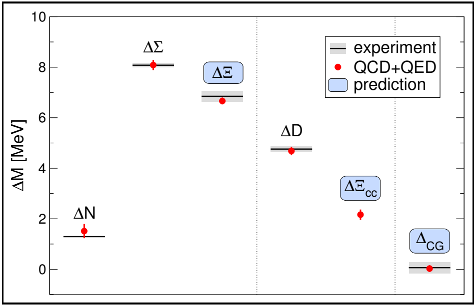
# Theoretical approaches




Source: J. Gasser et al., Eur. Phys. J. C **75**, 375 (2015)

- Chiral EFTs — Hill, Meißner, Pascalutsa, . . .
- Phenomenological Ansatzes — Pachucki, Walker-Loud, . . .
- Reggeon dominance hypothesis — Gasser, Leutwyler, . . .
- Lattice QCD $\rightarrow$ devoid of any model dependence

# Neutron-proton mass splitting



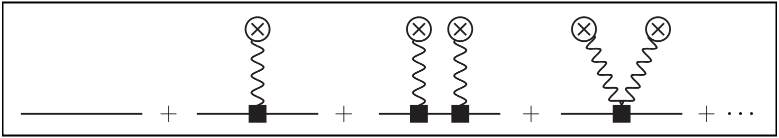

Source: S. Borsanyi et al., Science **347** (2015) 1452

| | $\Delta m$ [MeV] | QCD [MeV] | QED [MeV] |
|---|---|---|---|
| $\Delta N = n - p$ | 1.51(16)(23) | 2.52(17)(24) | -1.00(07)(14) |

- The fully unquenched lattice QCD + QED computation
  - ▷ 4 non-degenerate flavors, $m_\pi \approx 195$ MeV

BMW Collaboration

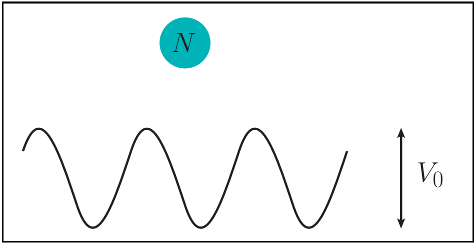
# External field method



Source: W. Detmold et al., Phys. Rev. D **73** (2006) 114505

- Two-point function in an external electromagnetic field $A_\mu(x)$
- $O(A^2)$ term $\rightarrow$ Compton scattering amplitude
- Uniform magnetic field $\rightarrow$ polarizabilities NPLQCD
- Static magnetic field $\rightarrow$ energy levels, measured on the lattice

# Field configuration




- Static periodic magnetic field $\mathbf{B} = (0, 0, B_3)$:

$$A_1 = \frac{B}{\omega}\sin(\omega x_2), \qquad A_0 = A_2 = A_3 = 0$$

- Frequency $\omega \neq 0$ → photon virtuality $q^2 = -\omega^2$
- Magnetic flux is quantized $\Rightarrow \omega = \frac{2\pi N}{L}, \qquad N \in \mathbb{Z}\backslash\{0\}$

# Energy shift

- For $\nu = 0$ and $q^2 < 0 \rightarrow S_1(q^2)$ is real
- If $V_0 = e^2 B^2 / 2m\omega^2$ is "small" $\Rightarrow$ perturbation theory
- The free energy spectrum:

$$w(\mathbf{k}_n) = \sqrt{m^2 + \mathbf{k}_n^2}, \qquad \mathbf{k}_n = \frac{2\pi \mathbf{n}}{L}, \qquad \mathbf{n} \in \mathbb{Z}^3$$

- Spin-averaged energy shift of the ground state ($\mathbf{k}_n = 0$):

$$\delta E = \frac{1}{2} \sum_s \delta E_s = \frac{B^2}{4m} S_1(-\omega^2)$$

- A remarkably simple expression!

# Outlook

- It is planned to test the formula on the lattice
- One can start with Compton scattering off pions
- The limit $\omega \to 0$ should be thoroughly studied
- Disconnected contributions have to be estimated
- Finite-volume effects should be considered

Work Ahead!