Anthropic Considerations in Big Bang Nucleosynthesis

Dr. Klaus Erkelenz Prize Colloquium

Helen Meyer

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Helmholtz-Institut für Strahlen- und Kernphysik

Dr. Klaus Erkelenz

CURRENT STATUS OF THE RELATIVISTIC TWO-NUCLEON ONE BOSON EXCHANGE POTENTIAL

K. ERKELENZ
Institut für Theoretische Kernphysik, Bonn, W.-Germany

Received April 1974

: Erkelenz, Phys. Rept. 1974

- one of the fathers of the Bonn potential for nucleon-nucleon interactions
- Thank you to Frau Dr. Gabriele Erkelenz, to the prize committee for this honor and to Prof. Dr. Dr. Ulf-G. Meißner for being my supervisor!

Motivation

- Fundamental constants: show up in every discipline of science
- We know them to precisions given units of parts per 10^9 [1]

permeability of free space	μ_0	$4\pi \times 10^{-7}$ N A^{-2} = 12.566 370 614 ... $\times 10^{-7}$ N A^{-2}	exact
fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	7.297 352 5664(17)$\times 10^{-3}$ = 1/137.035 999 139(31)†	0.23, 0.23
classical electron radius	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	2.817 940 3227(19)$\times 10^{-15}$ m	0.68
(e^- Compton wavelength)/2π	$\lambda\!\!\!^{-}_e = \hbar/m_e c = r_e \alpha^{-1}$	3.861 592 6764(18)$\times 10^{-13}$ m	0.45
Stefan-Boltzmann constant	$\sigma = \pi^2 k^4/60\hbar^3 c^2$	5.670 367(13)$\times 10^{-8}$ W m^{-2} K^{-4}	2300
Fermi coupling constant**	$G_F/(\hbar c)^3$	1.166 378 7(6)$\times 10^{-5}$ GeV^{-2}	510
weak-mixing angle	$\sin^2\widehat{\theta}(M_Z)$ ($\overline{\text{MS}}$)	0.231 22(4)††	1.7×10^5
$W^\pm$ boson mass	m_W	80.379(12) GeV/c^2	1.5×10^5

- Some theories predict changes in these constants over cosmological time scales

How fine-tuned is our universe?[2,3]

- How can we test this? $\Rightarrow$ Laboratory: Big Bang Nucleosynthesis (BBN)[4]

[1] PDG: Workman et al., 2022; [2] Dirac, 1973 and many others; [3] Meißner, Metsch, HM, 2025; [4] Olive, Steigman, and Walker, 2000; Iocco et al., 2009; Cyburt et al., 2016; Pitrou et al., 2018a

This talk

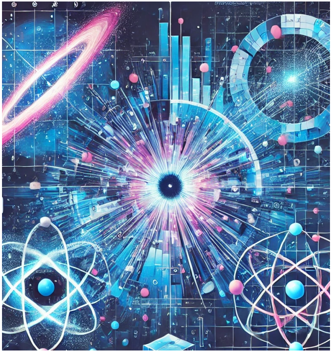
We have studied BBN under variation of

- the electromagnetic coupling constant α [1]
- the Higgs vacuum expectation value[2]
- the strange-quark mass[3]

Goal: find a bound on these variations through comparing calculations with experimental values for light element abundances

- Input from Nuclear Lattice EFT[4]: putting nucleon-nucleon interactions on the lattice

  (see talk by S. Elhatisari in last Dr. Klaus Erkelenz Price colloquium)



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[1] Meißner, Metsch, HM 2023; Bergström, Iguri, Rubenstein, 1999; Nollett, Lopez, 2002; Dent,Stern,Wetterich, 2007; Coc et al., 2007;
[2] Meißner, HM 2024; Burns et al., 2024; [3] Meißner, Metsch, HM 2025, [4] Lähde, Meißner 2019

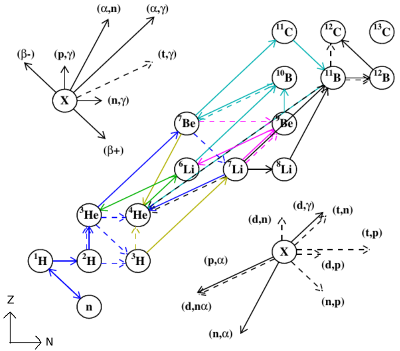
Introducing BBN – Evolution of Abundances

- abundance $Y_i = n_i / n_b$, with n_i density of nucleus i and n_b total baryon density
- Need to solve system of rate equations

$$\dot{Y}_i \supset -Y_i \Gamma_{i\to\ldots} + Y_j \Gamma_{j\to i+\ldots} + Y_k Y_l \Gamma_{kl\to ij} - Y_i Y_j \Gamma_{ij\to kl}$$

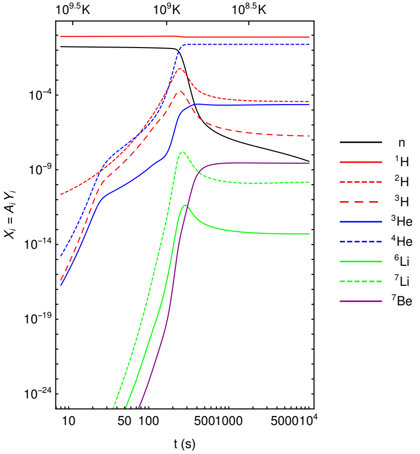
- Used different codes[1] to get an estimate of systematical errors

[1] PRIMAT: Pitrou et al., 2018b, AlterBBN: Arbey et al., 2020, PArthENoPE: Gariazzo et al., 2022, NUC123: Kawano, 1992 and PRyMordial: Burns, Tait, and Valli, 2023



: Taken from Pitrou et al., 2018a

Introducing BBN – The Timescales



: produced by PRIMAT

- $\boxed{t \lesssim 1\,\mathrm{s}}$

  Weak $n \leftrightarrow p$ reactions

  ☞ number density ratio $\frac{n_n}{n_p} = e^{-Q_n/T}$, Q_n: mass difference

  ☞ at 1 s or $T \approx 1$ MeV: freeze-out and free neutron decay

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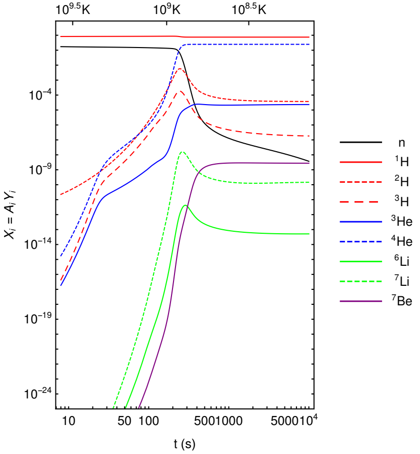

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- $t = 1\,\text{min}$

  Deuterium bottleneck: $n + p \to d + \gamma$ efficient

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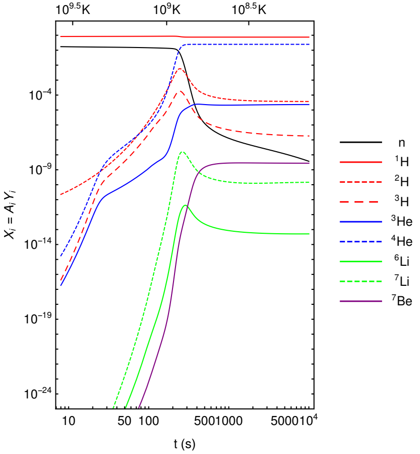

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  ☞ number density ratio $\frac{n_n}{n_p} = e^{-Q_n/T}$, Q_n: mass difference

  ☞ at 1 s or $T \approx 1\,\mathrm{MeV}$: freeze-out and free neutron decay

- $t = 1\,\mathrm{min}$

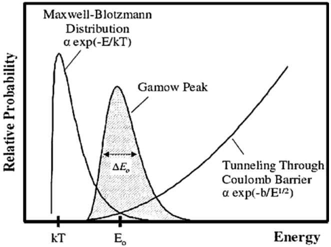
  Deuterium bottleneck: $n + p \to d + \gamma$ efficient

- $t \lesssim 3\,\mathrm{min}$

  Fusion of light elements (up to $^7\mathrm{Be}$)
  $\Rightarrow$ almost all n are bound in $^4\mathrm{He}$

Variation of α – What to consider

- Nuclear reaction rates: Coulomb barrier $\rightarrow$ energy-dependent penetration factor in cross section[1]
- Radiative capture



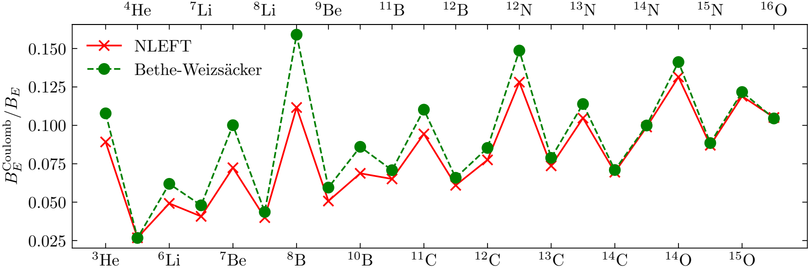
- $n \leftrightarrow p$ and β-decay rates: final (initial) state interactions between charged particles
- Indirect effects: binding energies[2] and Q_n (QED contribution)[3]

$$\Delta Q_n = Q_n^{\mathrm{QED}} \cdot \delta\alpha = -0.58(16)\,\mathrm{MeV} \cdot \delta\alpha$$

[1] Blatt and Weisskopf, 1979; [2] Elhatisari et al., 2024; [3] Gasser, Leutwyler, and Rusetsky, 2021

Coulomb contributions to binding energies

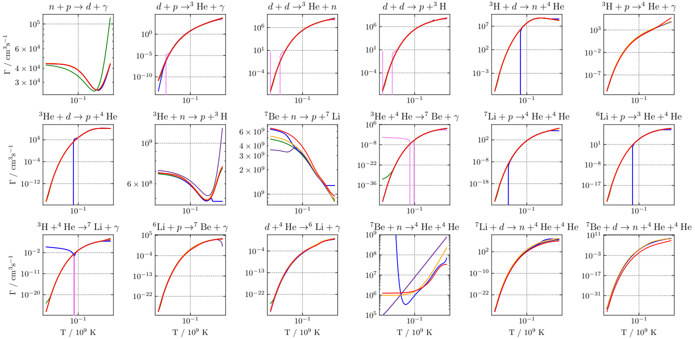
$$\Delta Q = \delta\alpha \left(-\sum_i B_E^{\mathrm{Coulomb},i} + \sum_j B_E^{\mathrm{Coulomb},j} \right)$$



$$\sigma(E,\alpha) \propto \underbrace{(E + Q(\alpha))^{p_\gamma}}_{\text{phase space}} \alpha^{q_\gamma} \frac{\sqrt{E_G^{\mathrm{in}}(\alpha)/E}}{\exp\left(\sqrt{E_G^{\mathrm{in}}(\alpha)/E}\right) - 1} \frac{\sqrt{E_G^{\mathrm{out}}(\alpha)/(E + Q(\alpha))}}{\exp\left(\sqrt{E_G^{\mathrm{out}}(\alpha)/(E + Q(\alpha))}\right) - 1}$$

Nuclear Reaction Rates – Leading Reactions

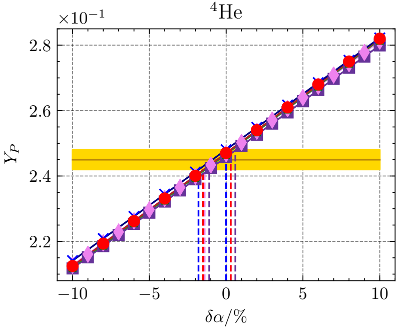
$$\Gamma_{ab\to cd}(T) = N_A \langle \sigma v \rangle \propto \int_0^\infty \mathrm{d}E\, \sigma_{ab\to cd}(E) \cdot E \cdot e^{-\frac{E}{k_B T}}$$

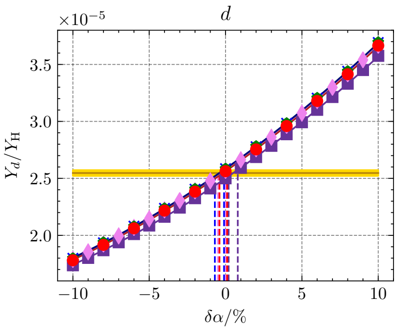


This work ;
PRIMAT ;
AlterBBN ;
PArthENoPE ;
NUC123 ;
NACRE II ;
(PRyMordial uses the PRIMAT rates)

Experimental constraints

- PDG[1]: reliable measurements for $^4\mathrm{He}$, d and $^7\mathrm{Li}$ (But: Lithium problem[2])





- 5 codes give similar results
- Only α-variation of $\boxed{|\delta\alpha| < 1.8\%}$ is consistent with experiment[3]

[1] Workman et al., 2022; [2] Fields, 2011; [3] Meißner, Metsch, HM, 2023

Higgs VEV Variation – What to consider

- QCD scale $\Lambda_{\mathrm{QCD}} \propto (1+\delta v)^{0.25}$ [1]
  - $\rightarrow$ nuclear rates assumed to scale in the same way
- Fermi constant $G_F \propto (1+\delta v)^{-2}$
- Change of electron and quark masses $\Rightarrow M_\pi$ through Gell-Mann-Oakes-Renner relation
  - $\rightarrow$ Q_n (QCD part)[2]
  - $\rightarrow$ nucleon mass and axial-vector coupling (from Lattice QCD or ChPT)
  - $\rightarrow$ nucleon-nucleon scattering parameters (low energy theorems)[3]
  - $\rightarrow$ Deuteron binding energy (next slide)

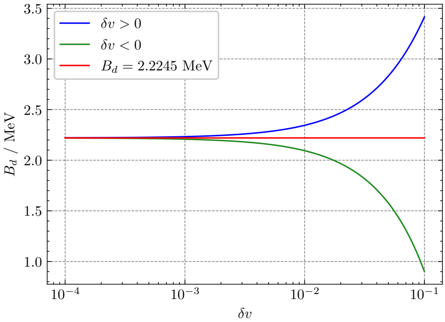
Changes in M_π (i.e., in the NN interaction!) will be dominant effect $\Rightarrow$ studied variations of strange quark mass individually in the next section!

[1] Burns et al., 2024, [2] Gasser, Leutwyler, and Rusetsky, 2021, [3] Baru et al., 2015, 2016

Deuteron Binding Energy

$B_d(M_\pi(v))$ can be parametrized by

- a OBE potential ⇒ as Dr. Erkelenz has shown in his 1974 report![1]
- a combination of OBE potentials with π-, σ- and ω-mesons[2,3]
- a fit to lattice QCD data ⇒ this is what we did (see plot)

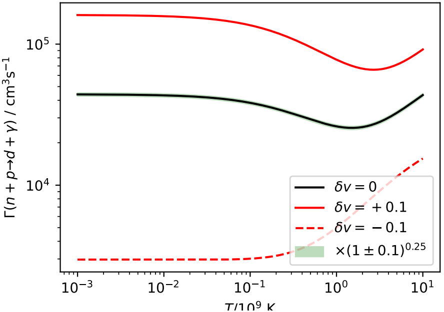


Main contribution from B_d: deuterium bottleneck $d + \gamma \rightarrow n + p$!

[1] Erkelenz, 1974; [2] Burns et al., 2024; [3] Meißner, 1988

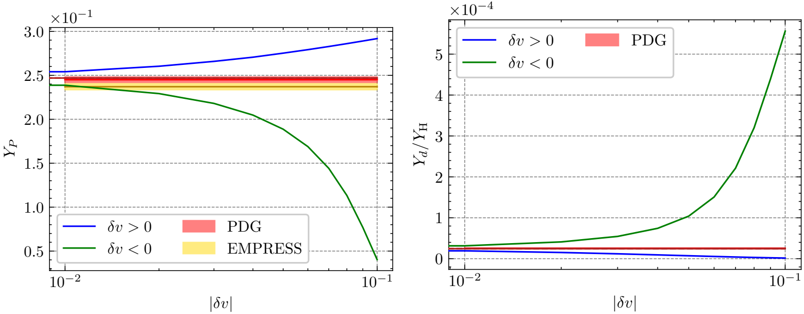
$n + p \to d + \gamma$

- $n + p \to d + \gamma$ rate[1] depends heavily on
  - deuteron binding energy
  - nucleon mass
  - nucleon-nucleon scattering parameters
- v-dependence much stronger than expected from $(1 + \delta v)^{0.25}$ [2]



[1] Rupak, 2000; [2] Burns et al., 2024

Experimental constraints



: PDG: Workman et al., 2022 ; EMPRESS: Matsumoto et al., 2022

- found more stringent 2σ-bound from deuterium abundance[1]:

$$-0.07\% \leq \delta v \leq -0.02\%$$

[1] HM, Meißner, 2024

Now the strange quark mass: where does strangeness appear in BBN?

Main contribution of m_s through strange quark σ-term[1,2,3]

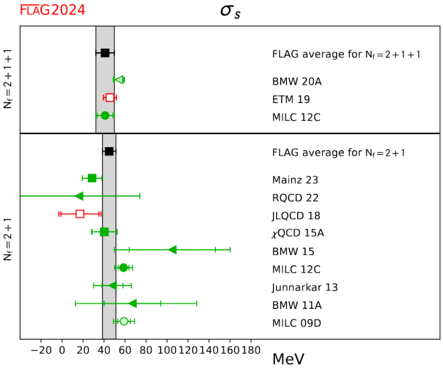
$$\sigma_s = \langle N | m_s \bar{s}s | N \rangle = 44.9(64)\,\text{MeV}$$ [4]

$\Rightarrow$ changes the nucleon mass m_N:

$$|\delta_{m_s}| = \frac{|\Delta m_N|}{\sigma_s}$$

Nucleon mass change in kinetic Hamiltonian affects

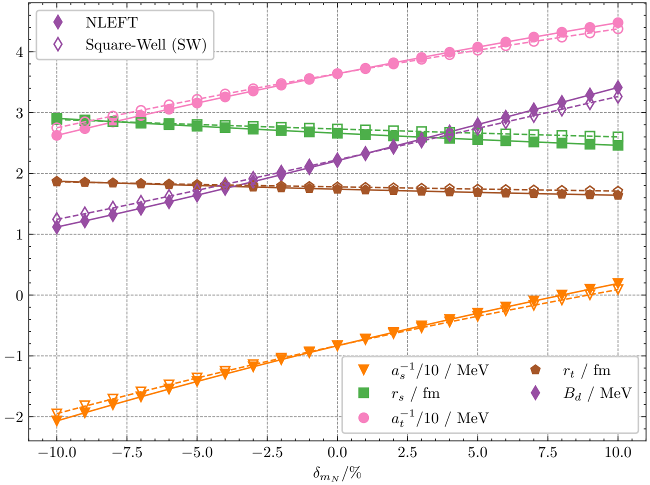
- nucleon-nucleon scattering observables
- nuclear binding energies



: taken from arxiv.org/pdf/2411.04268

[1] Collins, Duncan, Joglekar, 1977; [2] Crewther, 1972; [3] Nielsen, 1977; [4] FLAG collaboration, 2024 ($N_f = 2+1$)

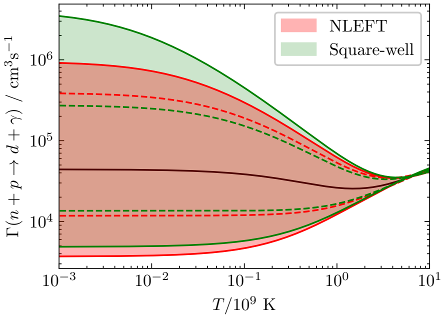
Nucleon-nucleon scattering



$n + p \rightarrow d + \gamma$

For $n + p \rightarrow d + \gamma$ there exists analytic cross section from pionless EFT[1]

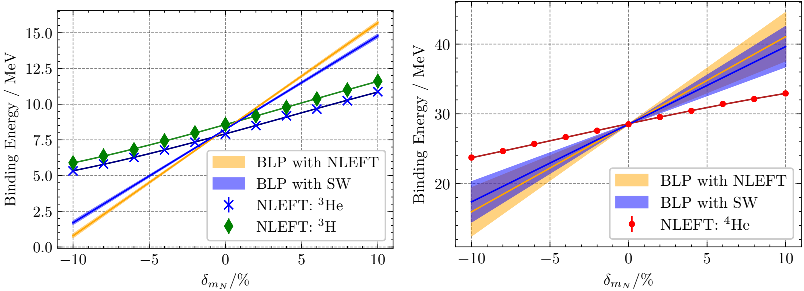
- change in scattering parameters has huge effect
- relevant temperature range: 1.25 to 10×10^9K
- main effect: backwards reaction (deuterium bottleneck)



[1] Rupak, 2000

Binding energies

Again: change in nuclear binding energies due δ_{m_N} in kinetic Hamiltonian



Alternatively, one defines (BLP)[1,2]

$$K^{m_N}_{B_{^3\mathrm{He}}} = K^{m_N}_{a_s}\, K^{a_s}_{B_{^3\mathrm{He}}} + K^{m_N}_{B_d}\, K^{B_d}_{B_{^3\mathrm{He}}}$$

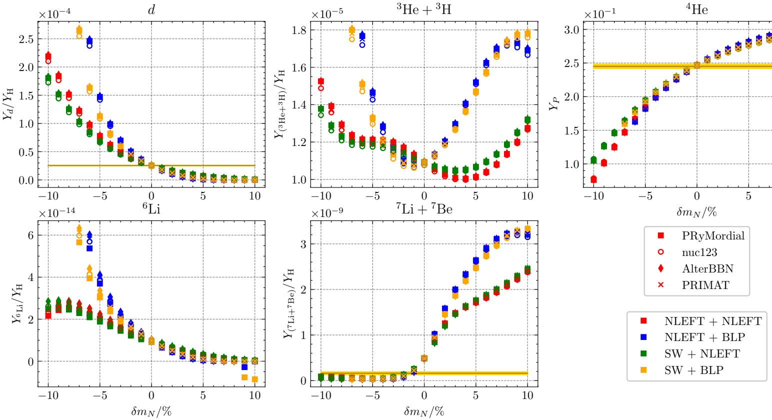
$$K^{m_N}_{B_{^4\mathrm{He}}} = K^{m_N}_{a_s}\, K^{a_s}_{B_{^4\mathrm{He}}} + K^{m_N}_{B_d}\, K^{B_d}_{B_{^4\mathrm{He}}}$$

$$K^{a_s}_{B_{^3\mathrm{He}}} = 0.12(1)\,, \qquad K^{B_d}_{B_{^3\mathrm{He}}} = 1.41(1)\,;$$

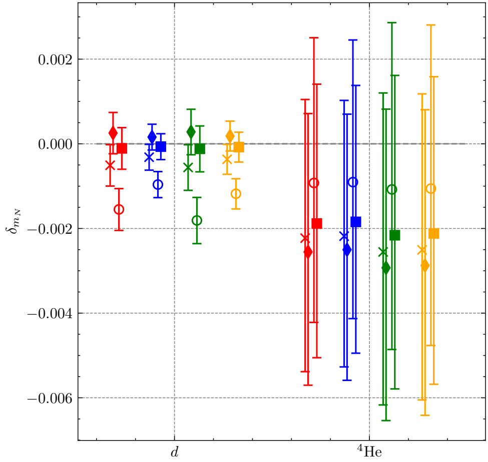
$$K^{a_s}_{B_{^4\mathrm{He}}} = 0.037(11)\,, \qquad K^{B_d}_{B_{^4\mathrm{He}}} = 0.74(22)\,.$$

[1] Berengut et al., 2013; [2] Bedaque, Luu, Platter, 2011

Results


d
$\times 10^{-4}$
Y_d/Y_H
^{3}He + ^{3}H
$\times 10^{-5}$
$Y_{(^3\mathrm{He}+^3\mathrm{H})}/Y_\mathrm{H}$
^{4}He
$\times 10^{-1}$
Y_P
^{6}Li
$\times 10^{-14}$
$Y_{^6\mathrm{Li}}/Y_\mathrm{H}$
^{7}Li + ^{7}Be
$\times 10^{-9}$
$Y_{(^7\mathrm{Li}+^7\mathrm{Be})}/Y_\mathrm{H}$
$\delta m_N/\%$
PRyMordial
nuc123
AlterBBN
PRIMAT
NLEFT + NLEFT
NLEFT + BLP
SW + NLEFT
SW + BLP

Constraints



- Constraints very narrow
- Now: deuterium constraints δ_{m_N} much more than ^{4}He
- $\Rightarrow$ upper bound for strange quark mass variation:

$$|\delta_{m_s}| = \frac{|\Delta m_N|}{\sigma_s} < 5.1\%$$

To summarize...

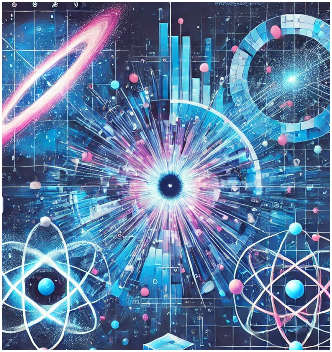
- simulated Big Bang Nucleosynthesis with 5 different codes as laboratory
- considered variation of fundamental constants and found[1,2,3]

$$|\delta\alpha| < 1.8\%$$
$$|\delta v| < 0.07\%$$
$$|\delta_{m_s}| < 5.1\%$$

to be consistent with measurements, using NLEFT as input

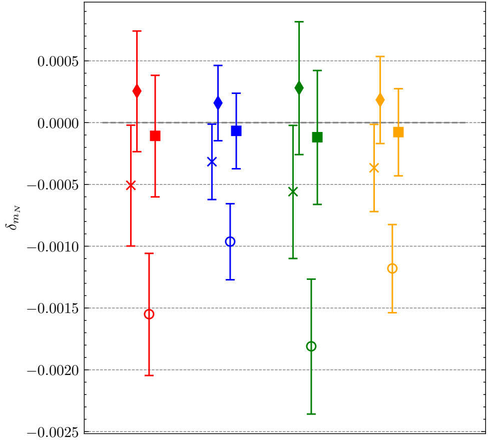
- Now: How fine-tuned is our universe?

[1] Meißner, Metsch, HM, 2023; [2] HM, Meißner, 2024; [3] Meißner, Metsch, HM, 2025



: Source : ChatGPT

Deuteron-deuteron reactions?



Constraints from d-abundance for strange quark mass: differences in the code bigger than range of possible variations!

Deuteron abundance is sensitive to choice of rates[1]

- $d(d,n)^3\mathrm{He}$
- $d(d,p)^3\mathrm{H}$

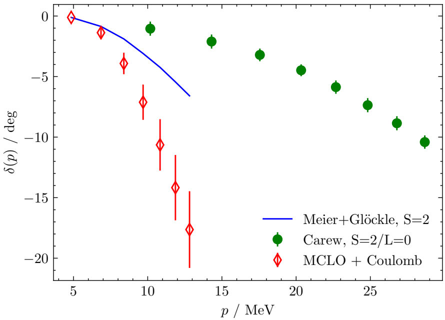
⇒ Goal: calculating these rates using NLEFT

[1] Pitrou et al., 2021

Some first immature results – Elastic $d - d$ scattering

Main problem: d is a very shallow bound state $\Rightarrow$ converges slowly!

- bound states that should not be there
- only MC-LO result plus added Coulomb potential: we want to go to N^3LO in chiral expansion!
- large errors



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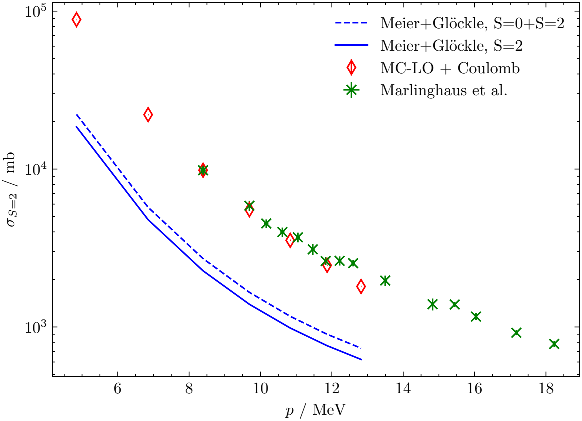

Outlook

- improve elastic $d-d$ scattering calculation
  - implemented NLEFT Monte Carlo code in a GPU version $\Rightarrow$ possibility to use computer time more efficiently[1]
  - Use exact four-body code with simple interaction for $d-d$ elastic scattering
- Next step to $d(d,n)^3\mathrm{He}$ and $d(d,p)^3\mathrm{H}$ is then small
- Excursion to Rome: BBN with a new gauge boson[2]
  - bounds on particle properties with Bayesian sampling $\Rightarrow$ improved error analysis

[1] Hildenbrand et al., 2025; [2] Esseili, Kribs, 2024

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[1] Hildenbrand et al., 2025; [2] Esseili, Kribs, 2024

Thank you for your attention!

Nuclear Reaction Rates – Coulomb Barrier

$$\boxed{\Gamma_{ab\to cd}(T) = N_A \langle \sigma v \rangle \propto \int_0^\infty \mathrm{d}E\, \sigma_{ab\to cd}(E) \cdot E \cdot e^{-\frac{E}{k_B T}}}, \quad E = \frac{1}{2}\mu_{ab} v^2$$

(1) Coulomb Barrier

Cross section is proportional to penetration factor [Blatt and Weisskopf, 1979]

$$\boxed{\sigma \propto v_0 = \frac{2\pi\eta}{e^{2\pi\eta} - 1}},$$

with Sommerfeld parameter

$$\eta = \frac{Z_a Z_b \alpha c}{\hbar v} = \frac{1}{2\pi}\sqrt{E_G/E},$$

and Gamow-energy

$$E_G = 2\mu_{ab} c^2 \pi^2 Z_a^2 Z_b^2 \alpha^2, \quad \mu_{ab} = \frac{m_a m_b}{m_a + m_b}$$

Nuclear Reaction Rates – Radiative Capture

(2) Radiative capture reactions

- Coupling $\propto e \Rightarrow$ Cross section $\sigma \propto \alpha \propto e^2$
- External capture processes [Christy and Duck, 1961]: parameterized in $f(\delta\alpha)$ [Nollett and Lopez, 2002]
- Assume dipole dominance
- For some reactions: Halo EFT cross sections $\Rightarrow$

α-dependence of cross section ($q_\gamma = 1$ for radiative capture, zero else)

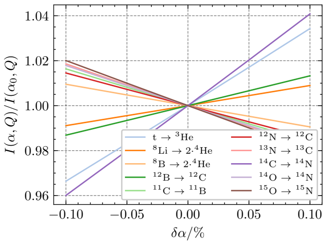
$$\sigma(\alpha, E) \propto \left(\frac{\sqrt{E_G^{\mathrm{in}}/E}}{e^{\sqrt{E_G^{\mathrm{in}}/E}} - 1} \right) \cdot \left(\frac{\sqrt{E_G^{\mathrm{out}}/(E+Q)}}{e^{\sqrt{E_G^{\mathrm{out}}/(E+Q)}} - 1} \right) \cdot (\alpha f(\delta\alpha))^{q_\gamma}$$

$$Q = m_a + m_b - m_c - m_d$$

Weak Rates – Fermi Function

β-decay rate (assume $|M_{fi}|^2$ to be p-independent) [Segrè, 1964]:

$$\lambda = \frac{g^2 |M_{fi}|^2}{2\pi^3 c^3 \hbar^7} \underbrace{\int_0^{p_{e,\max}} \left(W - \sqrt{m_e^2 c^4 + p_e^2 c^2}\right)^2 F(Z, \alpha, p_e) p_e^2 \, \mathrm{d}p_e}_{= I(\alpha, Q)},$$



$$p_{e,\max} = \frac{1}{c}\sqrt{W^2 - m_e^2 c^4}, \; W \approx M_a - M_b = Q$$

Fermi function (for $Z\alpha \ll 1$):

$$F(\pm Z, \alpha, \epsilon_e) \approx \frac{\pm 2\pi\nu}{1 - \exp(\mp 2\pi\nu)}, \quad \nu \equiv \frac{Z\alpha\epsilon_e}{\sqrt{\epsilon_e^2 - 1}}$$

Then:

$$\lambda(\alpha) = \lambda(\alpha_0)\frac{I(\alpha, Q)}{I(\alpha_0, Q)}$$

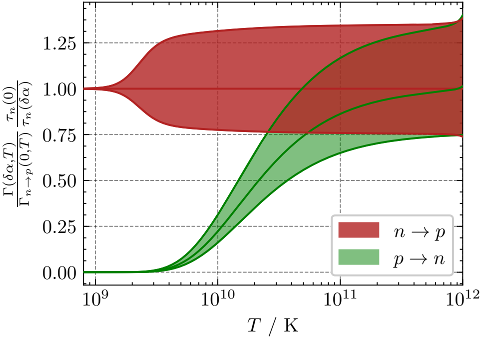
$n \leftrightarrow p$ Rates

Free neutron decay: lifetime

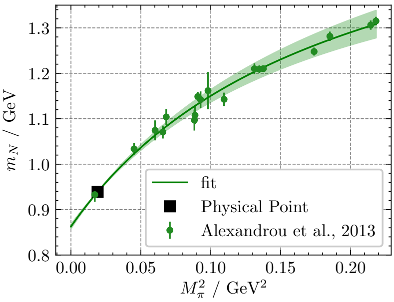
$$\tau_n(\alpha) = \tau_n(\alpha_0)\frac{I(\alpha_0, Q)}{I(\alpha, Q)}$$

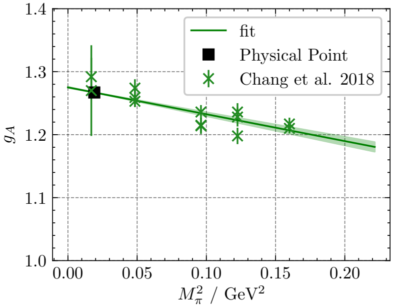
But: Ignored Fermi-Dirac distribution of neutrino and electron

$\Rightarrow$ temperature dependence in α-variation for high temperatures

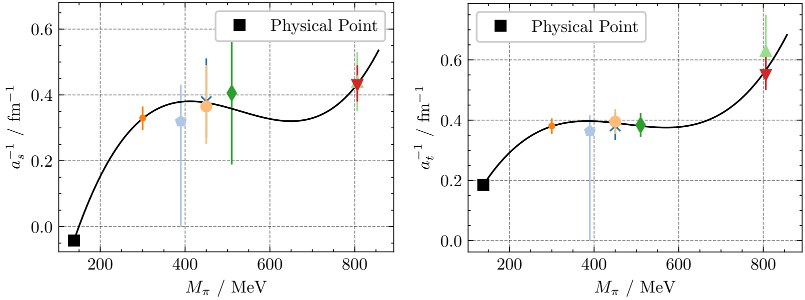


m_N and g_A


1.3
1.2
1.1
1.0
0.9
0.8
m_N / GeV
0.00
0.05
0.10
0.15
0.20
M_π^2 / GeV^2
fit
Physical Point
Alexandrou et al., 2013


1.4
1.3
1.2
1.1
1.0
g_A
0.00
0.05
0.10
0.15
0.20
M_π^2 / GeV^2
fit
Physical Point
Chang et al. 2018

m_N and g_A



Underlying QCD data from: Wagman et al., 2017; NPLQCD collaboration, 2012, 2013 and 2021; Amarasinghe et al., 2023; Berkowitz et al., 2017, Yamazaki et al., 2012 and 2015; Origonos et al., 2015