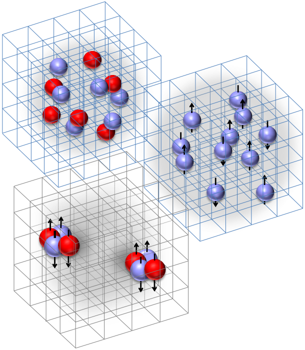
Advancing nuclear structure calculations with nuclear lattice simulations



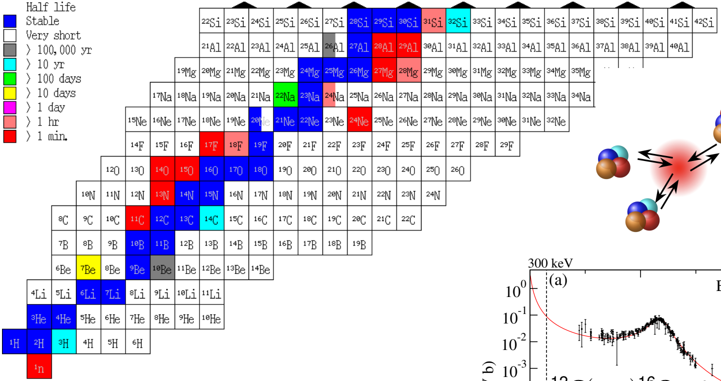
Serdar Elhatisari

(GIBTU & KFUPM)

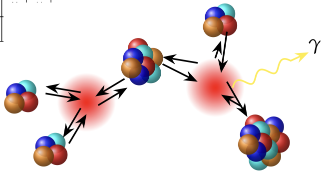
Dr. Klaus Erkelenz Prize Colloquium
HISKP, Uni-Bonn
December 10, 2024

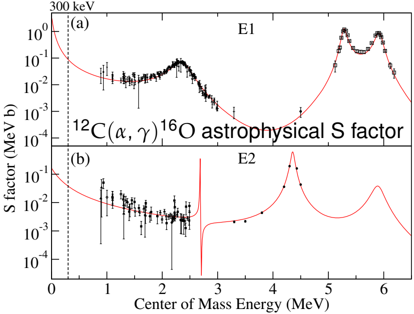
Special thanks to Dr. Gabriele Erkelenz
and the prize committe for the recognition.

Ab initio nuclear theory



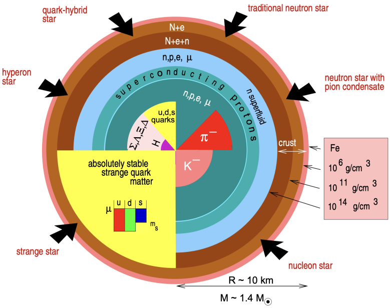
source://atom.kaeri.kr/old/ton/nuchart1.html



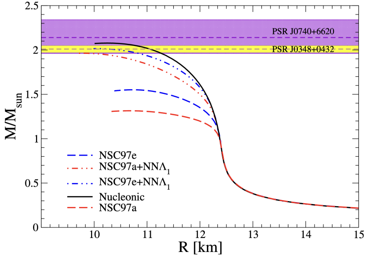


deBoer *et al.*, *Rev. Mod. Phys.* 89, 035007

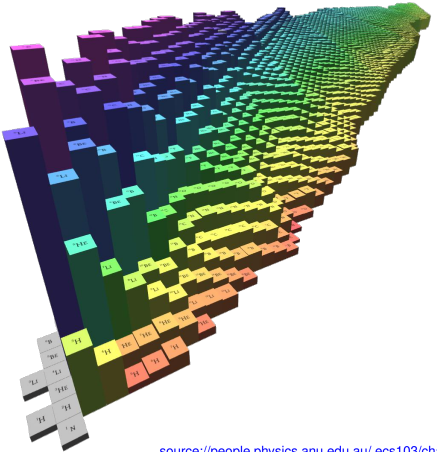
Ab initio nuclear theory: Towards neutron stars and hypernuclei



J. Weber (arXiv:astro-ph/0008376)

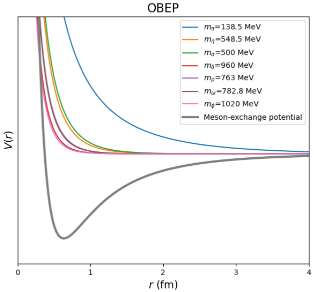


Logoteta, Vidana, Bombaci (Eur. Phys. J. A (2019) 55: 207)



source://people.physics.anu.edu.au/ ecs103/chart3d/

- **Pion-exchange potential:** Governs the long tail of nuclear forces, Yukawa (1935).
- **Meson-exchange potential:** Models intermediate-range nuclear forces, Bonn potential - Erkelenz (1974).
- **Meson-exchange model for *YN* interactions:** extension of Bonn potential, Jülich potential (1989).
- **Phenomenological potential models,** CD-Bonn, Nijmegen, AV18, Stony Brook, Paris, Urbana-Argonne, etc.



Contributions by Dr. Klaus Erkelenz:

→ A non-static OBEP for nuclear structure calculations, Erkelenz *et al.* (1969).

→ One-boson exchange potential and nuclear matter properties, Erkelenz *et al.* (1971).

→ Relativistic OBEP and two-nucleon data, Erkelenz *et al.* (1972).

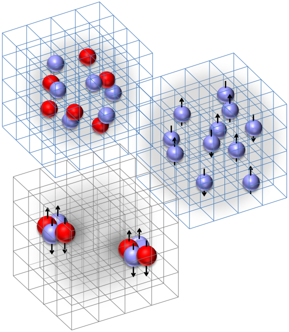
→ Relativistic OBEP and nuclear matter properties, Erkelenz *et al.* (1972).

→ Neutron matter with a relativistic OBEP, Erkelenz *et al.* (1973).

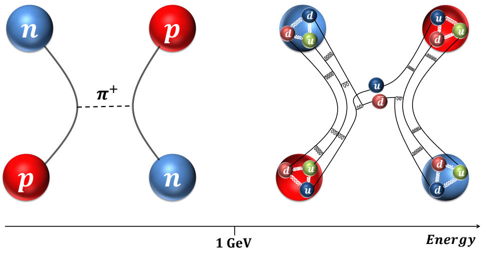
→ An improved relativistic OBEP for two-nucleon and infinite nuclear matter data, Erkelenz *et al.* (1974).

Outline

- Introduction
- Nuclear forces from QCD (EFTs)
- Lattice effective field theory
- Wavefunction matching method
- Recent progress
- Nuclear forces on the lattice
- Summary



- Quantum chromodynamics (QCD) describes the strong forces by confining quarks (and gluons) into baryons and mesons.
- Translating QCD directly into nuclear forces:



$(\Lambda_\chi \sim$ chiral limit$)$

$m_u, m_d, m_s \to 0$

"separation of scales"

- Effective theories provide the solution to bridge the gap between QCD and the nuclear interactions

S. Weinberg, Phys. Lett. B 251 (1990) 288, Nucl. Phys. B363 (1991) 3, Phys. Lett. B 295 (1992) 114.

Chiral effective field theory organizes the nuclear interactions as an expansion in powers of momenta and other low energy scales such as the pion mass (Q/Λ_χ).

The nuclear interactions as a series of increasing complexity:

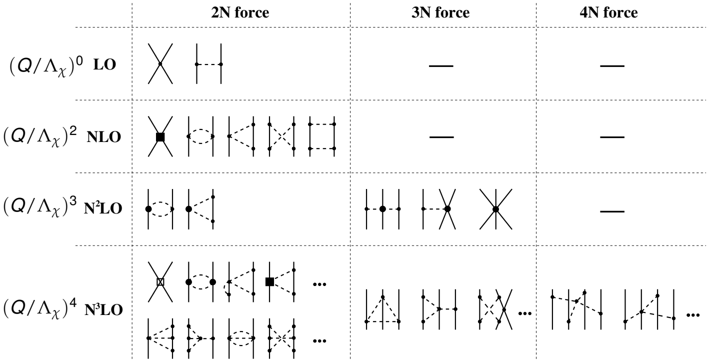
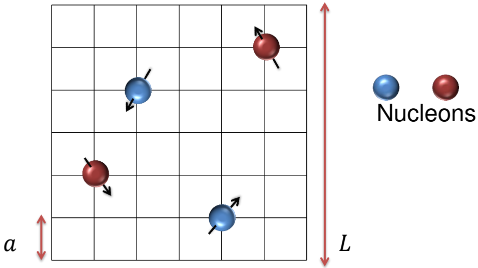


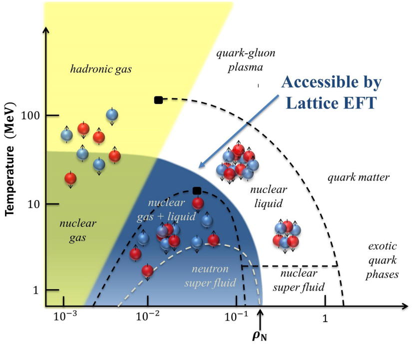
Fig. courtesy of E.Epelbaum

Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; Epelbaum et al. '98,'03,'05,'15; Kaiser '99-'01; Higa et al. '03; ...

Lattice effective field theory

- Lattice effective field theory is a powerful numerical method formulated in the framework of chiral effective field theory.





Lattice formulation of chiral EFT

- Lattice formulation of nuclear forces in the framework of chiral EFT:
  - a simpler decomposition into spin channels
  - accurate phase shifts and binding energies.

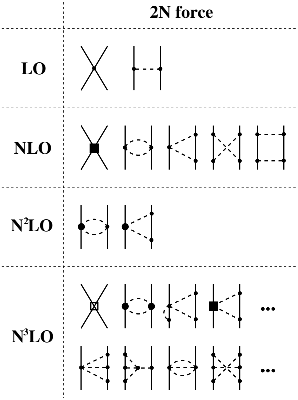
$$V^{S,I,J}_{L,L'}(\mathbf{n}) = \sum_{I_z,J_z}\sum_{S_z,L_z}\sum_{S'_z,L'_z}\left(\langle SS_z, LL_z|JJ_z\rangle \left[a(\mathbf{n})\ \nabla^{2M}\ R^*_{L,L_z}(\nabla)\ a(\mathbf{n})\right]^{s_{NL}}_{S,S_z,I,I_z}\right)^\dagger$$
$$\times \langle SS'_z, L'L'_z|JJ_z\rangle \left[a(\mathbf{n})\ \nabla^{2M}\ R^*_{L',L'_z}(\nabla)\ a(\mathbf{n})\right]^{s_{NL}}_{S,S'_z,I,I_z}$$

$$[a(\mathbf{n})\,a(\mathbf{n}')]^{s_{NL}}_{S,S_z,I,I_z} = \sum_{i,j,i',j'} a^{s_{NL}}_{i,j}(\mathbf{n})\,M_{ii'}(S,S_z)\,M_{jj'}(I,I_z)\,a^{s_{NL}}_{i,j}(\mathbf{n}')$$

Li, SE, Epelbaum, Lee, Lu, Meißner Phys. Rev. C 98, 044002 (2018)

Chiral EFT for nucleons: *NN* scattering phase shifts

- formulate the lattice action in the framework of chiral effective field theory
- fit the unknown coefficients of the short-range lattice interactions to empirical phase shifts



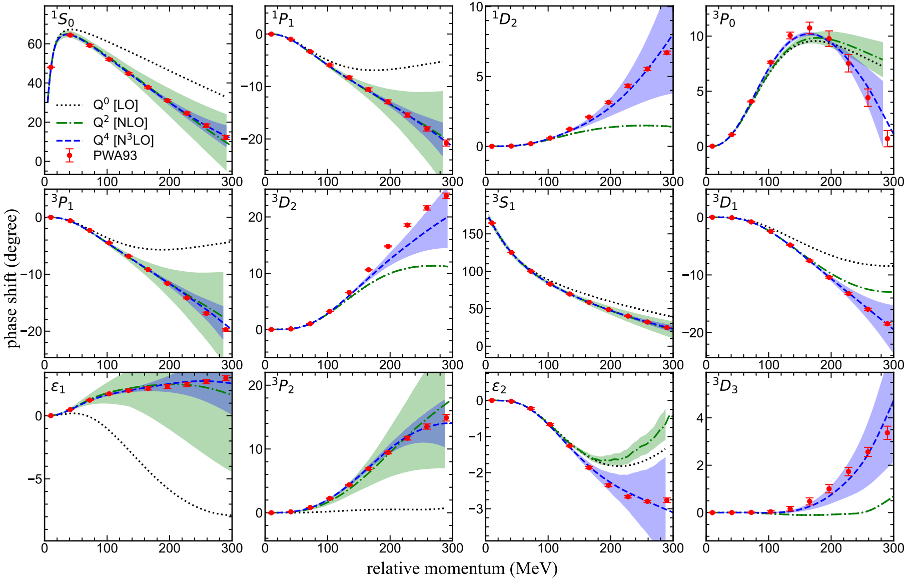
LO $\Longleftrightarrow$ $V_{0,{}^1S_0}$ $V_{0,{}^3S_1}$

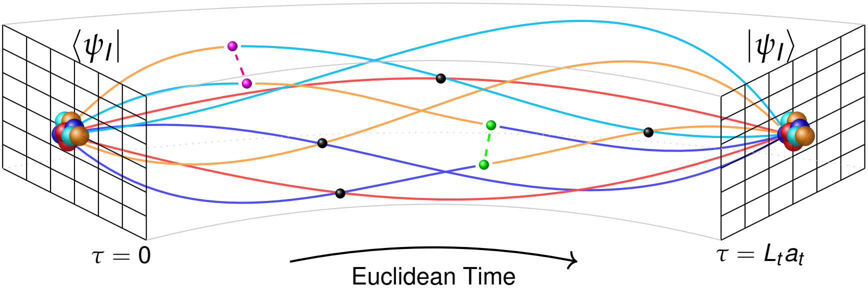
NLO $\Longleftrightarrow$ $V_{2,{}^1S_0}$ $V_{2,{}^3S_1}$ $V_{2,{}^3SD}$ $V_{2,{}^1P_1}$ $V_{2,{}^3P_0}$ $V_{2,{}^3P_1}$ $V_{2,{}^3P_2}$

N³LO $\Longleftrightarrow$ $V^{(1)}_{4,{}^1S_0}$ $V^{(2)}_{4,{}^1S_0}$ $V^{(1)}_{4,{}^3S_1}$ $V^{(2)}_{4,{}^3S_1}$ $V^{(1)}_{4,{}^3SD}$ $V^{(2)}_{4,{}^3SD}$ $V_{4,{}^1P_1}$ $V_{4,{}^3P_0}$ $V_{4,{}^3P_1}$ $V_{4,{}^3P_2}$ $V_{4,{}^3PF}$ $V_{4,{}^1D_2}$ $V_{4,{}^3D_1}$ $V_{4,{}^3D_2}$ $V_{4,{}^3D_3}$

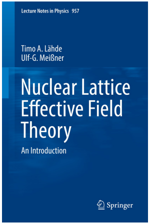
Ab initio nuclear theory: recent progress in NLEFT

$a = 1.32$ fm and $p_{\max} = \pi/a = 471$ MeV





- construct an initial/final state of nucleons, $|\psi_I\rangle$, as a Slater determinant of free-particle standing waves on the lattice.
- evolve nucleons forward in Euclidean time, $e^{-H_{\rm LO}\,\tau}\,|\psi_I\rangle$, where $\tau = L_t a_t$.
- The evolution in Euclidean time automatically incorporates the induced deformation, polarization and clustering.



Projection Monte Carlo uses a given initial state, $|\psi_I\rangle$, to evaluate a product of a string of transfer matrices $\hat{M}$.

$$Z(L_t) = \langle\psi_I| \underbrace{\hat{M} \quad \hat{M} \quad \ldots \quad \hat{M} \quad \hat{M}}_{\text{string of } L_t \text{ transfer matrices}} |\psi_I\rangle$$

In the limit of large Euclidean time the evolution operator $e^{-H_{\mathrm{LO}}\,\tau}$ suppress the signal beyond the low-lying states, and the ground state energy can be extracted by

$$\lim_{L_t\to\infty} \frac{\langle\psi_I|\, \hat{M}^{L_t/2}\, H_{\mathrm{LO}}\, \hat{M}^{L_t/2}\, |\psi_I\rangle}{\langle\psi_I|\, \hat{M}^{L_t}\, |\psi_I\rangle} = E_0$$

Lattice Monte Carlo calculations

perturbative higher order calculations

ho $=$ NLO, NNLO, $\cdots$

where the potential V_{ho} is treated perturbatively. Therefore, the higher order corrections to the ground state energy can be computed as,

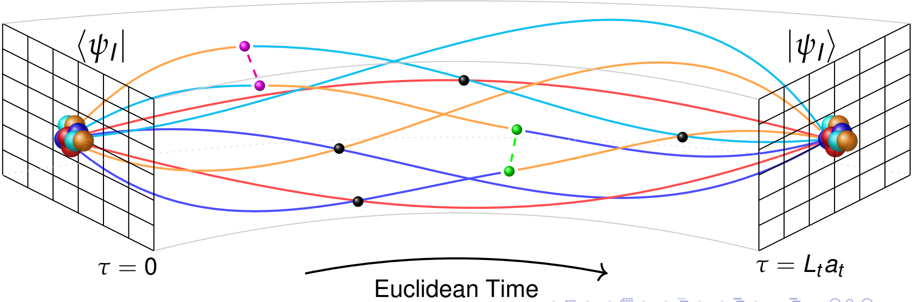
$$\Delta E_{\text{ho}} = \lim_{L_t \to \infty} \frac{\langle \psi_I | \hat{M}^{L_t/2} H_{\mathrm{LO}} \hat{M}^{L_t/2} | \psi_I \rangle}{\langle \psi_I | \hat{M}^{L_t} | \psi_I \rangle}$$

Auxiliary field Monte Carlo

Use a Gaussian integral identity

$$\exp\left[-\frac{C}{2}\left(N^{\dagger}N\right)^{2}\right] = \sqrt{\frac{1}{2\pi}}\int ds\,\exp\left[-\frac{s^{2}}{2}+\sqrt{-C}\,s\,\left(N^{\dagger}N\right)\right]$$

s is an auxiliary field coupled to the particle density. Each nucleon evolves as if a single particle in a fluctuating background of pion fields and auxiliary fields.

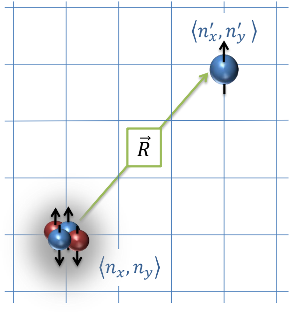


Scattering and reactions: Adiabatic projection method

The method constructs a low energy effective theory for the clusters

Use initial states parameterized by the relative spatial separation between clusters, and project them in Euclidean time.

$$|\psi_I^R\rangle = \sum_{\vec{r}} |\vec{r} + \vec{R}\rangle_1 \otimes |\vec{r}\rangle_2$$



$$|\psi_I^R\rangle_\tau = e^{-H\tau}\,|\psi_I^R\rangle$$ dressed cluster state

The adiabatic projection in Euclidean time gives a systematically improvable description of the low-lying scattering cluster states.
In the limit of large Euclidean projection time the description becomes exact.

SE & Lee. *PRC* 90 064001 (2014).

SE, Lee, Meißner & Rupak *EPJA* 52, 6, 174 (2016).

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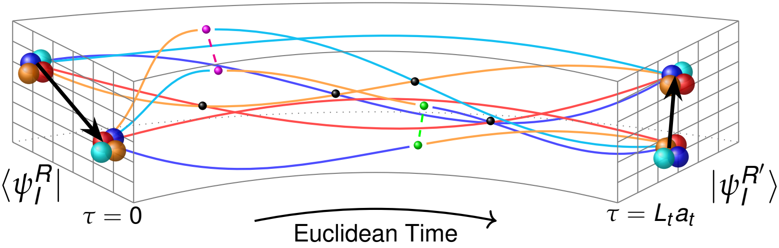
SE, Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, & Meißner. *Nature* 528, 111-114 (2015).

Adiabatic projection method



Hamiltonian matrix

$$[H_\tau]^{J,J_z}_{R,R'} = {}^{J,J_z}_{\tau}\langle \psi_I^R | H | \psi_I^{R'} \rangle^{J,J_z}_{\tau}$$

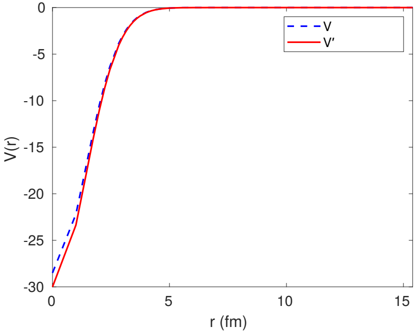
Norm matrix

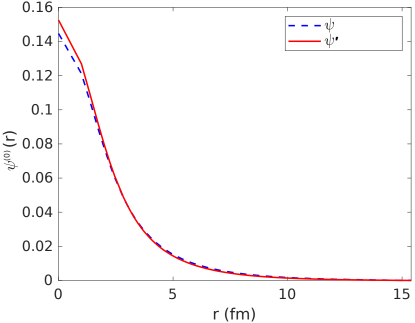
$$[N_\tau]^{J,J_z}_{R,R'} = {}^{J,J_z}_{\tau}\langle \psi_I^R | \psi_I^{R'} \rangle^{J,J_z}_{\tau}$$

$$[H^a_\tau]^{J,J_z}_{\vec{R},\vec{R}'} = \left[N_\tau^{-1/2} \; H_\tau \; N_\tau^{-1/2} \right]^{J,J_z}_{\vec{R}\,\vec{R}'}$$

Perturbative calculations

Toy model:



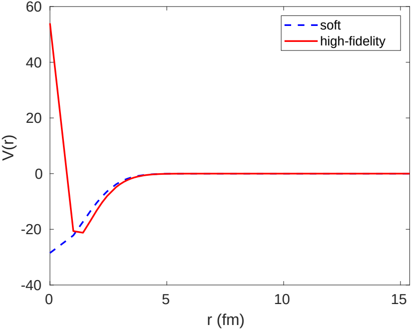


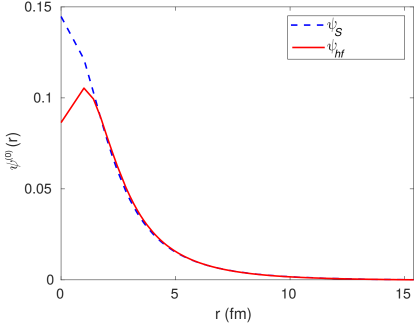
E	E'
−2.010472457971	−2.445743725635
1.775231321023	1.721517536958
6.206769197086	6.118307106128
12.776191791947	12.667625238436
21.337188185570	21.213065578266

Perturbative energies	
q	$\langle \psi^{(0)} \vert H' \vert \psi^{(q)} \rangle$
0	−2.43080610
1	−2.44610114
2	−2.44574140
3	−2.44575370

Perturbative calculations

Toy model:





E_{soft}	E_{hf}
−2.010472457971	−2.444693272597
1.775231321024	1.769682285996
6.206769197085	6.282284485051
12.776191791946	13.008087181009
21.337188185570	21.786534445492

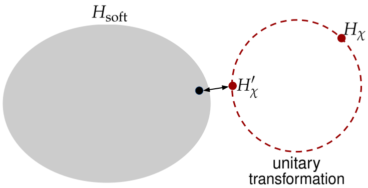
Perturbative energies	
q	$\langle \psi_S^{(0)} \| H \| \psi_S^{(q)} \rangle$
0	−1.74722993
1	−2.89957307
2	−2.10036797
3	−2.26376481

Wavefunction Matching

- ☐ H_χ : –severe sign oscillation, –derived from the underlying theory.
- ☐ $H_{\rm soft}$: –tolerable sign oscillation, –many-body observables with a fair agreement.

Can unitary transformation create a new chiral Hamiltonian which is (first order) perturbation theory friendly?

$$H'_\chi = U^\dagger \, H_\chi \, U$$



- ☐ Let $|\psi^0_\chi\rangle$ be the normalized lowest eigenstate of H_χ.
- ☐ Let $|\psi^0_{\rm soft}\rangle$ be the normalized lowest eigenstate of $H_{\rm soft}$.

$$U_{R',R} = \theta(r - R)\,\delta_{R',R} + \theta(R' - r)\,\theta(R - r)\;|\psi^\perp_\chi\rangle\,\langle\psi^\perp_{\rm soft}|$$

SE et al. *Nature* 630, 8015, 59-63 (2024)

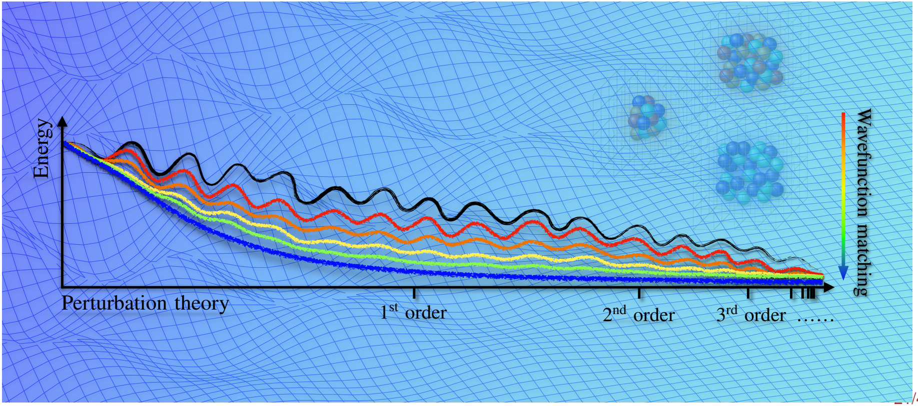
Wavefunction Matching

- ☐ H_{soft} : –tolerable sign oscillation, –many-body observables with a fair agreement.
- ☐ H_{χ} : –severe sign oscillation, –derived from the underlying theory.

Unitary transformation can create a new chiral Hamiltonian which is (first order) perturbative friendly

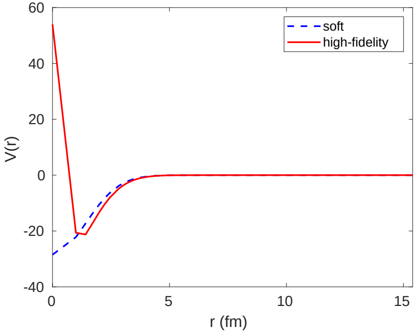
$$H'_{\chi} = U^{\dagger} H_{\chi} U \quad \rightarrow \quad H'_{\chi} = H_{\mathrm{soft}} + (H'_{\chi} - H_{\mathrm{soft}})$$

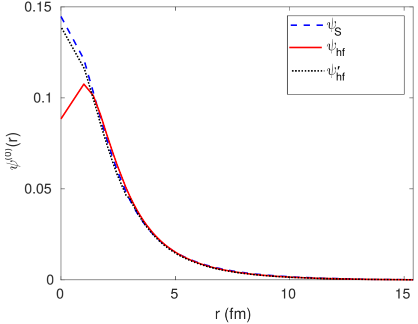
SE et al. *Nature* 630, 8015, 59-63 (2024)



Wavefunction Matching: Perturbative calculations

Toy model:



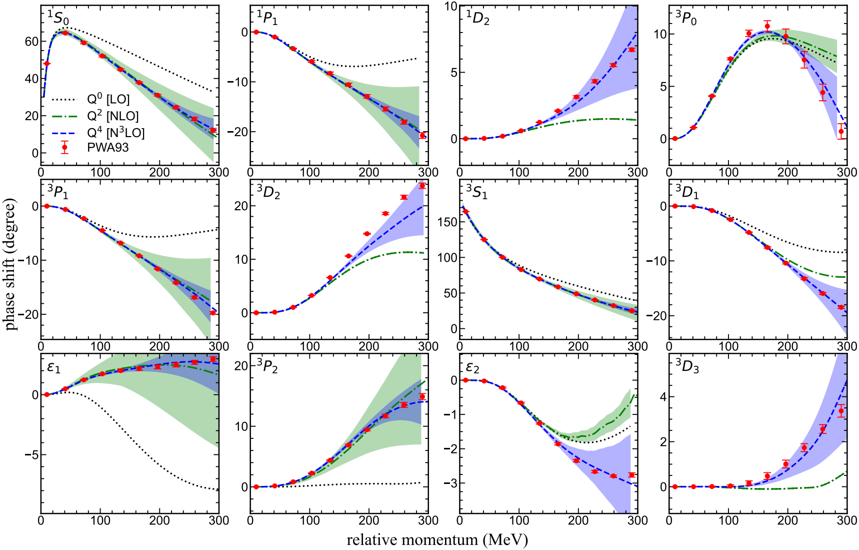


$E_{\rm hf}$	$E'_{\rm hf}$
−2.444693273	−2.444693273
1.769682286	1.769682286
6.282284485	6.282284485
13.008087181	13.008087181
21.786534446	21.786534446

q	$\langle\psi_S^{(0)}\|H'\|\psi_S^{(q)}\rangle$				
	$R = 0.00$	$R = 1.32$	$R = 1.86$	$R = 2.28$	$R = 3.22$ fm
0	−1.747230	−2.055674	−2.226685	−2.312220	−2.402507
1	−2.899573	−2.558509	−2.477194	−2.457550	−2.446214
2	−2.100368	−2.389579	−2.430212	−2.439585	−2.443339
3	−2.263765	−2.414809	−2.437676	−2.441072	−2.443233

Ab initio nuclear theory: recent progress in NLEFT

$a = 1.32$ fm and $p_{\rm max} = \pi/a = 471$ MeV

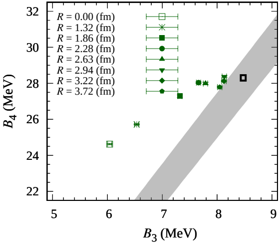


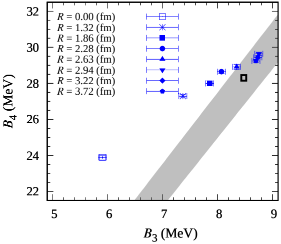
SE et al. *Nature* 630, 8015, 59-63 (2024)

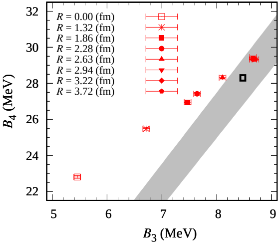
Ab initio nuclear theory: recent progress in NLEFT

$a = 1.32$ fm and $p_{\max} = \pi/a = 471$ MeV

Nuclei	B_{Q^0} MeV	B_{Q^2} MeV	B_{Q^4} MeV	Experiment
$E_{\chi,\mathrm{d}}$	1.7928	2.1969	2.2102	2.2246
$\langle\psi^0_{\mathrm{soft}}\vert\, H_{\chi,\mathrm{d}}\,\vert\psi^0_{\mathrm{soft}}\rangle$	0.4494	0.3445	0.6208	
$\langle\psi^0_{\mathrm{soft}}\vert\, H'_{\chi,\mathrm{d}}\,\vert\psi^0_{\mathrm{soft}}\rangle$	1.6496	1.9772	2.0075	







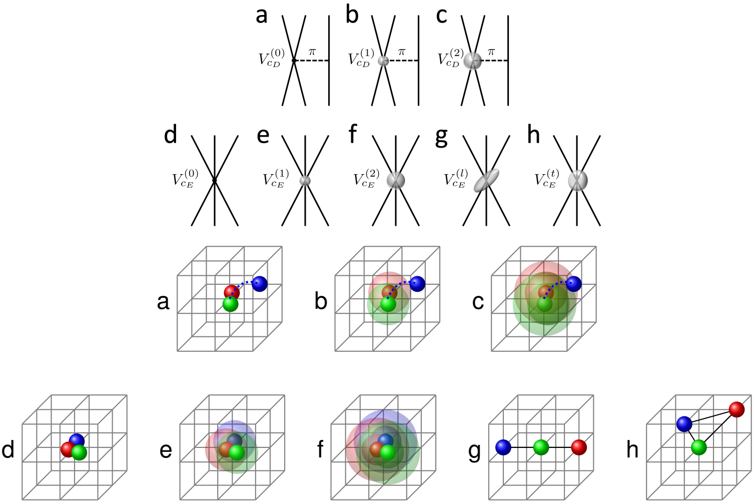
SE et al. *Nature* 630, 8015, 59-63 (2024)

Chiral interactions at N3LO – 2NFs + 3NFs

Work	Constraints	Predictions
NCSM, Barrett *et al.*, Nogga *et al.*	BE of ^{3}H and ^{4}He	Spectrum of ^{6}Li and ^{7}Li
NCSM, Navratil *et al.*	^{3}H, ^{6}Li, ^{10}B, ^{12}C	^{4}He, ^{6}Li, ^{10,11}B, ^{12,13}C
NCSM, Maris *et al.*, Roth *et al.*	BE of ^{3}H and ^{3}H β decay	Structures of $A = 7, 8$. ^{4}He, ^{6}Li, ^{12}C and ^{16}O
CC, Hagen *et al.*	BE of ^{3}H and ^{3}H β decay	EoS of nucleonic matter
BMBPT, Tichai *et al.*	BE of ^{3}H and ^{3}H β decay	BE of $^{16-26}$O, $^{36-60}$Ca and $^{50-78}$Ni
IT-NCSM, Roth *et al.*	BE of ^{3}H and ^{4}He, and ^{3}H β decay	BE of ^{4}He, ^{16}O, ^{40}Ca
CC, Roth *et al.*	BE of ^{3}H and ^{4}He, and ^{3}H β decay	BE of ^{16,24}O, ^{40,48}Ca
SCGF, Cipollone *et al.*	BE of ^{3}H and ^{4}He, and ^{3}H β decay	BE of ^{13,27}N, ^{14,28}O and ^{15,29}F
AFDMC, Lynn *et al.*	BE of ^{3}H and n-^{4}He P-wave phase shifts	EoS of nucleonic matter
MBPT, Bogner *et al.*, Hebeler *et al.*, Drischler *et al.*, Wienholtz *et al.*, Simonis *et al.*	BE ^{3}H and R_{c} of ^{4}He	symmetric and asymmetric NM, BE of $^{48-58}$Ca, spectrum of *sd*-shell nuclei with $8 \leq Z, N \leq 20$, BE and R_{c} of open- and closed-shell nuclei up to $A = 78$
NCCI, Epelbaum *et al.*, Maris *et al.*	BE of ^{3}H, *nd* spin-doublet scattering length and the *pd* differential cross section	the spectrum of light nuclei with $A = 3$–16, elastic *nd* scattering and in the deuteron breakup reactions, properties of the $A = 3, 4$ nuclei, and for spectra of p-shell nuclei up to $A = 16$, BE and R_{c} of the oxygen and calcium isotope chains
CC, Carlsson *et al.*, Ekström *et al.*, Hagen *et al.*	BE of ^{3}H, ^{3,4}He, ^{14}Li and ^{16,22,24,25}O	R_{C} and BE of nuclei up to ^{40}Ca, symmetric nuclear matter, neutron skin of ^{48}Ca, structure of ^{78}Ni
NCSM, IM-SRC, IM-NCSM, Hüther *et al.*	BE of ^{3}H and ^{16}O	R_{c} and BE of ^{4}He, $^{14-26}$O, $^{36-52}$Ca and $^{48-78}$Ni, the spectrum of ^{7}Li, ^{8}Be, ^{9}Be and ^{10}B
CC, Jiang *et al.*	properties of $A \leq 4$	properties of nuclei from $A = 16 - 132$

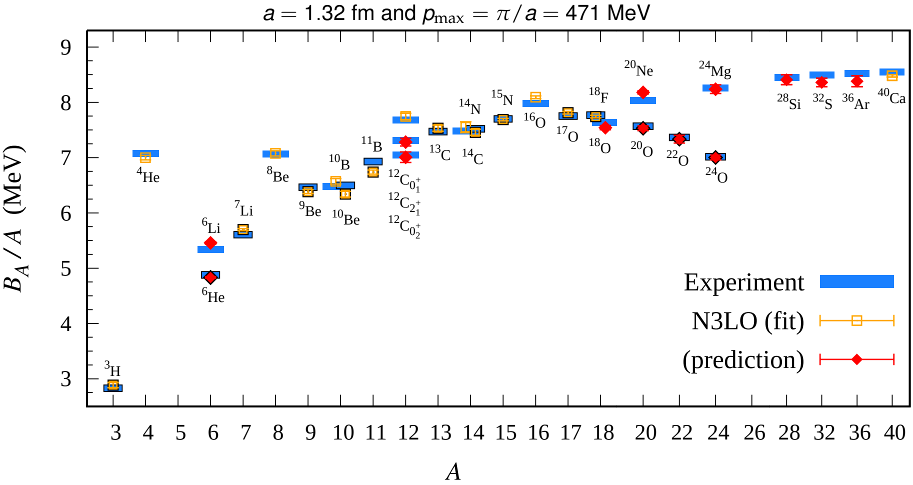
Ab initio nuclear theory: recent progress in NLEFT

$a = 1.32$ fm and $p_{\max} = \pi/a = 471$ MeV



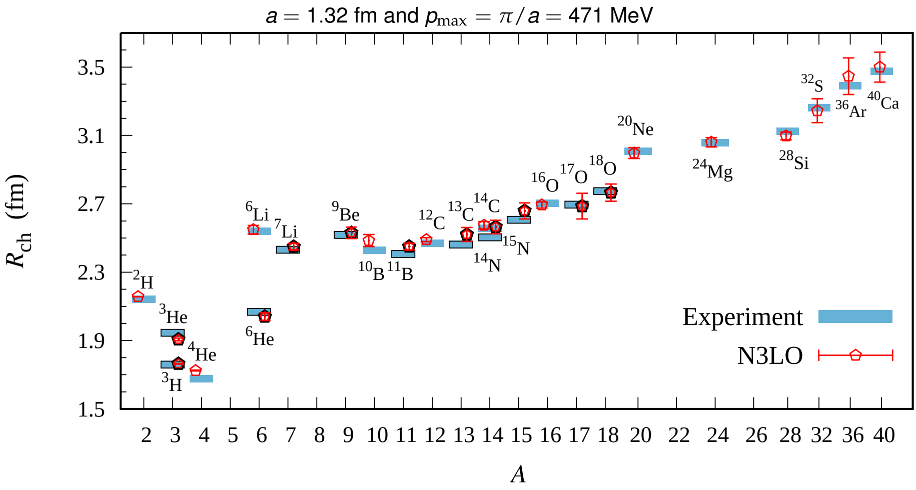
SE et al. *Nature* 630, 8015, 59-63 (2024)

SE and Meißner *in progress*.

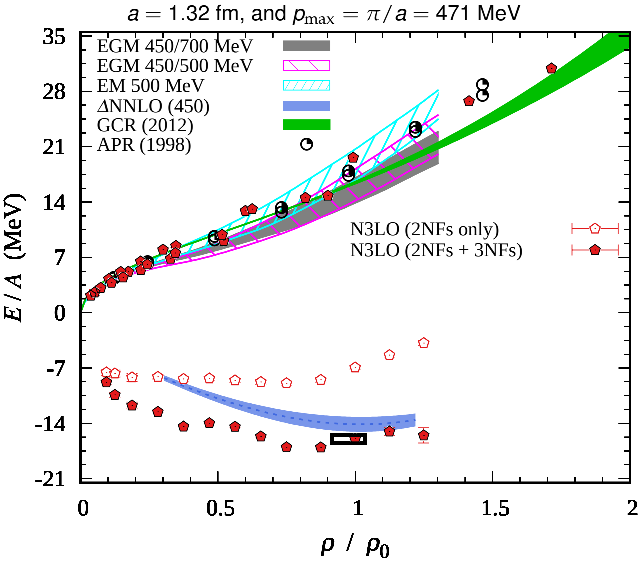


SE et al. *Nature* 630, 8015, 59-63 (2024)

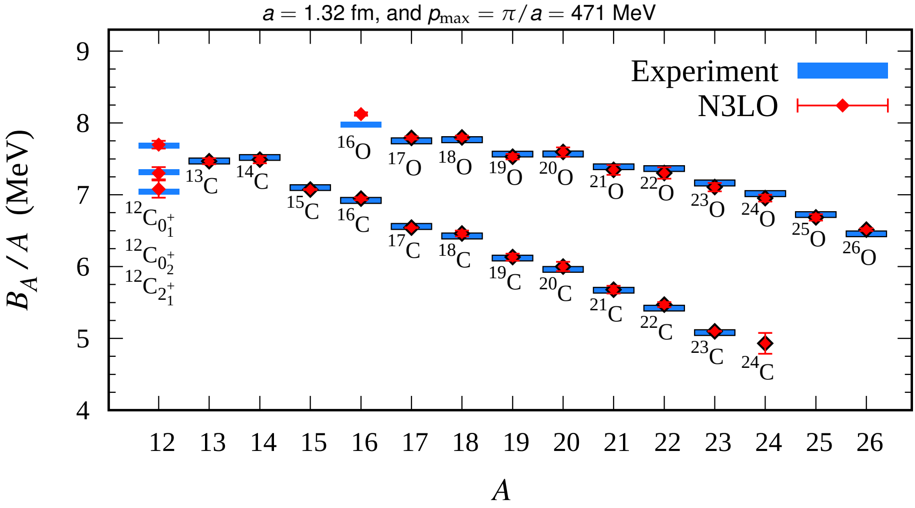
Ab initio nuclear theory: recent progress in NLEFT



SE et al. *Nature* 630, 8015, 59-63 (2024)



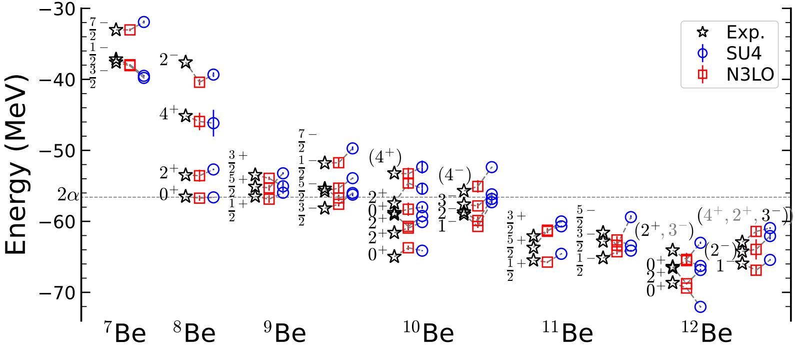
SE et al. *Nature* 630, 8015, 59-63 (2024)



work in progress

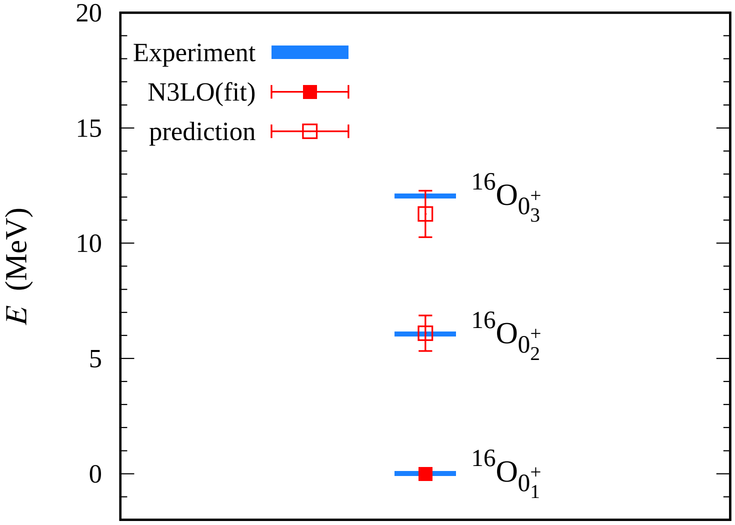
Ab initio nuclear theory: recent progress in NLEFT

$a = 1.32$ fm, and $p_{\max} = \pi/a = 471$ MeV

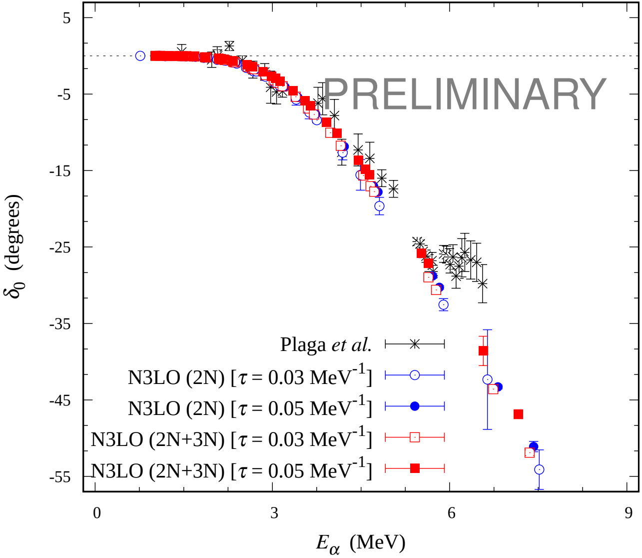


Shen, SE, Lee, Meißner and Ren, arXiv:2411.14935 [nucl-th]

Alpha-carbon scattering at N3LO



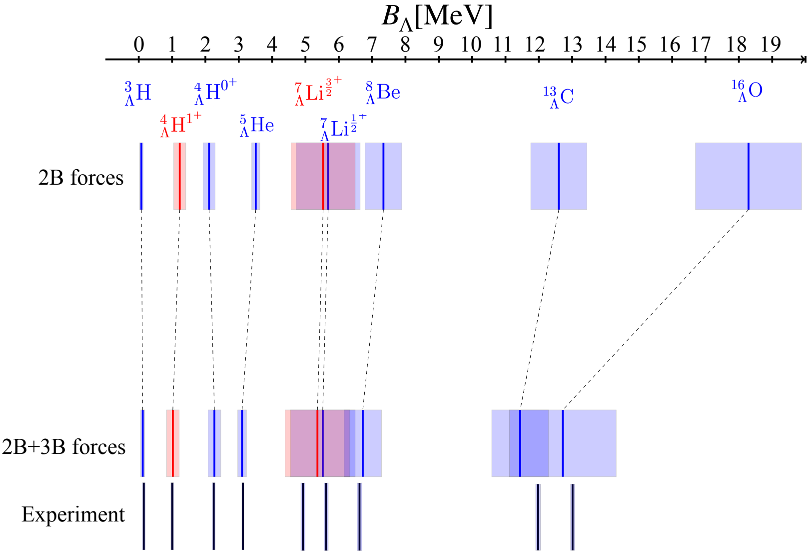
Ab initio alpha-carbon scattering at N3LO



SE, Hildenbrand, Meißner, ... NLEFT [*in progress*].

Ab initio nuclear theory: recent progress: towards hypernuclei

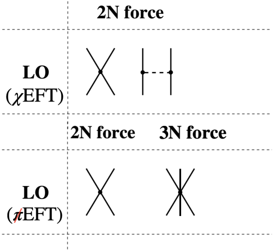
$a = 1.32$ fm and $p_{\max} = \pi / a = 471$ MeV



Chiral nuclear forces on the lattice

- ☐ H_S : tolerable sign oscillation, the g.s. wave function $|\psi_0^\mathrm{S}\rangle$.
- ☐ H_χ : severe sign oscillation, the g.s. wave function $|\psi_0\rangle$.

> Use $H'_\chi = U^\dagger H_\chi U$ and define $H'_\chi = H_\mathrm{S} + (H'_\chi - H_\mathrm{S})$



$$V_\mathrm{LO} = V^{s_\mathrm{NL}, s_\mathrm{L}}_{^1S_0} + V^{s_\mathrm{NL}, s_\mathrm{L}}_{^3S_1} + V_\mathrm{OPE}$$

$$V_\mathrm{LO} = V^{s_\mathrm{NL}, s_\mathrm{L}}_\mathrm{SU4} + V_\mathrm{OPE}$$

$$V^{\not\pi}_\mathrm{LO} = V^{C_2, s_\mathrm{NL}, s_\mathrm{L}}_\mathrm{SU4} + V^{C_3}_\mathrm{SU4}$$

- ☑ Does every chiral EFT interaction give well controlled and reliable results for heavier systems?
- ☑ Is the convergence of higher-order terms under control?

SE, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, Lee, Rupak, *PRL* 117, 132501 (2016)

Degree of locality of nuclear forces

Nucleus	**A (LO)**	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
^{3}H	-7.82(5)	-7.78(12)	-7.82(5)	-7.78(12)	-8.482
^{3}He	-7.82(5)	-7.78(12)	-7.08(5)	-7.09(12)	-7.718
^{4}He	-29.36(4)	-29.19(6)	-28.62(4)	-28.45(6)	-28.296
^{8}Be	**-58.61(14)**	-59.73(6)	-56.51(14)	**-57.29(7)**	-56.591
^{12}C	**-88.2(3)**	-95.0(5)	-84.0(3)	**-89.9(5)**	-92.162
^{16}O	**-117.5(6)**	-135.4(7)	-110.5(6)	**-126.0(7)**	-127.619
^{20}Ne	**-148(1)**	-178(1)	-137(1)	**-164(1)**	-160.645

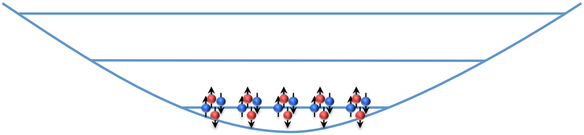
$$\frac{E_{^8\mathrm{Be}}}{E_{^4\mathrm{He}}} = 1.997(6)$$

$$\frac{E_{^{12}\mathrm{C}}}{E_{^4\mathrm{He}}} = 3.00(1)$$

$$\frac{E_{^{16}\mathrm{O}}}{E_{^4\mathrm{He}}} = 4.00(2)$$

$$\frac{E_{^{20}\mathrm{Ne}}}{E_{^4\mathrm{He}}} = 5.03(3)$$

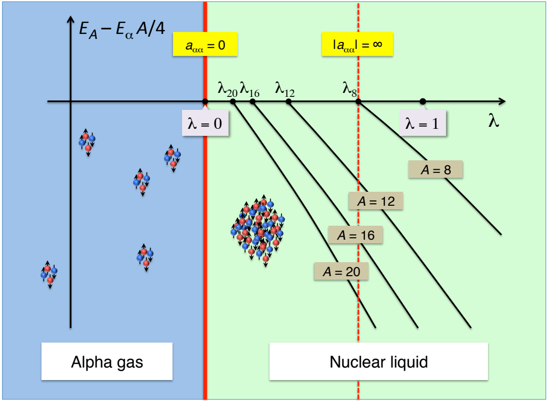
Bose condensate of alpha particles!



SE, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, Lee, Rupak, *PRL* 117, 132501 (2016)

Nuclear binding near a quantum phase transition

Consider a one-parameter family of interactions: $V = (1-\lambda)\, V^{A}_{\rm LO} + \lambda\, V^{B}_{\rm LO}$



There is a quantum phase transition at the point where the α-α scattering length $a_{\alpha\alpha}$ vanishes, and it is a first-order transition from a Bose-condensed α-particle gas to a nuclear liquid.

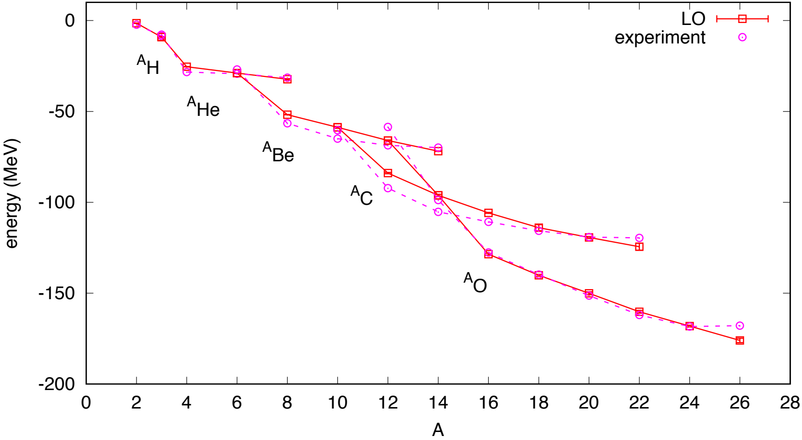
SE, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, Lee, Rupak, *PRL* 117, 132501 (2016)

Stoff, *Phys. Rev. A* 49 (1994) 3824

Ground state energies at LO

We can probe the degree of locality only by many-body calculations, and we consider an SU4-symmetric potential,

$$V_{\mathrm{LO}} = V_{\mathrm{SU4}}^{s_{\mathrm{NL}}, s_{\mathrm{L}}} + V_{\mathrm{OPE}} + V_{\mathrm{Coulomb}}$$

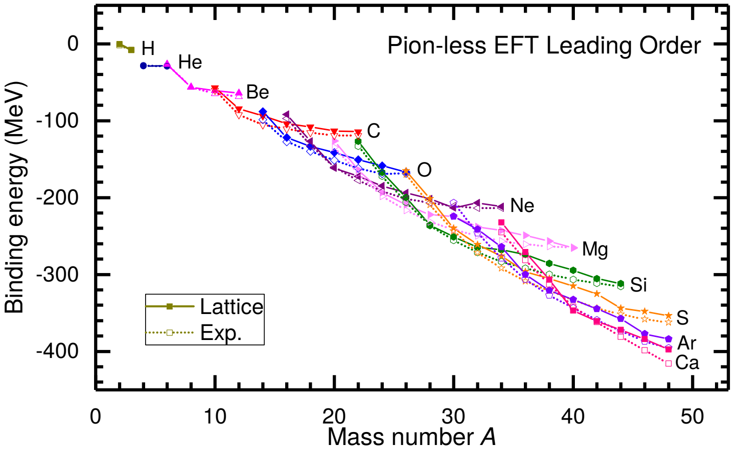


SE, Epelbaum, Krebs, Lähde, Lee, Li, Lu, Meißner, Rupak, *PRL* 119, 222505 (2017)

Essential elements for nuclear binding

Consider the following potential in the framework of pionless effective field theory to probe the degree of locality from many-body calculations,

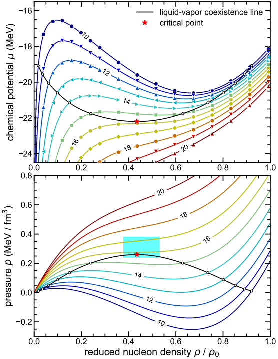
$$V_{\not\pi}^{\mathrm{LO}} = V_{\mathrm{SU4}}^{C_2, s_{\mathrm{NL}}, s_{\mathrm{L}}} + V_{\mathrm{SU4}}^{C_3} + V_{\mathrm{Coulomb}}$$

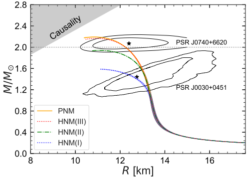


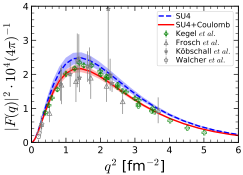
Lu, Li, SE, Lee, Epelbaum, Meißner, *Phys. Lett. B*, 797, 134863 (2019)

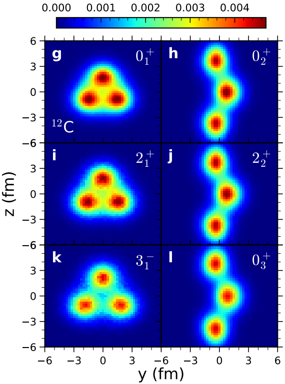
Essential elements for nuclear binding

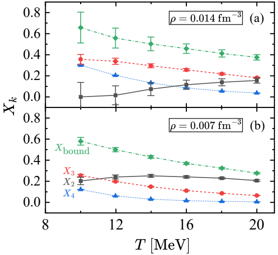
- ☐ *Ab-initio* nuclear thermodynamics,
  Lu, Li, SE, Lee, Drut, Lahde, Epelbaum, Meißner, *Phys. Rev. Lett.* 125, 192502 (2020)
- ☐ Emergent geometry and duality in the carbon nucleus,
  Shen, SE, Lahde, Lee, Lu, Meißner, *Nature Commun.* 14, 2777 (2023)
- ☐ *Ab-initio* study of nuclear clustering in hot dilute nuclear matter,
  Ren, SE, Lahde, Lee, Meißner, PLB 850, 138463 (2024)
- ☐ *Ab-initio* calculation of the alpha-particle monopole transition form factor,
  Meißner, Shen, SE, Lee, PRL 132, 6, 062501 (2024)
- ☐ *Ab-initio* calculation of hyper-neutron matter,
  Tong, SE, Meißner, arXiv:2405.01887











Summary

- Nuclear forces in the framework of chiral effective field theory are well-established, and it is very important time for *ab initio* methods to make predictions in many-nucleon system using these forces.
- Understanding of the connection between the degree of locality of nuclear forces and nuclear structure has led to a more efficient set of lattice chiral EFT interactions.
- Improving QMC calculations with perturbation theory for many-body systems in nuclear physics is crucial to be able to use more realistic interactions in *ab initio* nuclear theory. Phys. Rev. Lett. 128, 242501 (2022)
- A recently developed method so called the wave function matching provides a rapid convergence in perturbation theory for many-body nuclear physics. Using this new method now we are able to calculate the nuclear binding energies, neutron matter, symmetric nuclear matter and charge radii of nuclei simultaneously in very good agreements with the experimental results.

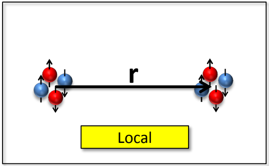
Thanks!

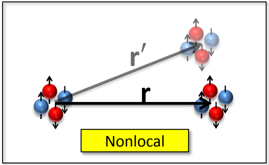
Extras

	2N force
LO	

$$V_{\mathrm{LO}} = V_{{}^1S_0}^{s_{\mathrm{NL}}, s_{\mathrm{L}}} + V_{{}^3S_1}^{s_{\mathrm{NL}}, s_{\mathrm{L}}} + V_{\mathrm{OPE}}$$

$$U(r) = V(r, r')\,\delta(r - r') \qquad U(r, r') = V(r, r')$$





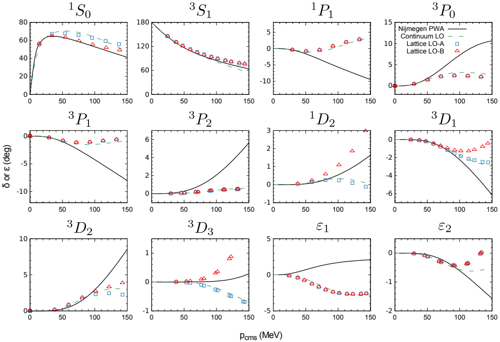
- ☐ Does every chiral EFT interaction give well controlled and reliable results for heavier systems?
- ☐ Is the convergence of higher-order terms under control?

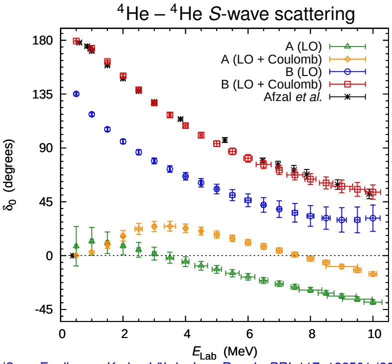
SE, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, Lee, Rupak, *PRL* 117, 132501 (2016)

Degree of locality of nuclear forces - I

$$V_{\mathrm{LO}}^{\mathrm{A}} = V_{^1S_0,Q^0}^{s_{\mathrm{NL}}} + V_{^3S_1,Q^0}^{s_{\mathrm{NL}}} + V_{\mathrm{OPE}}$$

$$V_{\mathrm{LO}}^{\mathrm{B}} = V_{^1S_0,Q^0}^{s_{\mathrm{NL}},s_{\mathrm{L}}} + V_{^3S_1,Q^0}^{s_{\mathrm{NL}},s_{\mathrm{L}}} + V_{\mathrm{OPE}}$$



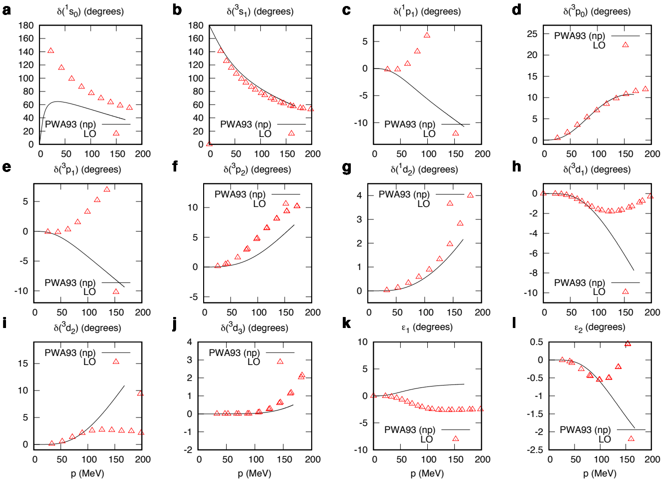


SE, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, Lee, Rupak, *PRL* 117, 132501 (2016)

Degree of locality of nuclear forces – II

We can probe the degree of locality only by many-body calculations, and we consider an SU4-symmetric potential,

$$V_{\rm LO} = V_{\rm SU4}^{s_{NL}, s_L} + V_{\rm OPE}$$

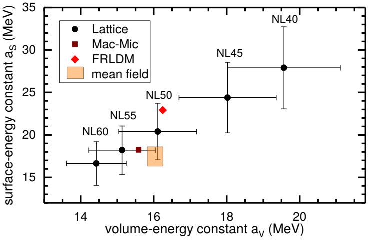


Consider the following potential in the framework of pionless effective field theory to probe the degree of locality from many-body calculations,

$$V_{\not\pi} = V_{\rm SU4}^{C_2, s_{NL}, s_L} + V_{\rm SU4}^{C_3} + V_{\rm Coulomb}$$

- ☐ C_2, $s_{\rm L}$, and C_3 are tuned to get the few-body physics correct for given $s_{\rm NL}$,
- ☐ This is repeated for $s_{\rm NL}$ = 0.1 - 0.6,
- ☐ For $A \geq 16$, the binding energies are well-parameterized with the Bethe-Weizsäcker mass formula;

$$B(A) = a_V\, A - a_S\, A^{2/3} + E_{\rm Coulomb} + (\text{symmetry} + \text{pairing} + \text{shellcorrection} + \ldots)$$



Lu, Li, SE, Lee, Epelbaum, Meißner, Phys. Lett. B, 797, 134863 (2019)

Essential elements for nuclear binding

- ☐ a lattice action with minimum number of parameters (four) which describes neutron matter up to saturation density and the ground state properties of nuclei up to calcium. $a = 1.32$ fm, $s_{\rm L} = 0.0609$ (l.u.), and $s_{\rm NL} = 0.5$ (l.u.)

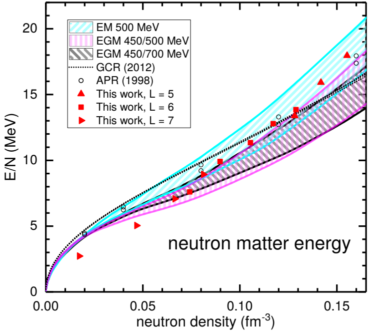
Lu, Li, SE, Lee, Epelbaum, Meißner, *Phys. Lett. B*, 797, 134863 (2019)

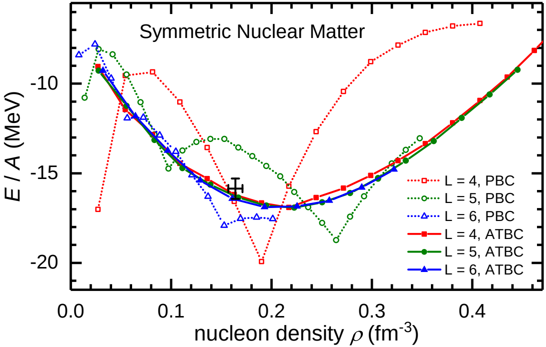
	B	Experiment	$R_{\rm ch}$	Experiment
^{3}H	8.48(2)	8.48	1.90(1)	1.76
^{3}He	7.75(2)	7.72	1.99(1)	1.97
^{4}He	28.89(1)	28.3	1.72(1)	1.68
^{16}O	121.9(1)	127.6	2.74(1)	2.70
^{20}Ne	161.6(1)	160.6	2.95(1)	3.01
^{24}Mg	193.5(2)	198.3	3.13(1)	3.06
^{28}Si	235.8(4)	236.5	3.26(1)	3.12
^{40}Ca	346.8(6)	342.1	3.42(1)	3.48

Essential elements for nuclear binding

- a lattice action with minimum number of parameters (four) which describes neutron matter up to saturation density and the ground state properties of nuclei up to calcium. $a = 1.32$ fm, $s_{\rm L} = 0.061$ (l.u.), and $s_{\rm NL} = 0.5$ (l.u.)

Lu, Li, SE, Lee, Epelbaum, Meißner, *Phys. Lett. B*, 797, 134863 (2019)





Lu, Li, SE, Lee, Drut, Lahde, Epelbaum, Meißner, *Phys. Rev. Lett.* 125, 192502 (2020)

Ab initio nuclear theory: recent progress: towards hypernuclei

$a = 1.32$ fm and $p_{\rm max} = \pi/a = 471$ MeV

Hypernucleus	Experiment	N3LO (2N+3N) and LO (ΛN)	N3LO (2N+3N) and LO (ΛN+ΛNN)
${}^{3}_{\Lambda}$H	0.16 ± 0.04	0.08 ± 0.05	0.12 ± 0.06
${}^{4}_{\Lambda}\mathrm{H}^{0^+}$	2.25 ± 0.042	2.11 ± 0.18	2.258 ± 0.19
${}^{4}_{\Lambda}\mathrm{H}^{1^+}$	1.01 ± 0.046	1.23 ± 0.18	1.012 ± 0.19
${}^{5}_{\Lambda}$He	3.102 ± 0.03	3.51 ± 0.12	3.10 ± 0.13
${}^{7}_{\Lambda}\mathrm{Li}^{\frac{1}{2}^+}$	5.62 ± 0.06	5.68 ± 0.96	5.52 ± 0.97
${}^{7}_{\Lambda}\mathrm{Li}^{\frac{3}{2}^+}$	4.93 ± 0.06	5.53 ± 0.96	5.36 ± 0.97
${}^{9}_{\Lambda}$Be	6.61 ± 0.07	7.34 ± 0.55	6.72 ± 0.55
${}^{13}_{\Lambda}$C	11.96 ± 0.07	12.60 ± 0.84	11.44 ± 0.84
${}^{16}_{\Lambda}$O	13.00 ± 0.06	18.29 ± 1.59	12.72 ± 1.61

Hildenbrand, SE, Ren, Meißner, Eur. Phys. J. A 60, 215 (2024) Jun 25, 2024