High-quality nuclear forces from chiral EFT

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Dr. Klaus Erkelenz Kolloquium
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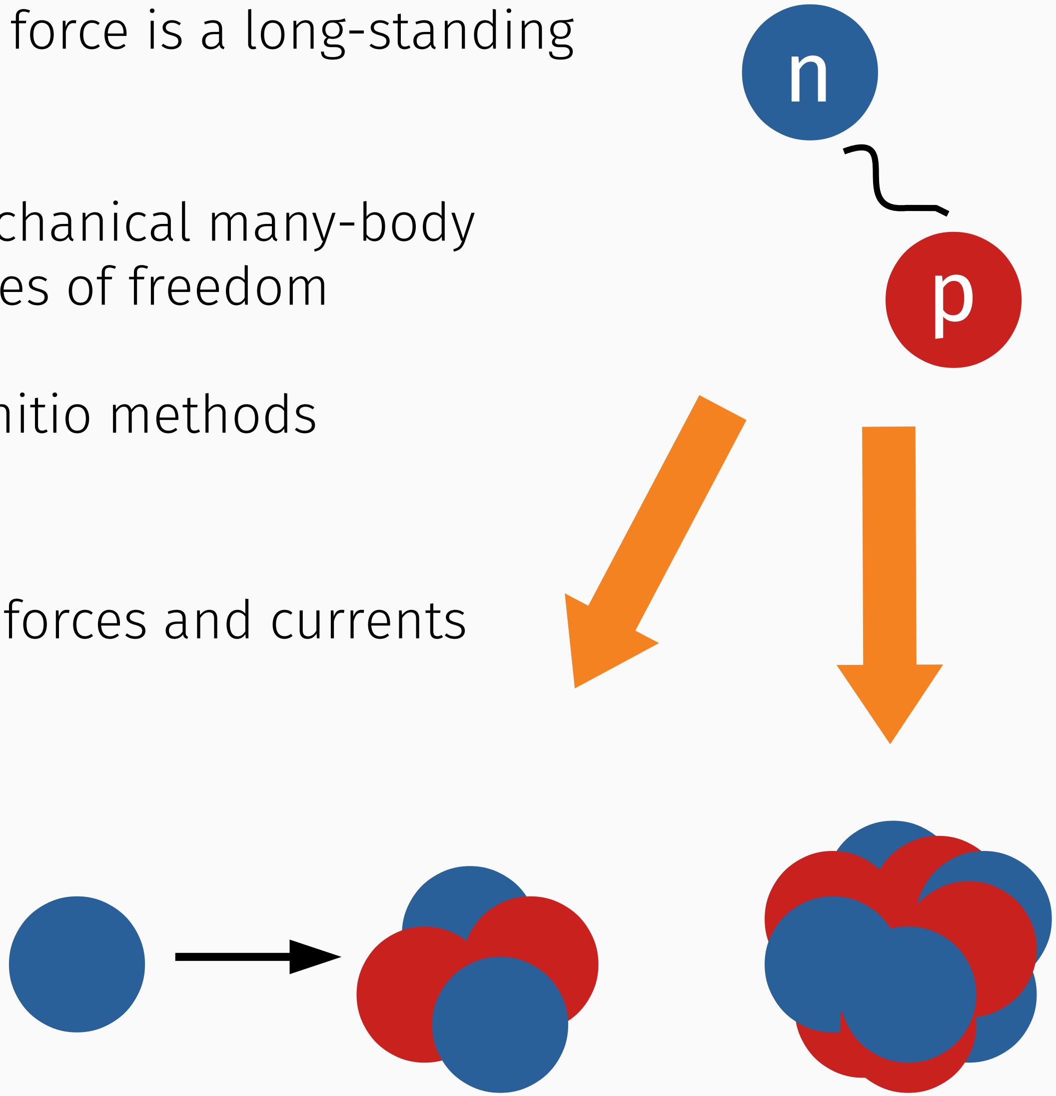
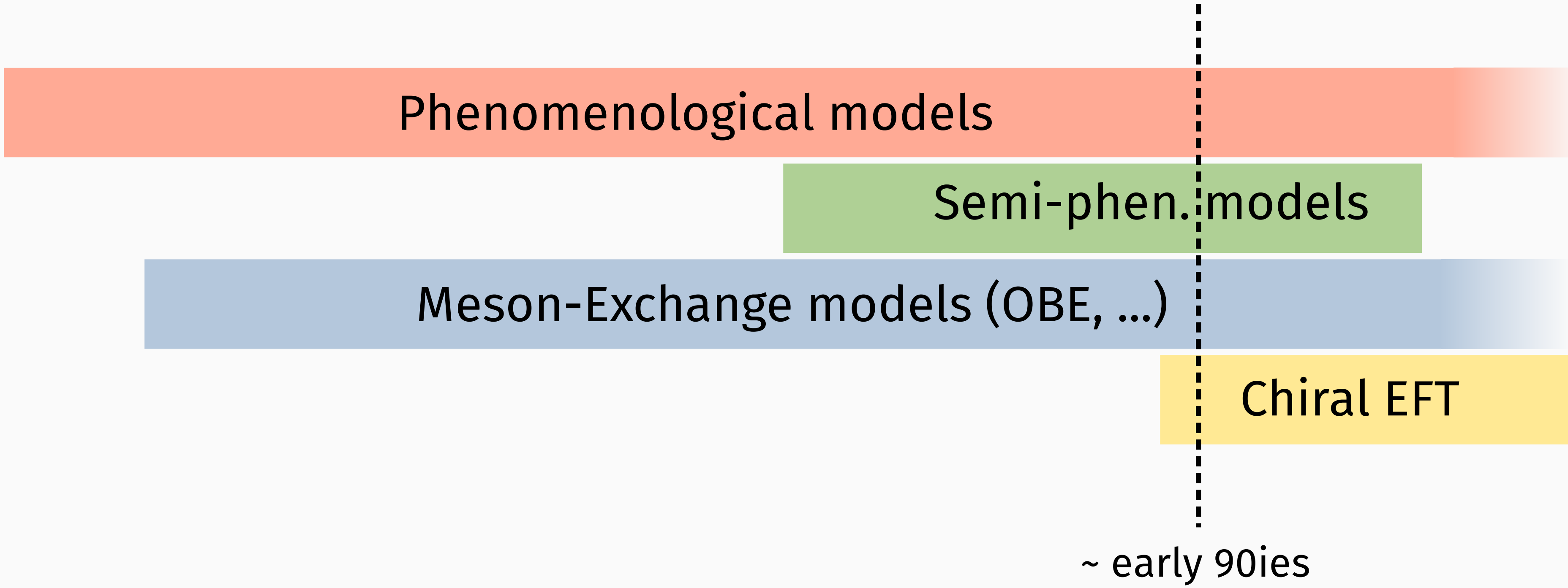
Motivation

- Proper description of the strong force is a long-standing problem
- Formulate nuclear quantum-mechanical many-body problem with nucleons as degrees of freedom
- Computational advances in ab initio methods
  - **Precise forces required**
- Need for consistent many-body forces and currents



A short history of NN potentials



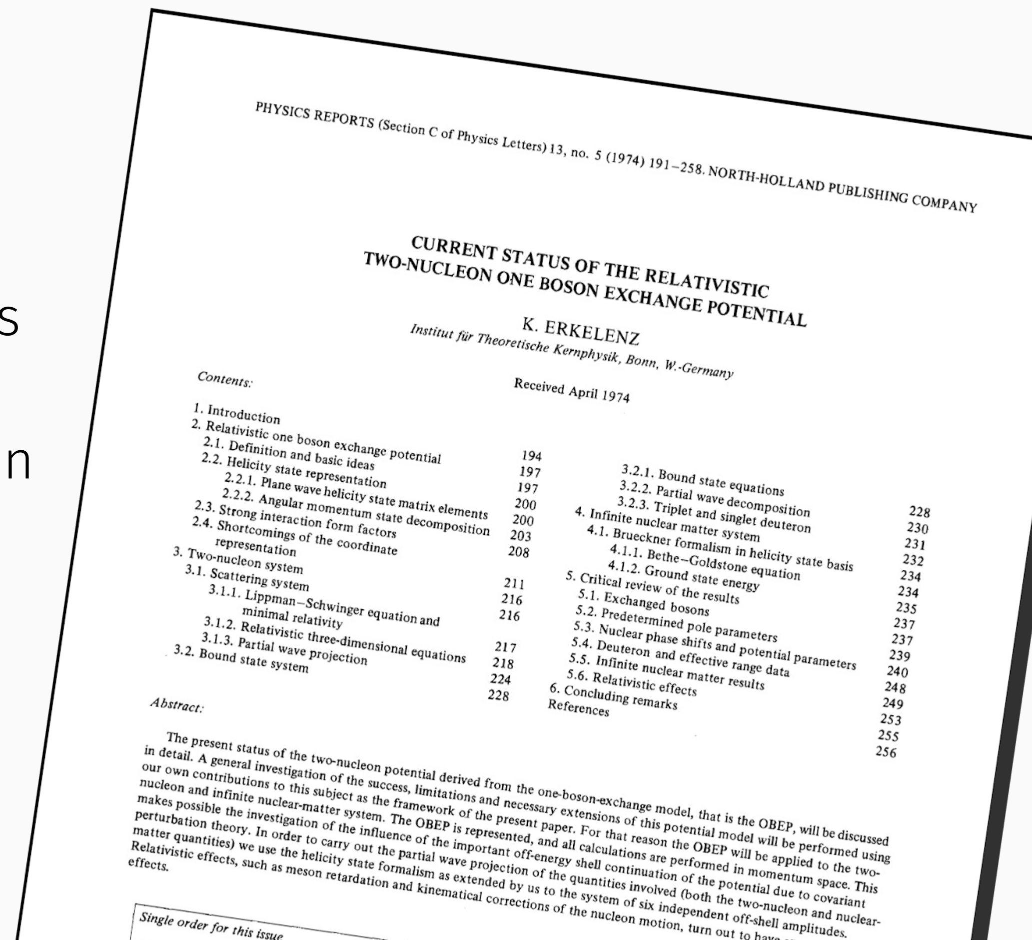
- 1935: One-Pion-Exchange (Yukawa)
- 1960ies: Phenomenological potentials & One-Boson-Exchange (OBE)
- 1970ies: Bonn Potential (Erkelenz)
- 1990ies: Nijmegen PWA & semi-phenomenological potentials achive $\chi^2/\text{datum} \sim 1$ description, Chiral EFT as QCD-inspired approach

Erkelenz‘ work in the light of today

Then: **OBE model** ⟷ Now: **Chiral EFT**

Driven by the same goals as today:

- Rejection of phenomenological models
  - too many parameters
  - Performance beyond the two-nucleon system
  - Derivation of potential from field theory



PHYSICS REPORTS (Section C of Physics Letters) 13, no. 5 (1974) 191–258. NORTH-HOLLAND PUBLISHING COMPANY

CURRENT STATUS OF THE RELATIVISTIC TWO-NUCLEON ONE BOSON EXCHANGE POTENTIAL

K. ERKELENZ
Institut für Theoretische Kernphysik, Bonn, W.-Germany

Received April 1974

Contents:

Abstract:

The present status of the two-nucleon potential derived from the one-boson-exchange model, that is the OBEP, will be discussed in detail. A general investigation of the success, limitations and necessary extensions of this potential model will be performed using our own contributions to this subject as the framework of the present paper. For that reason the OBEP will be applied to the two-nucleon and infinite nuclear-matter system. The OBEP is represented, and all calculations are performed in momentum space. This makes possible the investigation of the influence of the important off-energy shell continuation of the potential due to covariant perturbation theory. In order to carry out the partial wave projection of the quantities involved (both the two-nucleon and nuclear-matter quantities) we use the helicity state formalism as extended by us to the system of six independent off-shell amplitudes. Relativistic effects, such as meson retardation and kinematical corrections of the nucleon motion, turn out to have … effects.

Single order for this issue

Chiral nuclear EFT

- Degrees of freedom: Nucleons & Pions

symmetries

QCD

Construct most general Langrangians:

$$\mathcal{L} = \mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \mathcal{L}_{NN}^{(4)} + \ldots$$

derive perturbatively, e.g.

πN amplitude

NN potential

arrange in powers of **Q**

(power counting)

Chiral nuclear EFT

- Degrees of freedom: Nucleons & Pions

symmetries ← QCD

Construct most general Langrangians:

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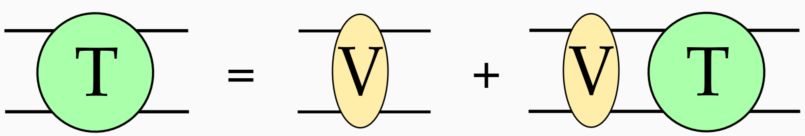
arrange in powers of **Q** (power counting)

derive perturbatively, e.g.

πN amplitude | **NN potential**

Nuclear forces require non-perturbative resummation

- Iterations UV-divergent →apply regulator



Expansion in Q →

- Systematic improvement
- Uncertainty estimation

Hierarchy of nuclear forces

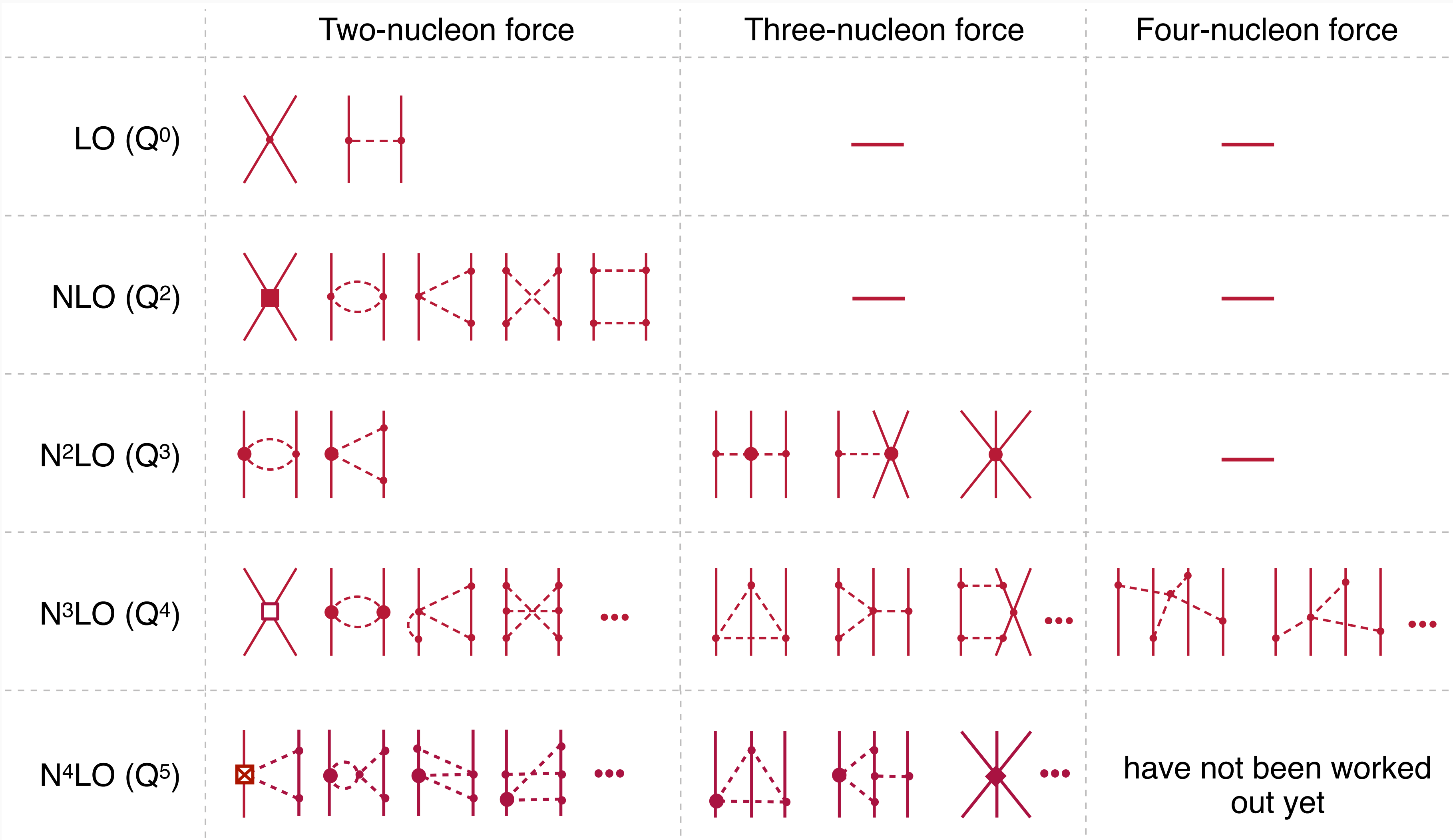
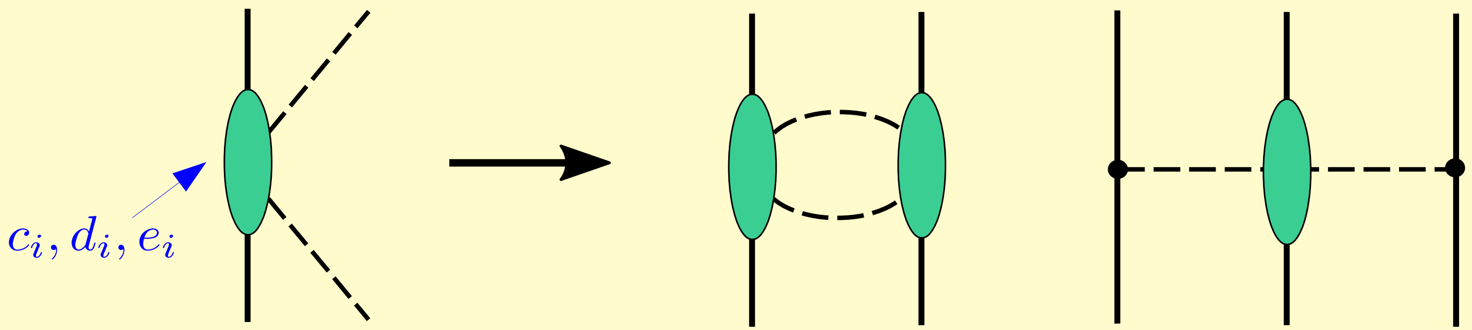


Figure courtesy of E. Epelbaum

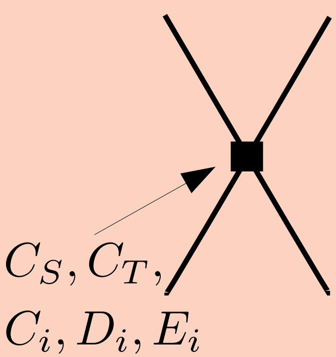
Parameters in chiral nuclear EFT

Consistent derivation of many-body forces & currents from same Lagrangian:

Long-range part of nuclear forces due to pion exchanges:



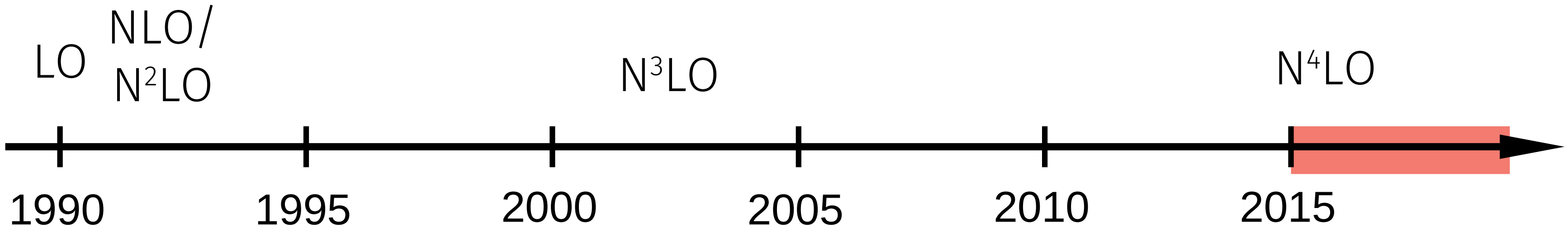
- **πN LECs** from recent Roy-Steiner eq. analysis of πN scattering **[Phys. Rev. Lett. 115, 192301]** (Dr. Klaus Erkelenz Preis 2015)
- Parameter-free for nuclear forces



Short-range part of 2NF due to contact interactions

- **NN LECs** determined from NN scattering data, B_d=2.224575(9) MeV and b_{np}=-3.7405(9) fm
- Use self-consistent 2013 Granada database **[Phys. Rev. C 88.064002]**

Current generation of chiral NN potentials



New generation of N^4LO forces

- Semilocal coordinate-space regularized (EKM) [EPJA 51 (2015) 53; PRL 115 (2015) 122301]
- Semilocal momentum-space regularized (SMS), this work
- Nonlocal momentum-space regularized (EMN) [PRC 96 (2017) 024004]

Also:

- Interactions fitted to properties of heavier nuclei (e.g. NNLOsat [PRC 91 (2015) 051301]), Delta-full,...

Important differences in regularization & fitting protocol!

Regularization of SMS forces

“Semilocal momentum-space regularized (SMS) NN forces“

Non-local regulator for short-range contacts (same as EKM)

$$\langle p'|V_{\text{cont}}|p\rangle_{\text{reg.}} = \langle p'|V_{\text{cont}}|p\rangle \exp\left[-\frac{p'^2+p^2}{\Lambda^2}\right]$$

+

New **local** regularization for long-range potential *in momentum space*

Regularization of long-range part

EKM potential used local regularization in coordinate space:

$$V_{\pi,\mathrm{reg}}(\vec{r}) = V_\pi(\vec{r}) \left[1 - e^{-\frac{r^2}{R^2}}\right]^n$$

✓ Minimization of long-range cutoff artifacts
✗ inconvenient for currents & N^{3}LO 3NF

New local regularization in momentum space (inspired by **Annals Phys. 208, 253**):

Main idea: regularize static pion-propagators with gaussian form factor $\frac{1}{\vec{l}^2 + M_\pi^2} \to \frac{1}{\vec{l}^2 + M_\pi^2} e^{-\frac{\vec{l}^2 + M_\pi^2}{\Lambda^2}}$

e.g. $$V_{1\pi,\mathrm{reg.}}(q) \propto \frac{1}{q^2 + M_\pi^2} e^{-\frac{q^2 + M_\pi^2}{\Lambda^2}} = \frac{1}{q^2 + M_\pi^2} + \text{short-range terms}$$

- Polynomial short-range contribution chosen such that $V(r)|_{r=0} = 0$ and $\left.\frac{d^n V}{dr^n}(r)\right|_{r=0} = 0$

  ✓ similar to previous coordinate-space regulator
  ✓ scheme extendable to many-body forces & currents

χ^2/datum (Λ=450 MeV)

E_{lab} bin	LO	NLO	N^2LO	N^3LO	N^4LO	N^4LO^+
neutron-proton scattering data						
0–100	73	2.2	1.2	1.07	1.07	1.07
0–200	62	5.4	1.7	1.09	1.08	1.06
0–300	75	14	4.2	2.01	1.16	1.06
proton-proton scattering data						
0–100	2290	10	2.2	0.90	0.88	0.86
0–200	1770	90	37	1.99	1.42	0.95
0–300	1380	90	41	3.43	1.67	1.00

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	2 + 1 IB LECs	+7 LECs	No new LECs	+12 LECs	+1 IB LEC	+4 LECs

- (Almost) parameter-free improvements show importance of chiral 2π-exchange

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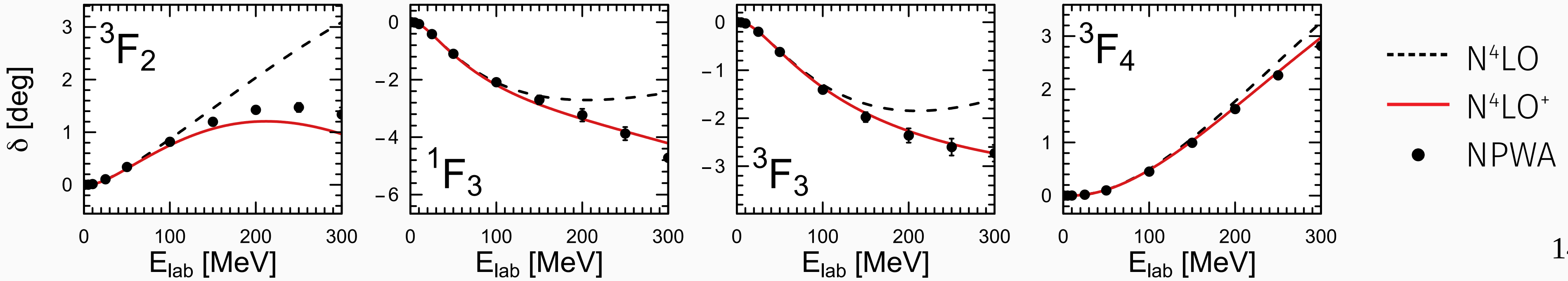
2 + 1 IB LECs — +7 LECs — +12 LECs — +4 LECs

N^{4}LO$^+$ = N^{4}LO + leading F-Wave contacts: $\langle {}^SF_j,\, p'|V_{\text{cont}}|{}^SF_j,\, p\rangle = E_{i_F} p^3 p'^3$



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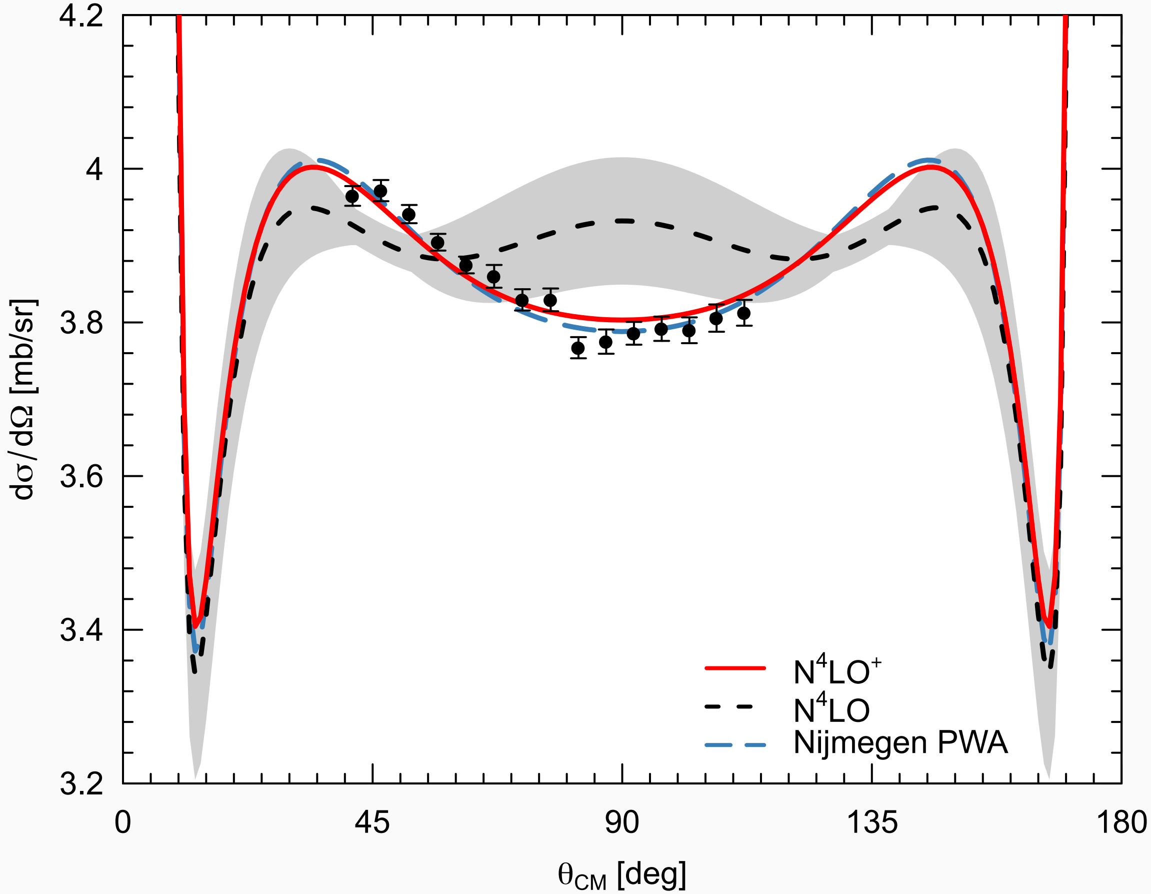
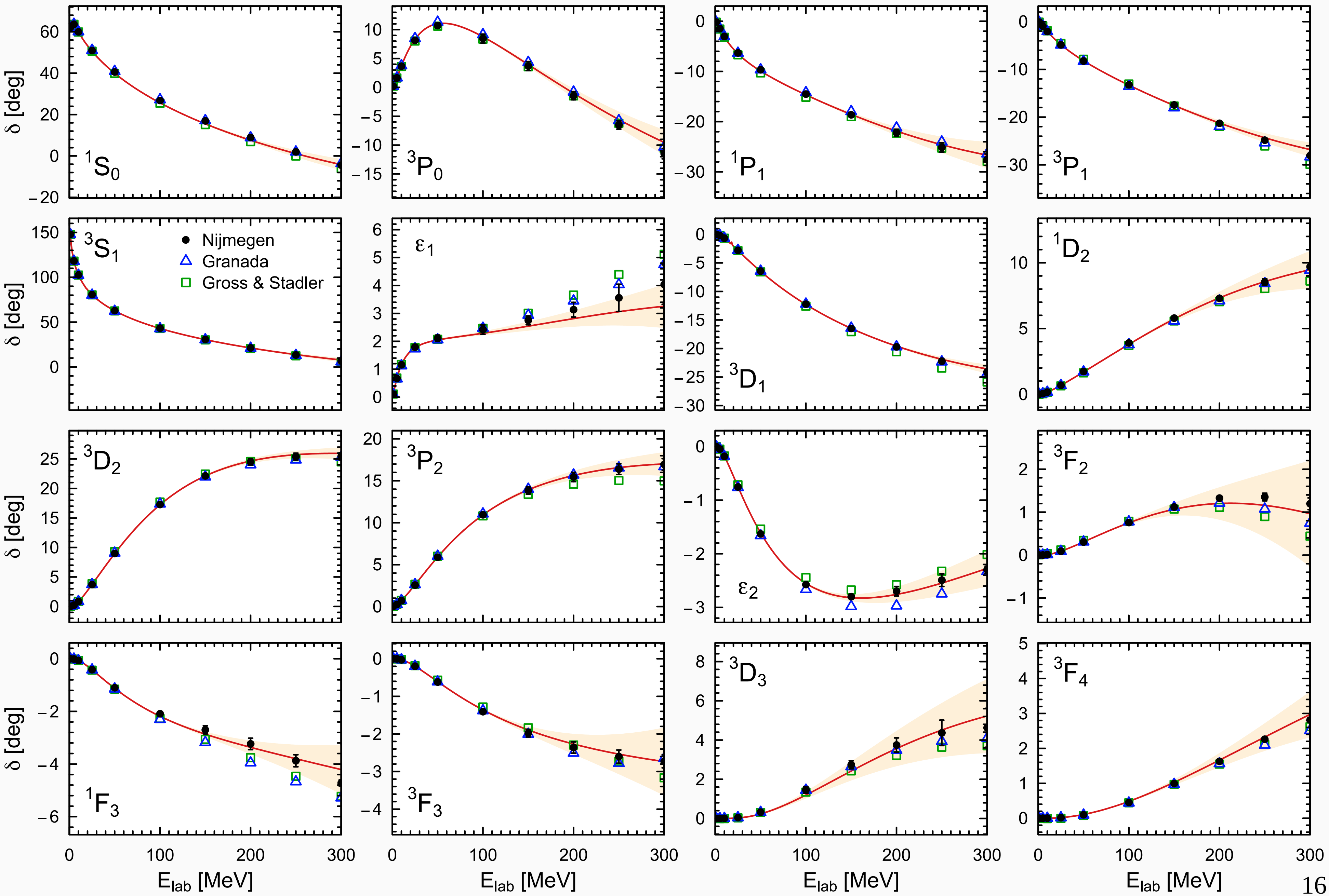


About this dataset:

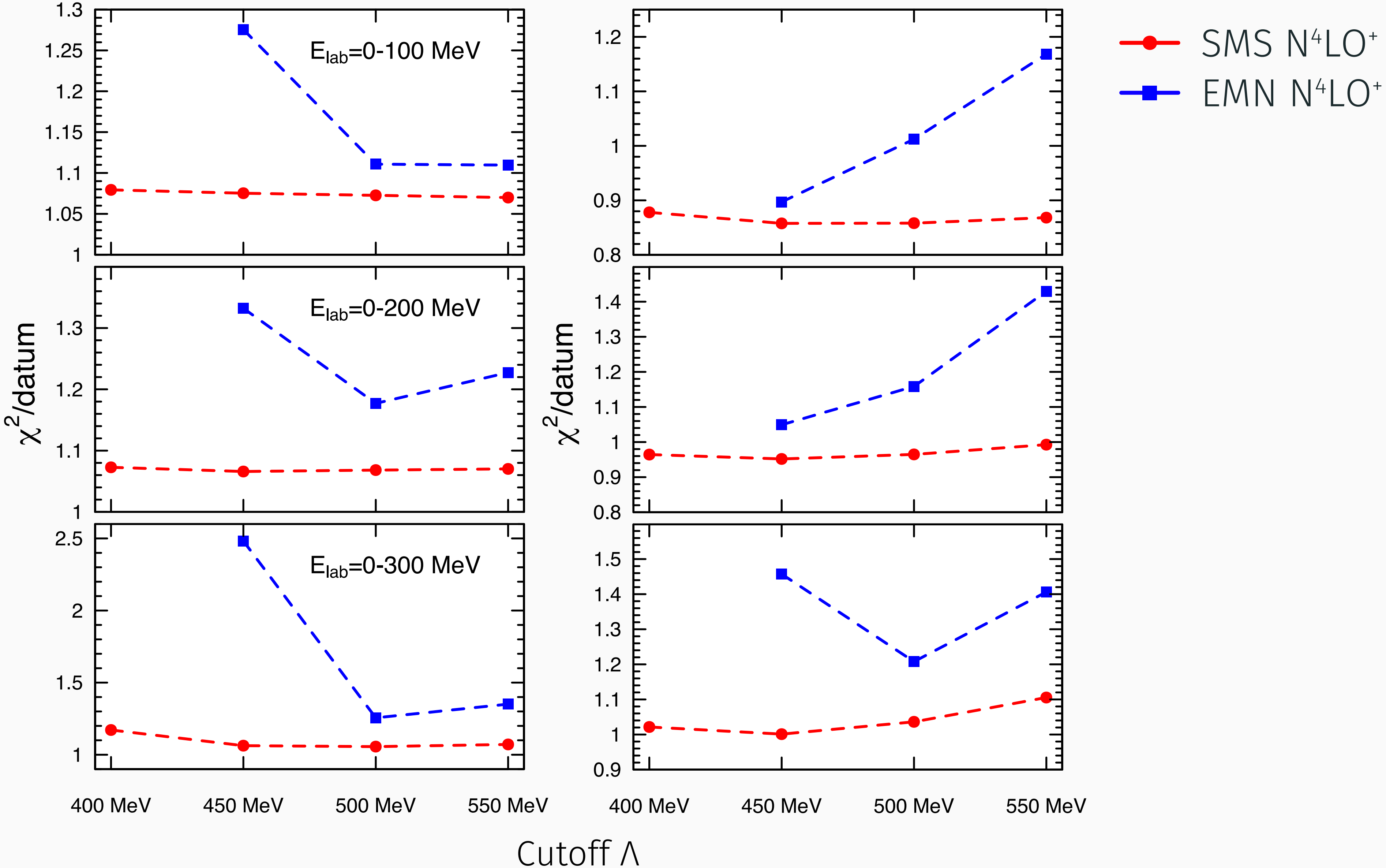
- differential cross section dσ/dΩ
- E_{lab} = 144.1 MeV
- experimental errors ~ 0.5%

- Need accurate F-Waves (in particular $^3F_2$) at energies ~ 150 MeV to describe such observables well
- Estimated truncation error and naturalness of F-Wave LECs suggest no failure of power counting

Phaseshifts N⁴LO⁺ (Λ = 450 MeV)



χ^2/datum Cutoff Dependence



Best precision for Λ = 450 MeV in 2N system

Quantile-Quantile plot

Check assumption: $r_i = \frac{O_i^{\text{exp}} - O_i^{\text{th}}}{\Delta O_i^{\text{exp}}} \sim \mathcal{N}(0,1)$

- Plot $F_{\text{emp}}^{-1}(x)$ vs $F_{\text{th}}^{-1}(x)$

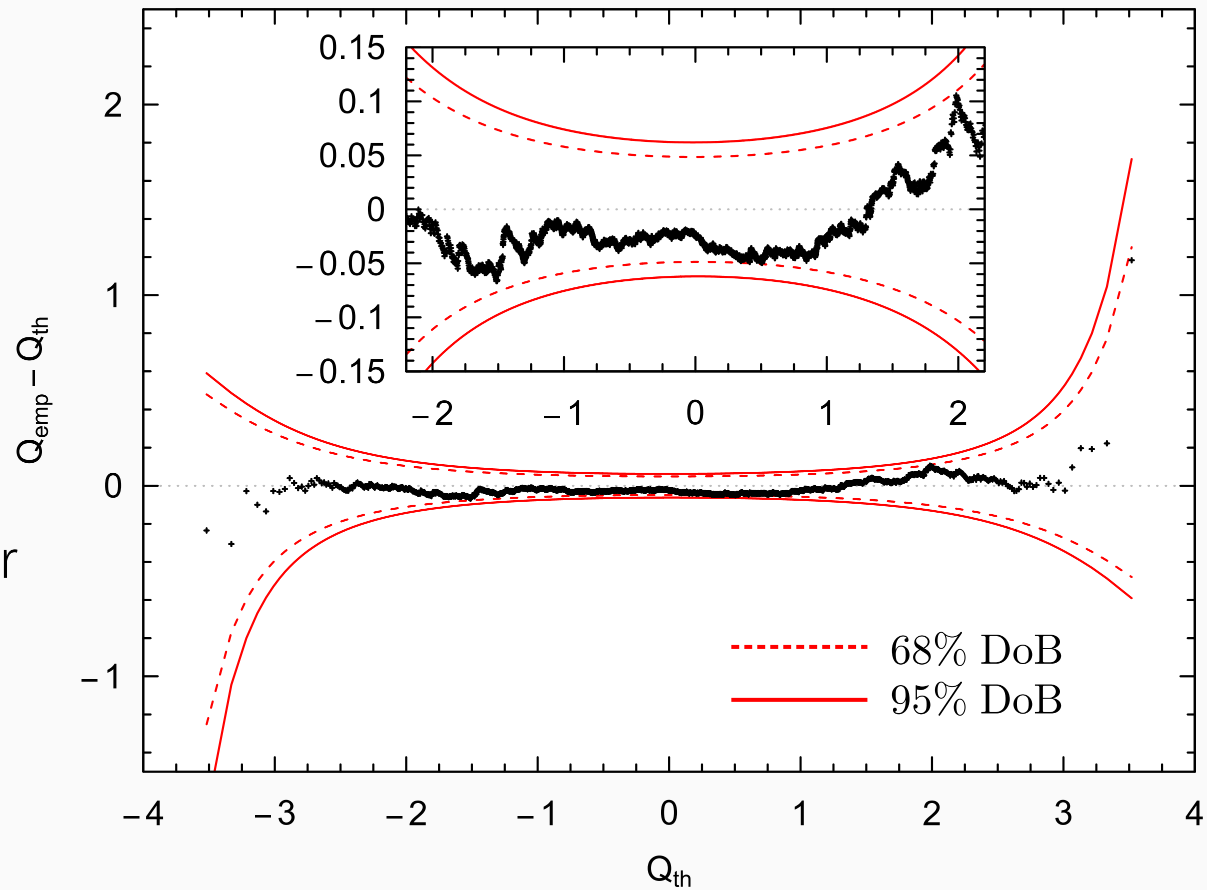
$$F_{\text{emp}}(x) = \frac{1}{N}\sum_{i=1}^{N}\theta(x - r_i)$$

$$F_{\text{th}}(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} dt e^{-t^2/2}$$

- Tail-sensitive error bands for Q-Q plot **Am. Stat. Phys. 67, 249**

➡ High quality of fit confirmed at N^{4}LO$^+$

$\Lambda = 450\,\text{MeV}$



χ^2 Comparison with other potentials

E_{lab} bin	CD Bonn	Nijm I	Nijm II	Reid93	Idaho $\mathrm{N^3LO}^*$	EMN $\mathrm{N^4LO}^{+*}$	SMS $\mathrm{N^4LO}^{+\dagger}$
neutron-proton scattering data							
0–100	1.07	1.06	1.07	1.08	1.17	1.11	1.07
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0–300	1.01	1.03	1.05	1.04	1.39	1.21	1.00
No. of Parameters	43	41	47	50	28+x	29+x	27+1

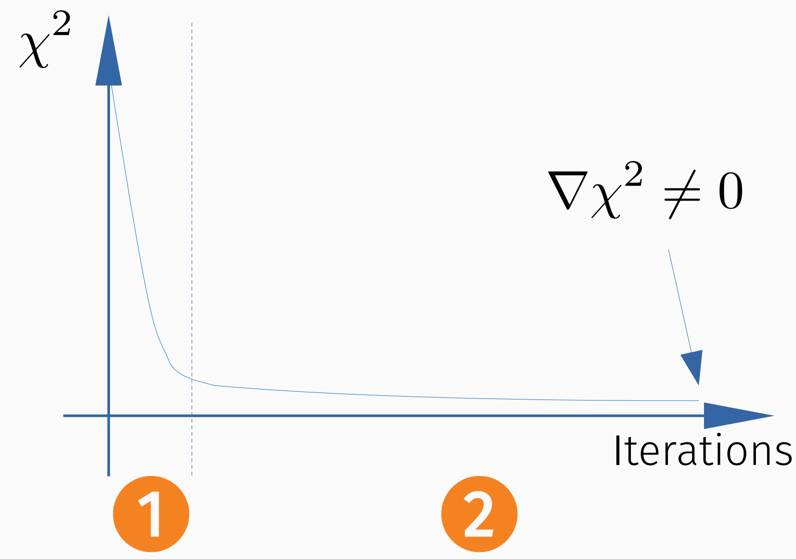
$^*\Lambda = 500$ MeV , $\quad ^\dagger\Lambda = 450$ MeV

SMS $\mathrm{N^4LO^+}$ achieves same precision as high-precision phenomenological potentials with notably less parameters

Redundant Contact Interactions at Q⁴

Peculiar convergence behavior of fits:

→Strong correlations?



Look at S-wave contact potentials:

pure off-shell contributions

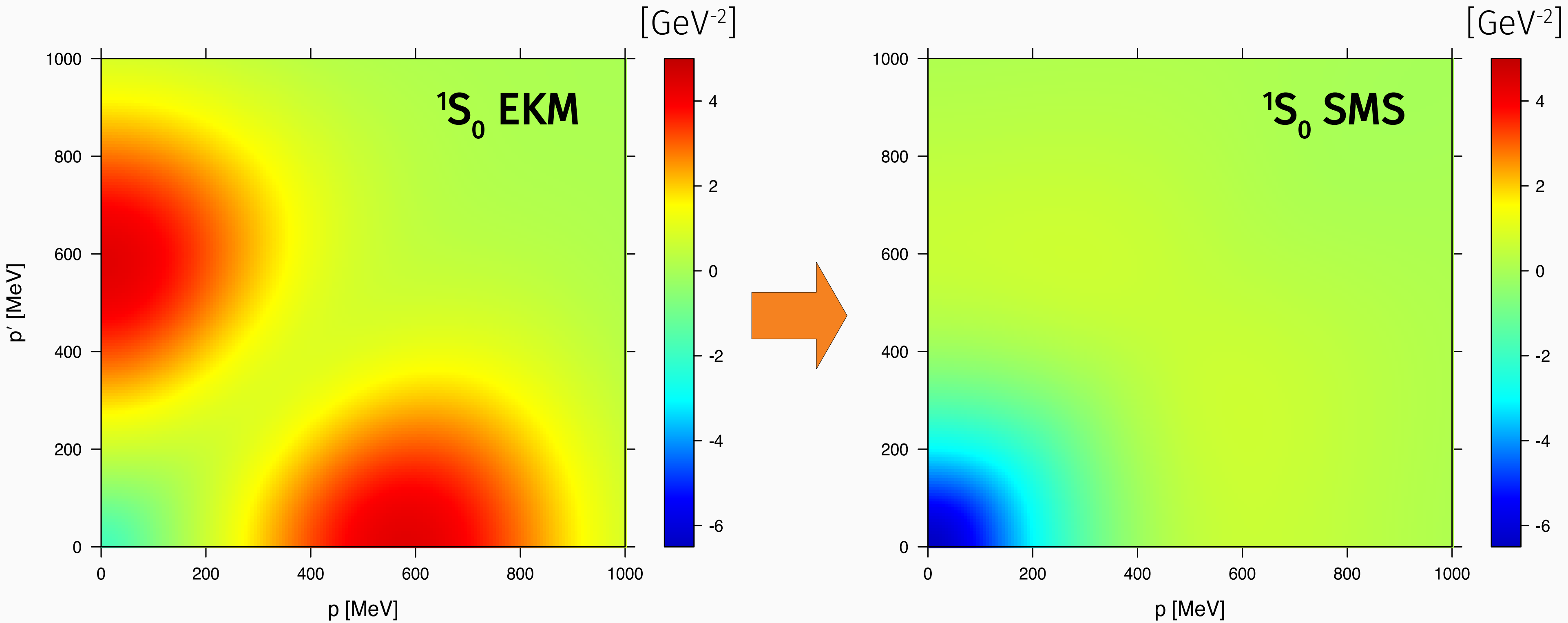
$$\langle {}^1S_0,\, p'|V_{\rm cont}|{}^1S_0,\, p\rangle = \tilde{C}_{1S0} + C_{1S0}(p^2+p'^2) + D_{1S0}p^2p'^2 + D^{\rm off}_{1S0}(p'^2-p^2)^2$$
$$\langle {}^3S_1,\, p'|V_{\rm cont}|{}^3S_1,\, p\rangle = \tilde{C}_{3S1} + C_{3S1}(p^2+p'^2) + D_{3S1}p^2p'^2 + D^{\rm off}_{3S1}(p'^2-p^2)^2$$
$$\langle {}^3S_1,\, p'|V_{\rm cont}|{}^3D_1,\, p\rangle = C_{\epsilon 1}p^2 + D_{\epsilon 1}p^2p'^2 + D^{\rm off}_{\epsilon 1}p^2(p'^2-p^2)$$

Can be changed by unitary transformation

→ affects only values of other contact LECs modulo higher order terms

Redundant Contact Interactions at Q^4

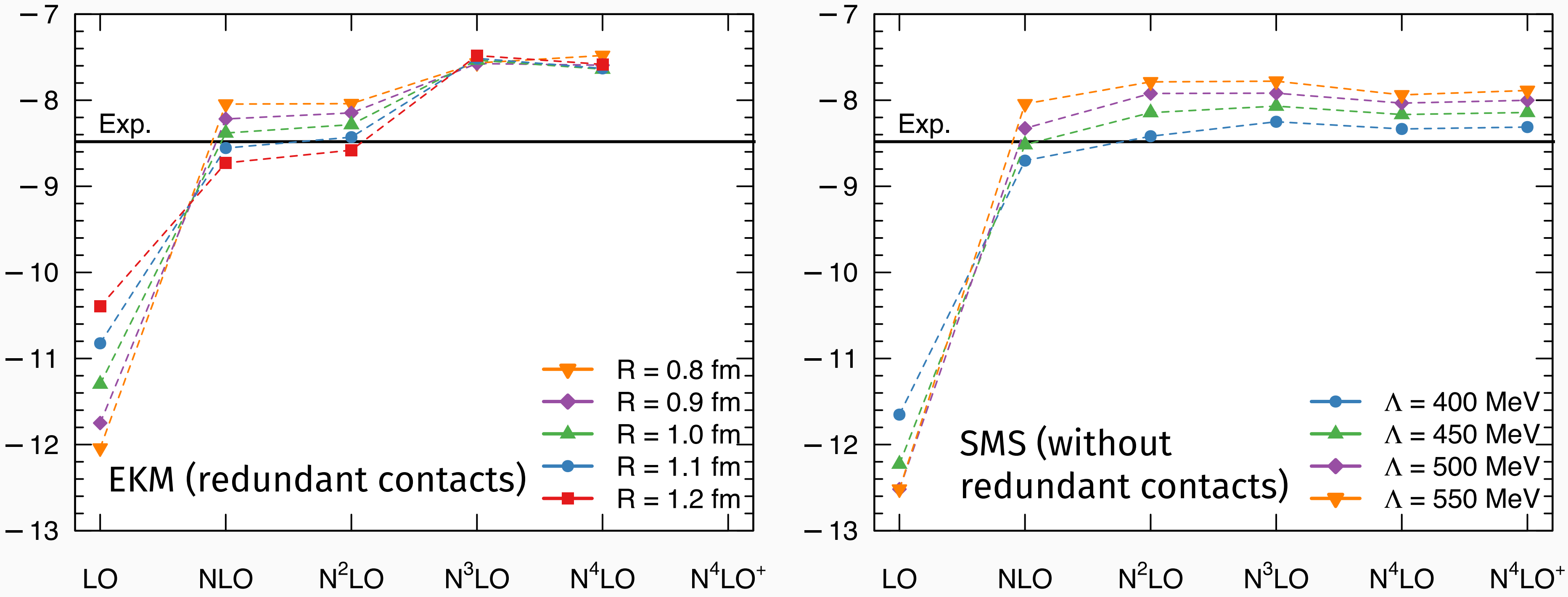
Choice: $D^{\text{off}}_{1S0} = D^{\text{off}}_{3S1} = D^{\text{off}}_{\epsilon 1} = 0$



- Impact on 2N observables negligible
- Leads to softer, more perturbative interactions (confirmed in Weinberg eigenvalue analysis)
- Fast convergence of fits, stable LECs

Triton binding energy

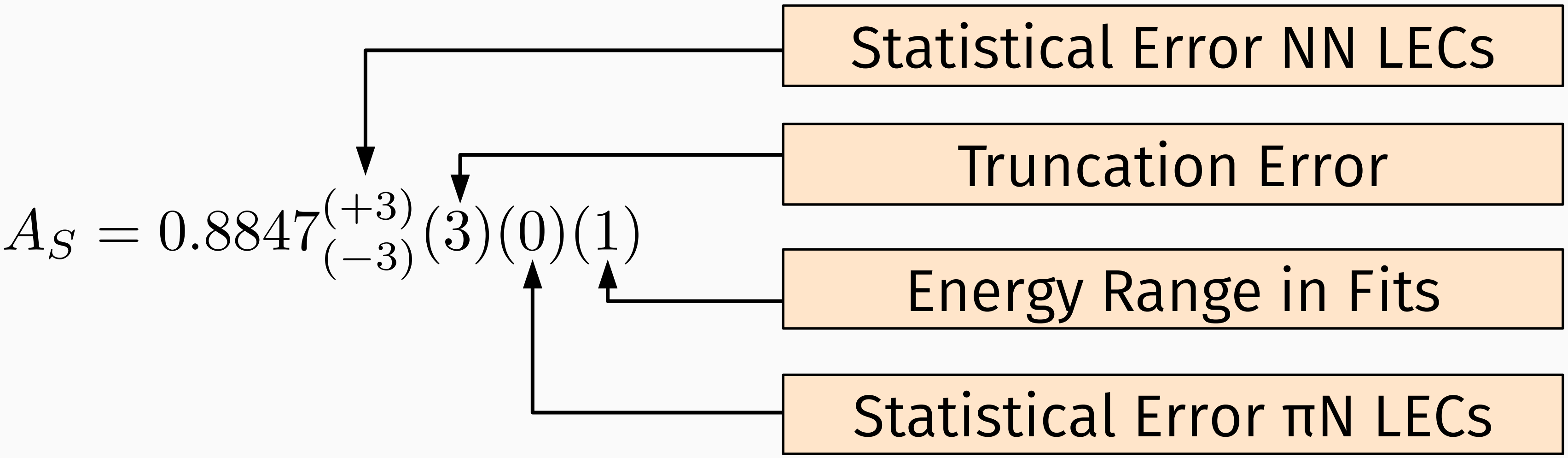
Faddeev equation without 3NF:



- Unitary transformation also induces short-range 3NF
- Smaller 3NF strength to compensate Triton underbinding with new choice
- Caution: 3NF contributions are generally not smaller than before

Uncertainty Estimation

$$A_S = 0.8847^{(+3)}_{(-3)}(3)(0)(1)$$



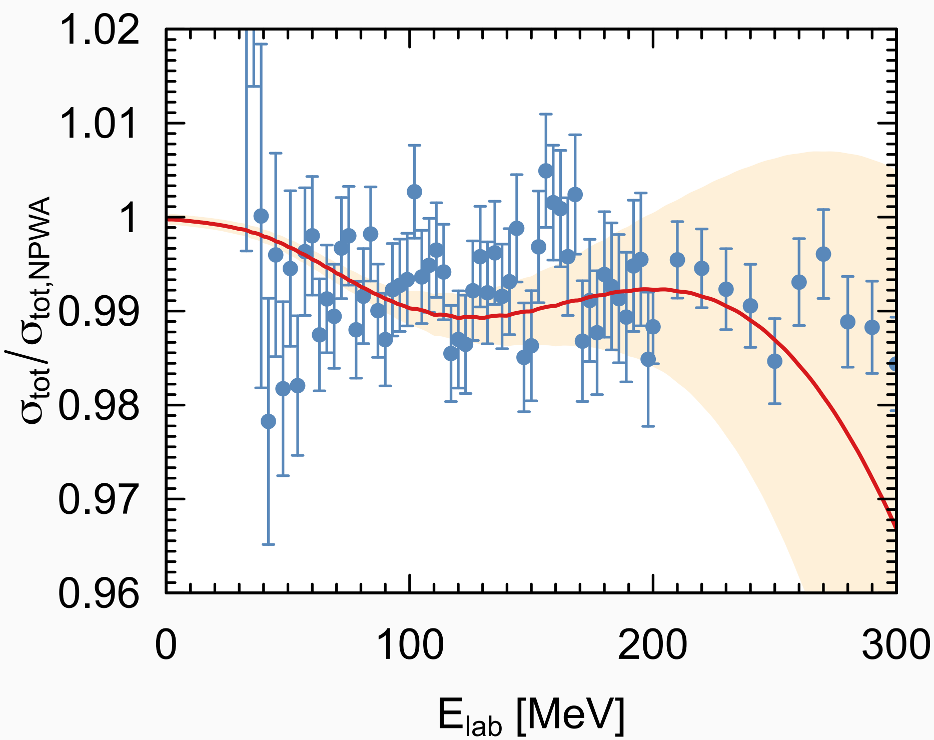
- Statistical errors of NN LECs based on covariance matrix of the fit and second-order Taylor expansion of observables **[Phys. Rev. X 6.011019]**
- Recently switched to Bayesian model for estimation of truncation errors **[Phys. Rev. C 92.024005, Phys. Rev. C 96.024003]**
- For Energy Range Errors, compare observables from fits up to E_{lab} = 220, 260 & 300 MeV
- Finally, propagate statistical uncertainty of Roy-Steiner πN LECs
  → usually smallest error

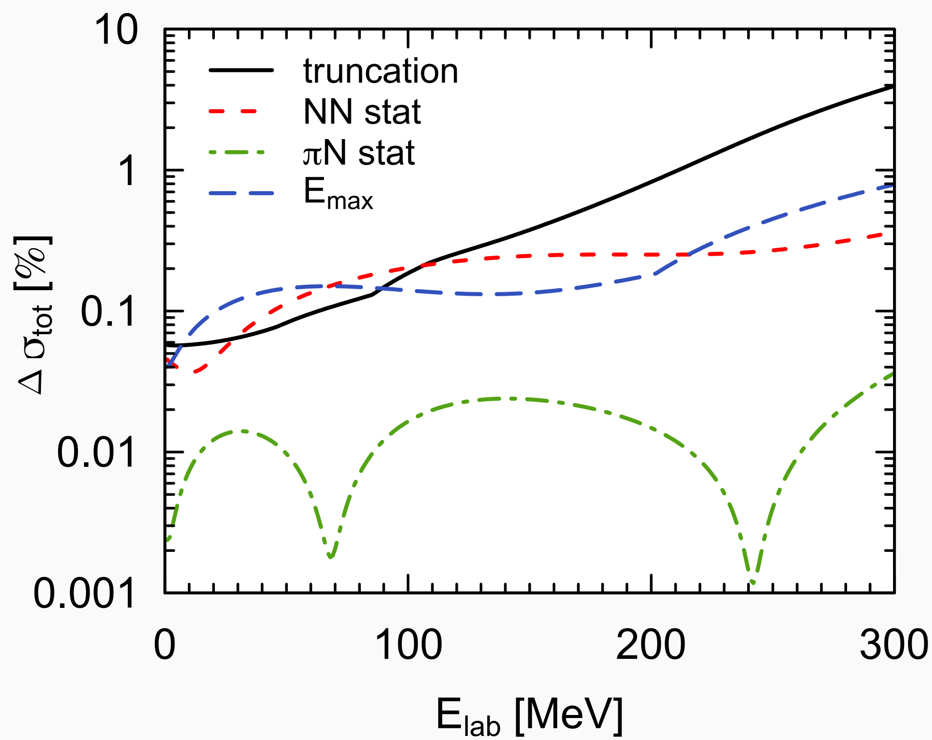
Uncertainty Estimation

$$A_S = 0.8847^{(+3)}_{(-3)}(3)(0)(1)$$

- Statistical Error NN LECs
- Truncation Error
- Energy Range in Fits
- Statistical Error πN LECs

np total cross section:





Isospin-breaking (IB) 2N force

- SMS potential achieves χ^2/datum ~ 1
- But isospin-breaking (IB) limited to pion-mass splitting in One-pion-exchange and charge-dependent short-range interactions in $^1S_0$
- Chiral EFT allows systematic incorporation of IB effects:

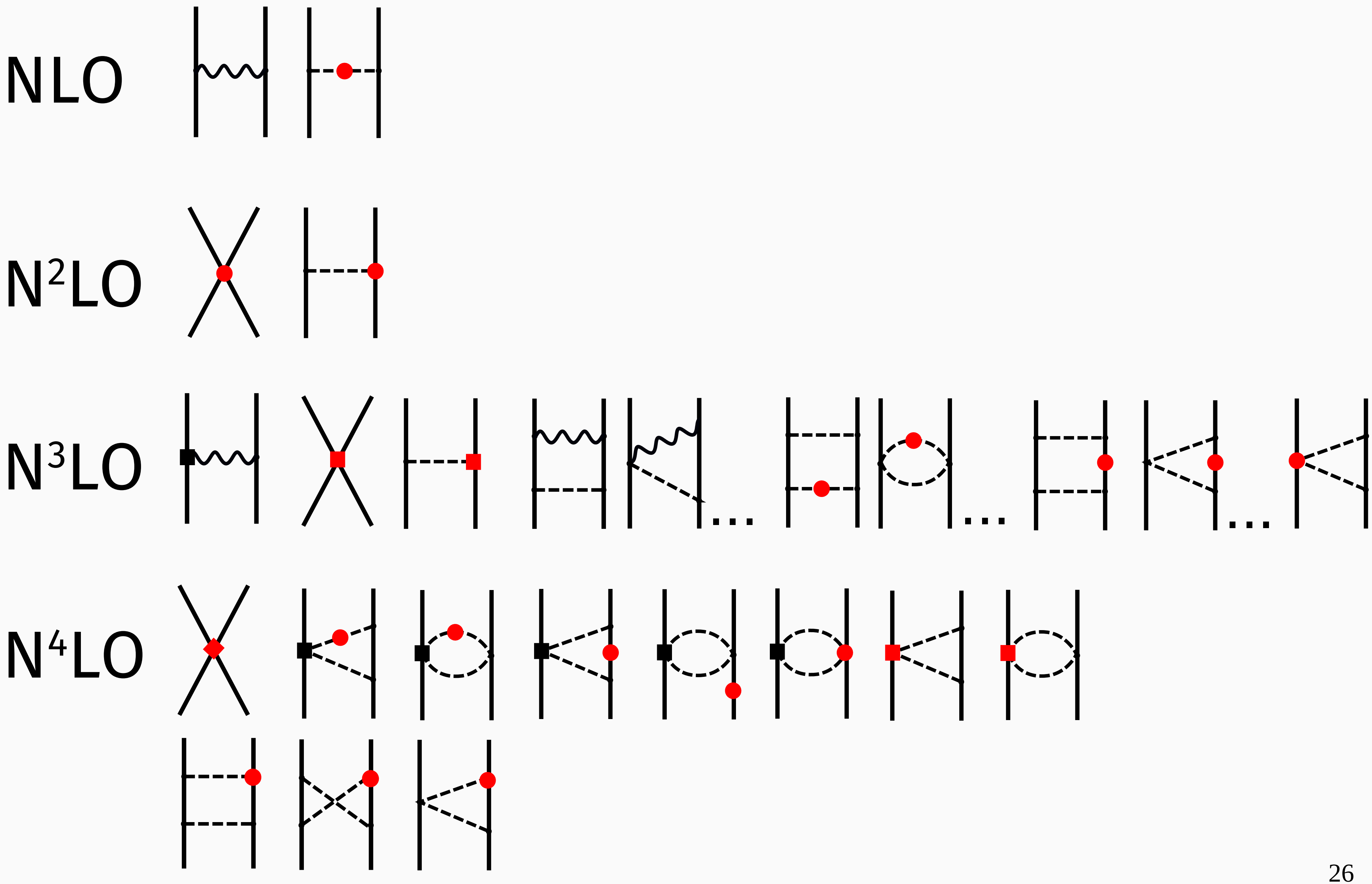
Standard Model:
- strong IB due to $m_d \neq m_u$
- electromagnetic IB due to $q_d \neq q_u$

↓

ChEFT: String of IB interaction vertices between nucleons and pions $\propto \epsilon = \frac{m_d - m_u}{m_d + m_u}$ and e

- Count IB contributions in terms of chiral expansion parameter [Epelbaum & Meißner '04]

Overview of isospin-breaking contributions



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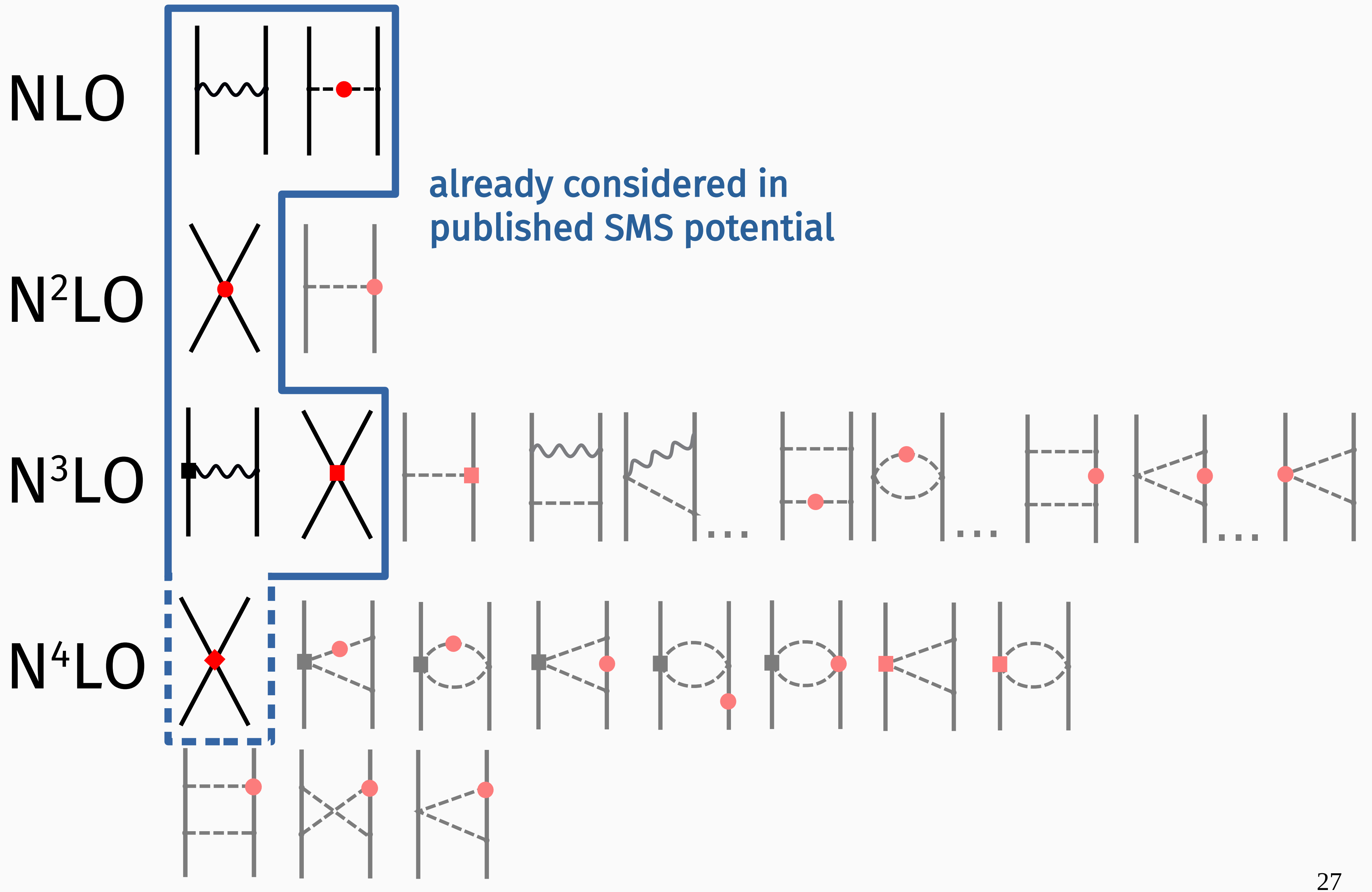

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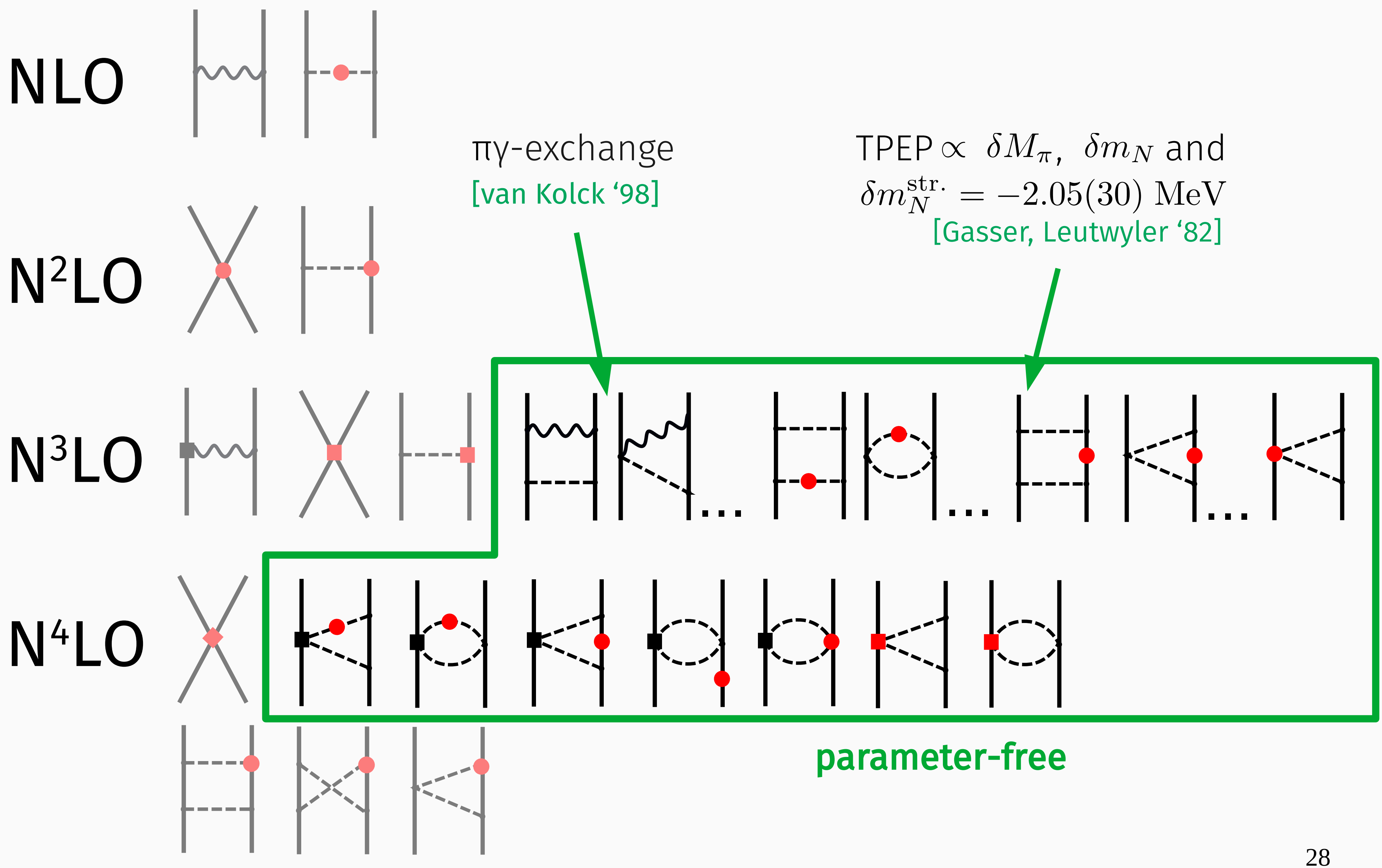

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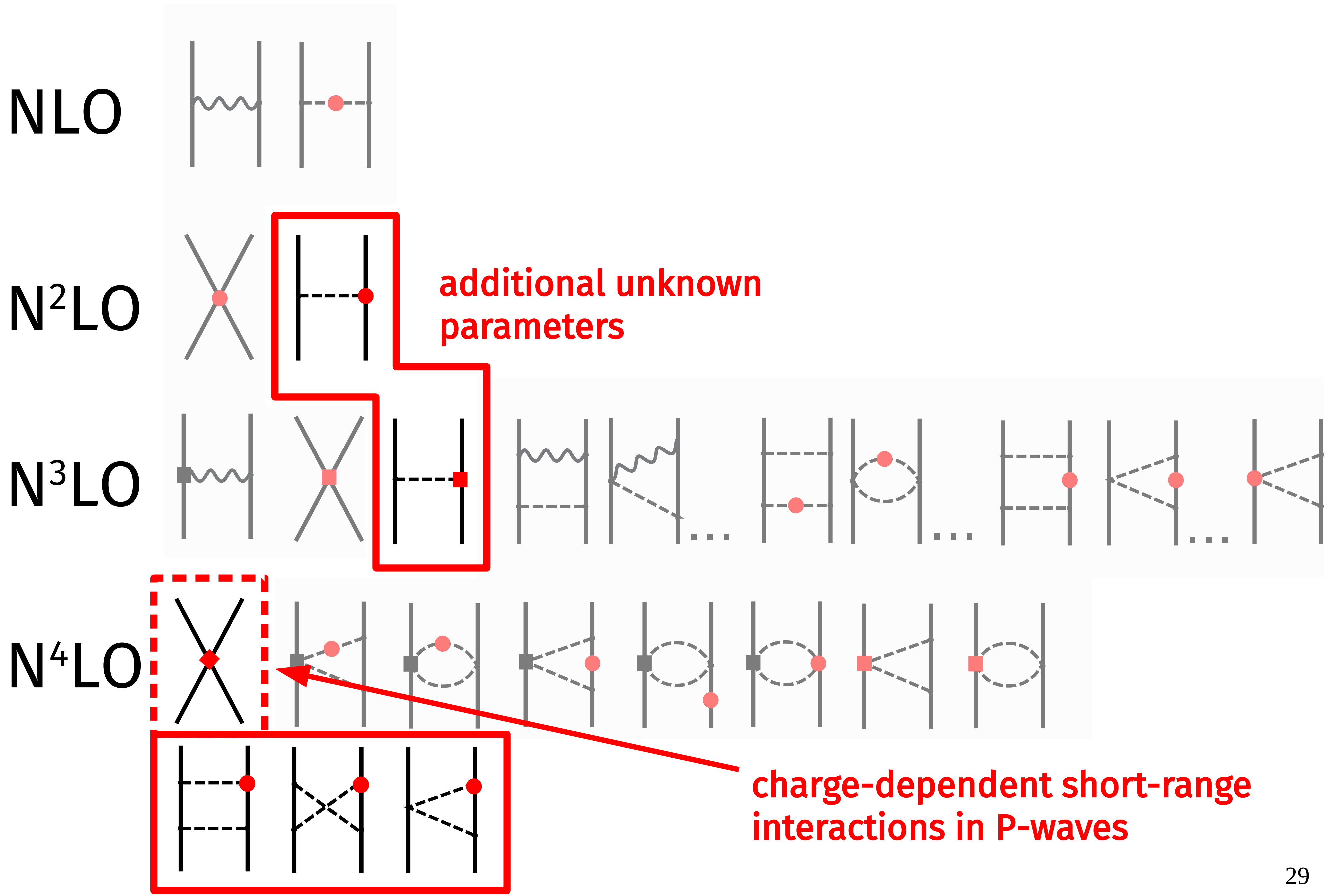
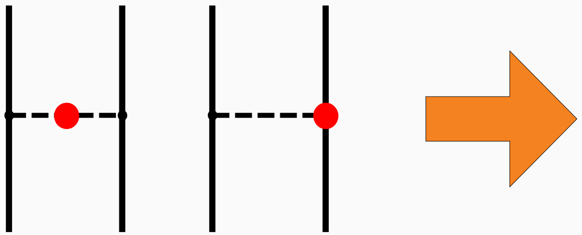


Charge-dependent πNN couplings



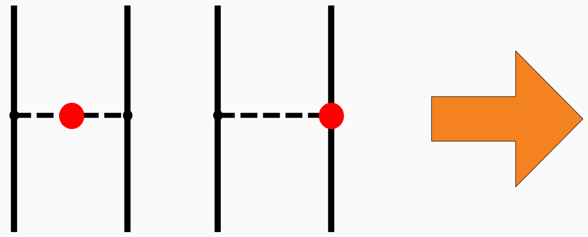
General OPE without isospin limit:

$$V_{1\pi}(pp) = f_p^2\, V_\pi(M_{\pi^0})$$
$$V_{1\pi}(np) = -f_0^2\, V_\pi(M_{\pi^0}) + (-1)^{t+1} 2 f_c^2\, V(M_{\pi^\pm})$$
$$V_{1\pi}(nn) = f_n^2\, V_\pi(M_{\pi^0})$$

with $V_\pi(M_i) = -\frac{4\pi}{M_{\pi^\pm}^2} \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + M_i^2}$, $f_0^2 = f_p f_n$

Long-standing question regarding charge-dependence of the πNN coupling constant!

Charge-dependent πNN couplings



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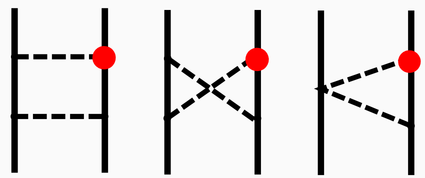
\+ 

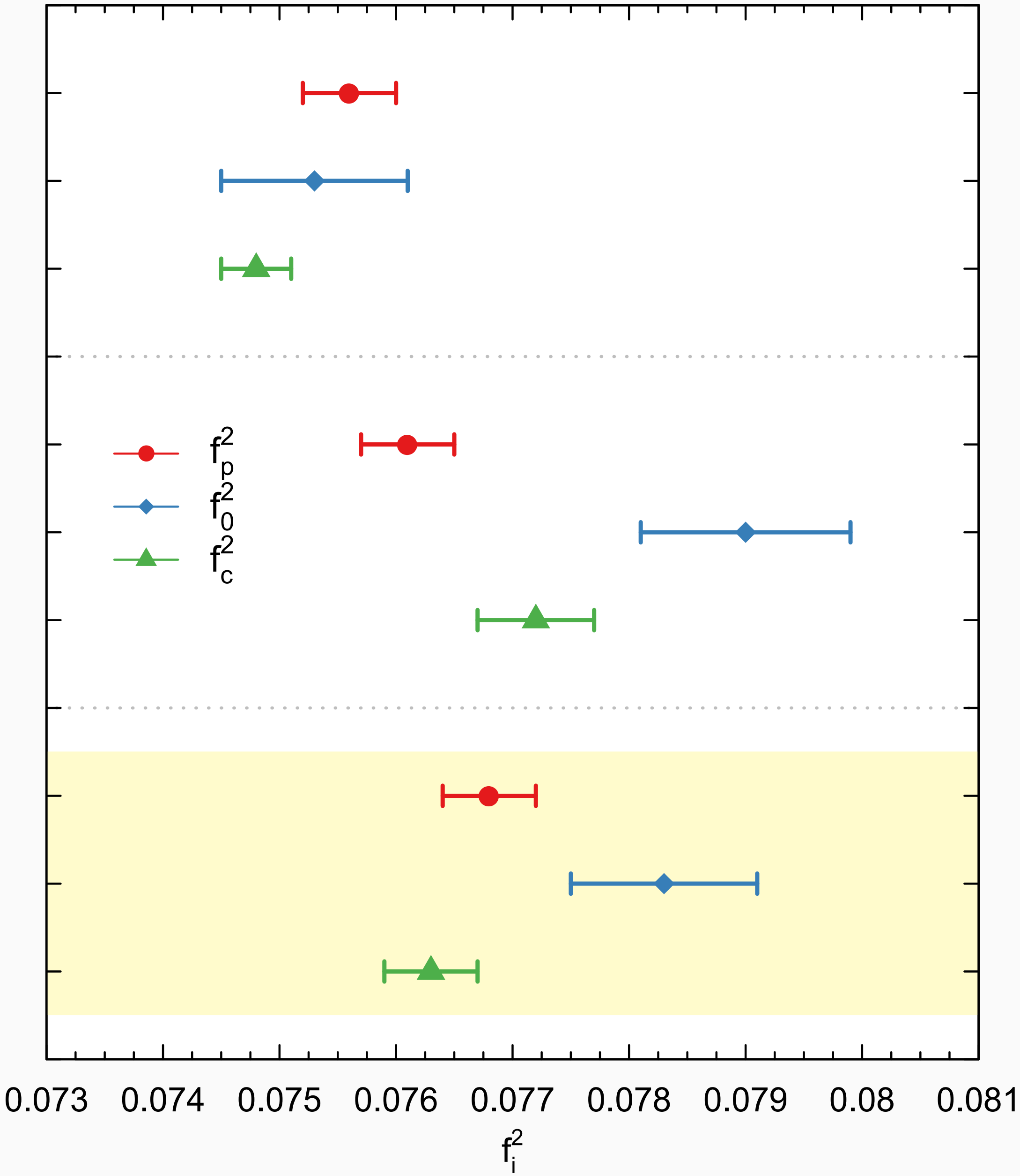
IB two-pion exchange $\propto \mathcal{O}(\Delta f)$

Determine f_i's from NN data

- Combined fit of np- and pp- scattering data from self-consistent 2013 Granada database for E_{lab} = 0 – 280 MeV
- Integrate („marginalize") over Λ and short-range LECs:

$$p(\{f_i^2\}|D) = \int d\Lambda\, dC\, \frac{p(D|\{f_i^2\},\{C_i\},\Lambda)\, p(\{f_i^2\},\{C_i\},\Lambda)}{p(D)}$$

Results for πNN couplings (preliminary)



Nijmegen
[Phys Rev Lett 82, 4992 (1999),

Phys Rev C 44, R1258 (1991),

Phys Rev C 47, 512 (1993)]

Granada '17 [Phys Rev C 95, 064001]

this work

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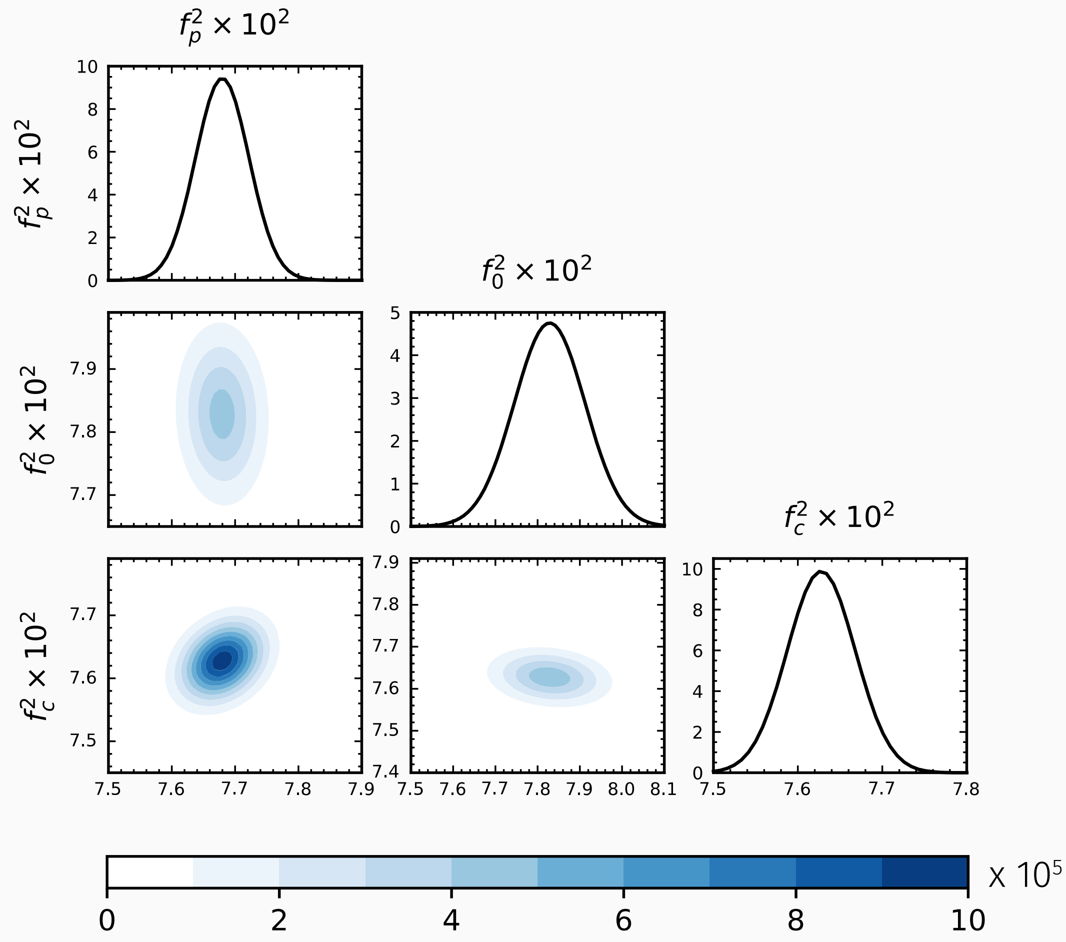
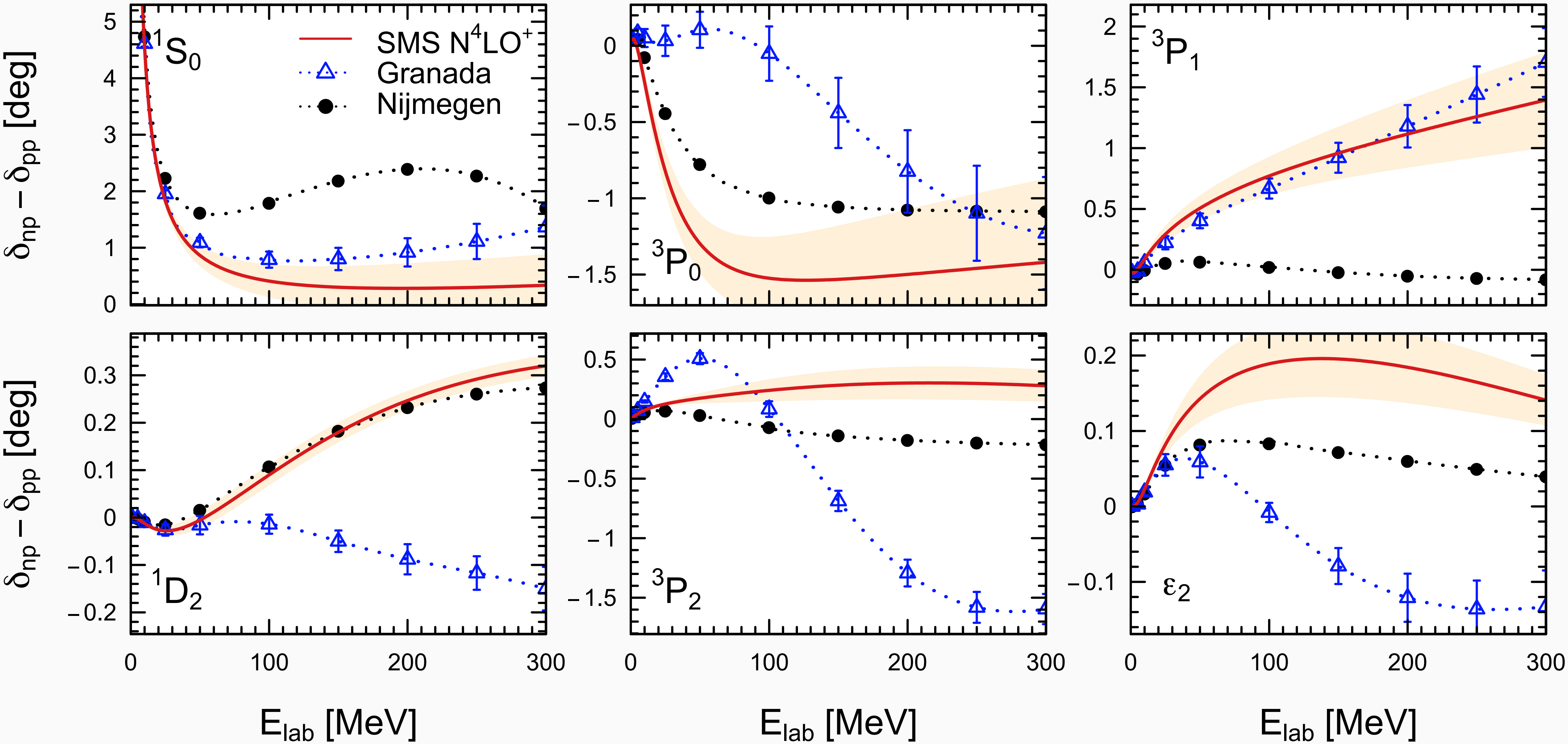


- $p(\{f_i^2\}|D)$ is gaussian to a high degree of approximation
- Integrand over cutoff is strongly peaked at Λ = 453 MeV
- Implicit dependence on data selection by Granada group

What is the impact on 2N data (total IB)?

	Without additional IB effects		With additional IB effects	
χ^2/datum E_{lab} = 0-280 MeV:	**1.022**	→	**1.015**	$(\Lambda = 450\ \mathrm{MeV})$
			+ 6 parameters	

np-pp phaseshift difference (lower partial waves)



preliminary

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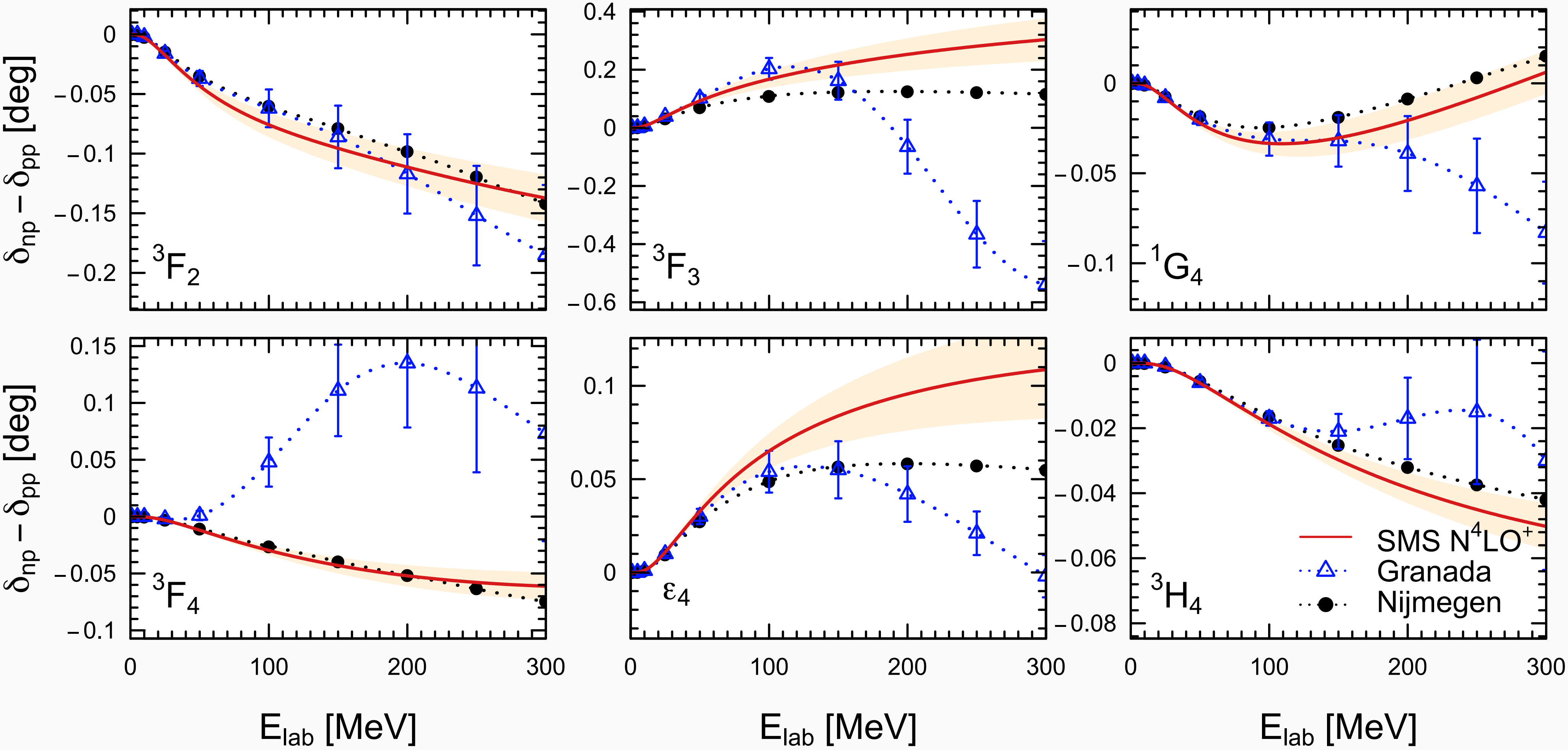


preliminary

Summary

1. The N4LO+ SMS potential is currently the most precise interaction from chiral EFT, rivaling high-quality semi-phenomenological potentials
2. Local regularization in momentum space preserves long-range behavior of the interactions and paves the way for consistent regularization of higher-order 3NF & currents
3. Softer forces due to removal of redundant off-shell contributions to the Q^4 contact potential
4. Complete treatment of isospin-breaking effects up to N^4LO
5. Extraction of charge-dependent πNN couplings using Bayesian methods

Database

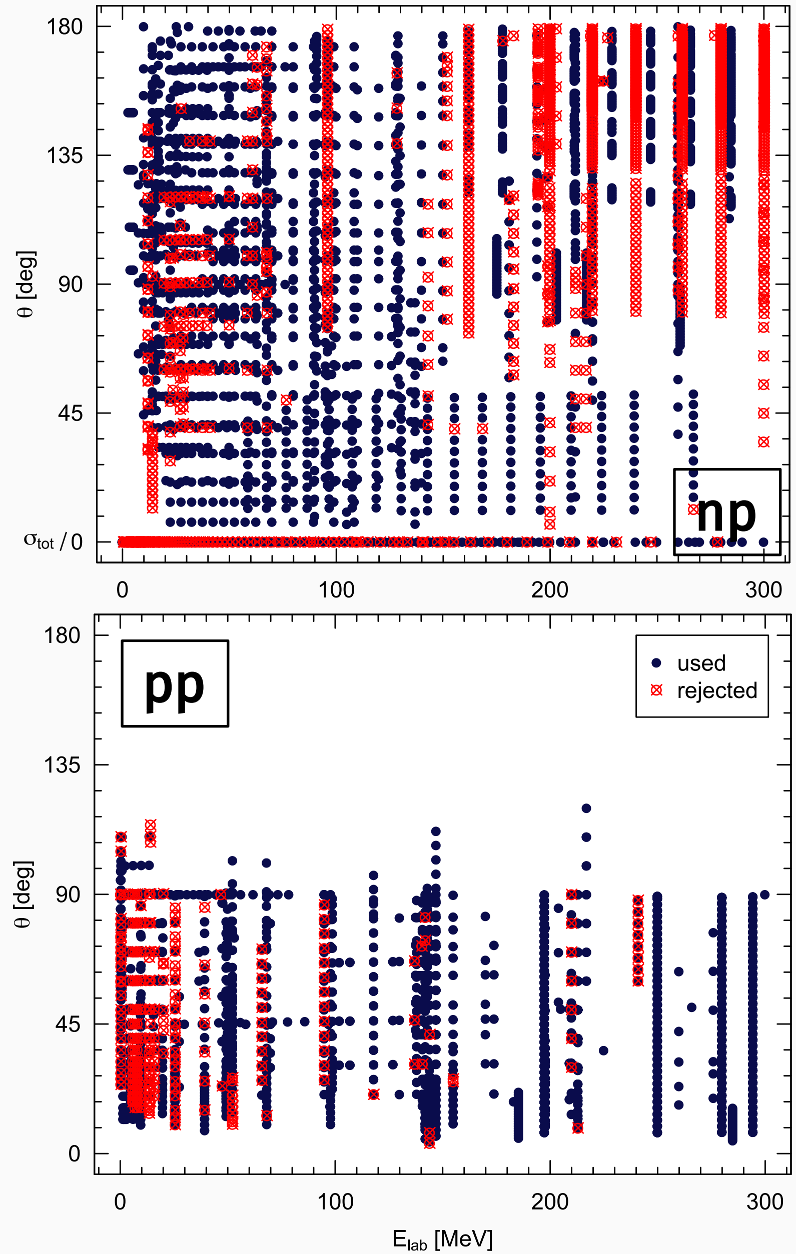
Use self-consistent 2013 Granada database
[Phys. Rev. C 88.064002]

- Includes scattering data from 50ies up to 2013
- uses ”3σ-criterion” to reject non-normal-distributed data
- rejection rate 0-300 MeV: np: 31%, pp: 11%

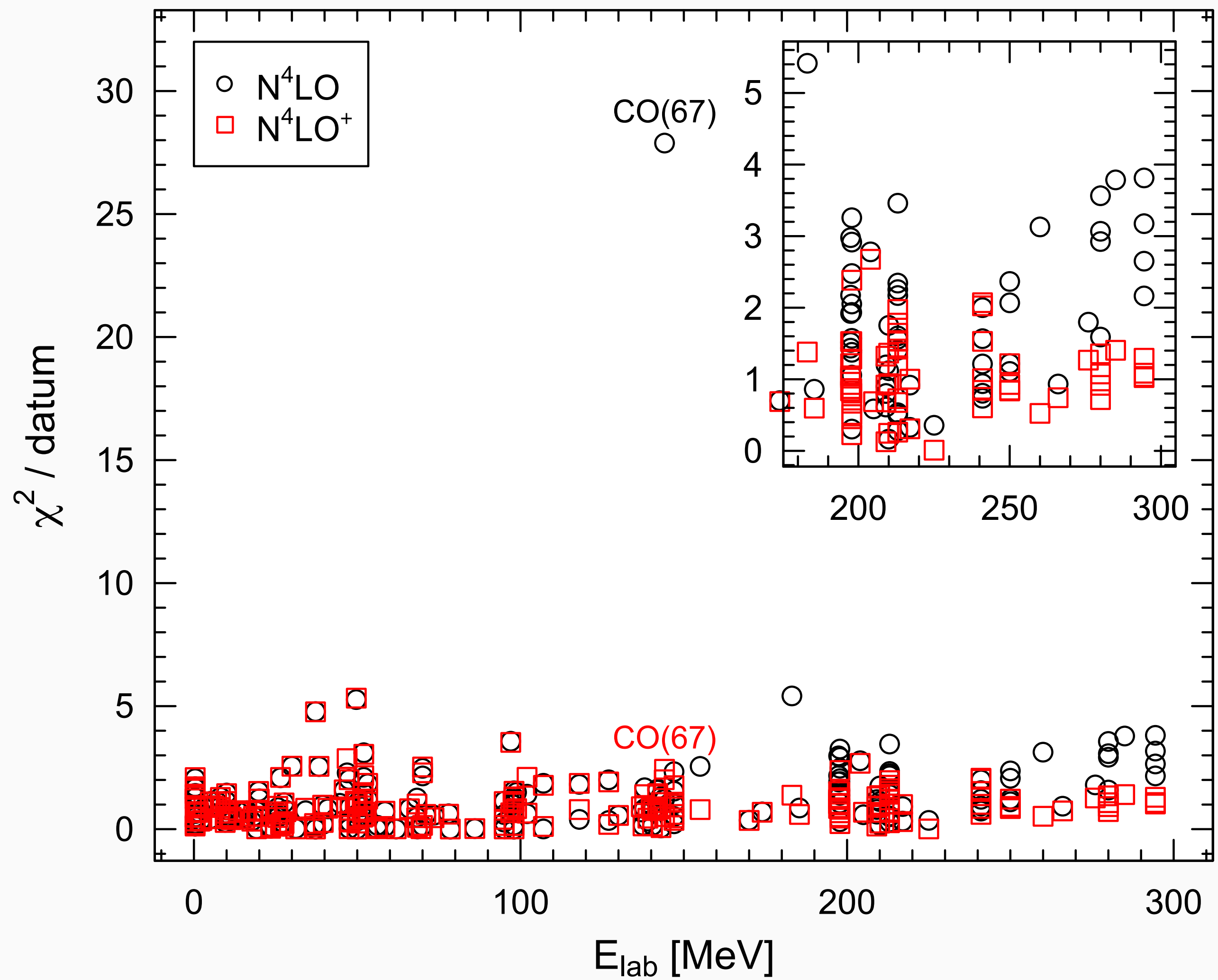
Comparison between theory and experiment via standard χ^2 approach:

$$\chi_j^2 = \sum_{i=1}^{n_j} \left(\frac{O_i^{\mathrm{exp}} - Z O_i^{\mathrm{theo}}}{\delta O_i} \right)^2 + \left(\frac{Z-1}{\delta_{\mathrm{sys}}} \right)^2$$

- Z (inverse relative norm) is chosen to minimze χ^2_j



F-Wave sensitivity of pp data



- Higher accuracy also improves extraction of other LECs

Repulsive WE

$$\lim_{\epsilon \to 0} G_0(E + i\epsilon) V |\psi_i(E)\rangle = \eta_i(E) |\psi_i(E)\rangle$$