Roy–Steiner-equation analysis of pion–nucleon scattering

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MH, JRE, B. Kubis, U.-G. Meißner, PRL 115 (2015) 092301, 192301, 1510.06039

From meson-exchange models to chiral forces

- Pion postulated to explain **nuclear force**
  Yukawa 1935, Nobel prize 1949

$$V(r) \sim \frac{e^{-M_\pi r}}{r}$$

  $\hookrightarrow$ small mass $M_\pi \Leftrightarrow$ **long range** $\sim 1/M_\pi$

- **Intermediate-range** of the *NN* potential: exchange of heavier mesons $\sigma, \rho, \omega, \phi, \ldots$
- Idea of **meson-exchange potentials**: fit coefficients of meson-exchange operators to experiment
- Pioneered by Erkelenz 1974: **Bonn potential**
- Phenomenological potentials: CD Bonn, AV18, ...
  $\hookrightarrow$ describe *NN* data with $\chi^2/\text{dof} \sim 1$

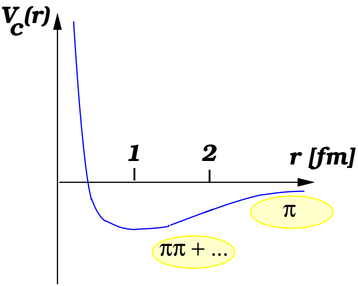


Figure courtesy of U.-G. Meißner 0811.1338

From meson-exchange models to chiral forces

- Phenomenological potentials
  - Beyond single-meson exchange?
  - **Hierarchy** of multi-nucleon forces?
  - **Consistency** between *NN* and 3*N*?
- Chiral Effective Field Theory (ChEFT)
  - Based on **chiral symmetry** of QCD
  - **Power counting**
  - Systematically improvable
  - Same accuracy with less parameters

    ↪ **low-energy constants**

↪ modern theory of nuclear forces

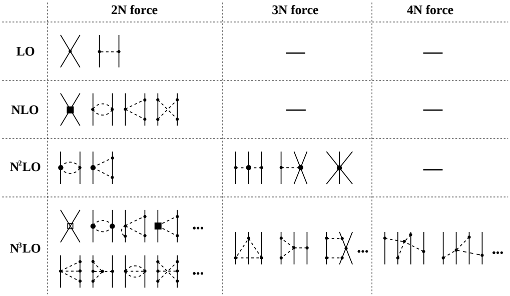


Figure courtesy of E. Epelbaum 1011.1343

Consequences of chiral symmetry

- **Chiral symmetry** of QCD

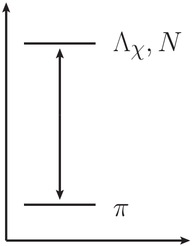
$$\mathcal{L}_{\text{QCD}} = \bar{q}_L i\not{D} q_L + \bar{q}_R i\not{D} q_R - \bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M} q_L - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

- Expansion in momenta p/Λ_χ and quark masses $m_q \sim p^2$

  $\hookrightarrow$ **scale separation**

- Relates different processes by **low-energy theorems**
  - Leading order: F and M, determined by F_π and M_π

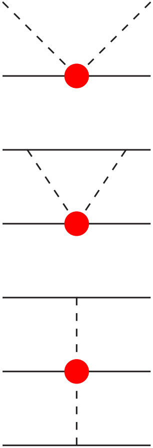
    $\hookrightarrow$ predict $\pi\pi$ scattering
  - Higher orders: $M_\pi \leftrightarrow \pi\pi$ scattering $\leftrightarrow$ scalar radius
  - Nucleon sector: πN coupling $\leftrightarrow g_A$, $m_N \leftrightarrow \pi N$ scattering



Pion–nucleon scattering and nuclear forces

- πN scattering appears as **subprocess** in NN and $3N$ forces $\hookrightarrow$ **long-range part** of potential
- At given chiral order: same low-energy constants as in πN
  $\hookrightarrow$ **parameter-free prediction**
  $\hookrightarrow$ reduces number of fit parameters
- Interpretation:
  - 1π exchange: πN coupling constant
  - 2π exchange: σ, ρ, ...
  - 3π exchange: ω, ...

  $\hookrightarrow$ πN scattering relevant for **2π channel**



Scalar content of the nucleon

Decomposition of the nucleon mass

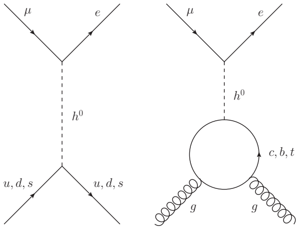
$$m_N = \langle N | \underbrace{\frac{\beta_{\text{QCD}}}{2g} F^a_{\mu\nu} F_a^{\mu\nu}}_{\text{trace anomaly}} + \underbrace{m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + \cdots}_{\text{Higgs}} | N \rangle$$

- Mass largely generated by gluon field energy via the **trace anomaly of the QCD energy- momentum tensor** $\theta^\mu_\mu \neq 0$
- Contribution from u- and d-quarks

$$\sigma_{\pi N} = \langle N | \hat{m}(\bar{u}u + \bar{d}d) | N \rangle$$

- $\sigma_{\pi N}$ related to πN scattering via low-energy theorem Cheng, Dashen 1971
- Challenges
  - Amplitude in unphysical region: **analytic continuation**
  - Isoscalar amplitude: chirally suppressed, $\pi\pi$ rescattering strong, isospin breaking large

Why care about $\sigma_{\pi N}$?

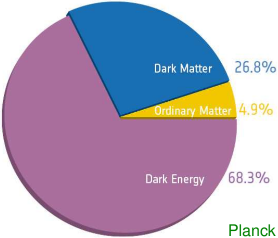


- Scalar coupling of the nucleon

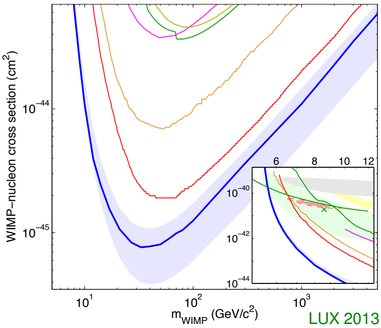
$$\langle N|m_q\bar{q}q|N\rangle = f_q^N m_N \qquad\qquad N \in \{p, n\}$$

$\hookrightarrow$ Dark Matter, $\mu \to e$ conversion in nuclei

- Condensates in nuclear matter
- *CP*-violating πN couplings, EDMs

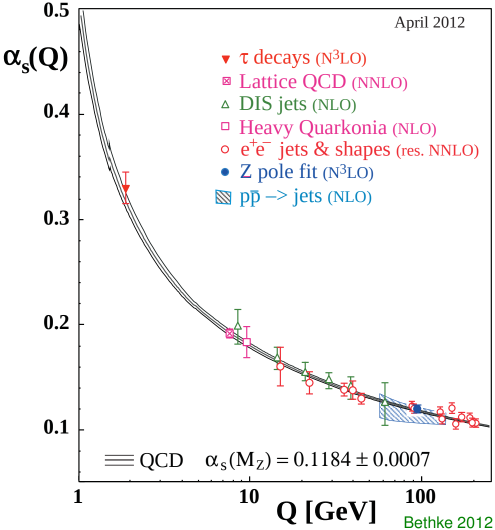


Planck 2013



LUX 2013

Running coupling of QCD



Bethke 2012

- Asymptotic freedom

$$\beta_{\rm QCD} = \mu \frac{\partial}{\partial \mu} g = -\left(11 - \frac{2n_f}{3}\right) \frac{g^3}{16\pi^2} + \mathcal{O}(g^5)$$

Gross, Politzer, Wilczek 1973 (Nobel prize 2004)

- QCD strongly coupled at low energies

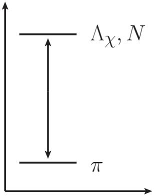
  $\Rightarrow$ Perturbation theory fails

  $\Rightarrow$ **Need non-perturbative methods**

Non-perturbative methods for low-energy QCD

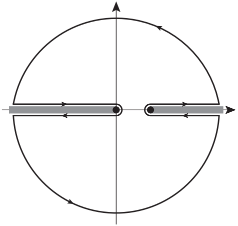
1. **Effective field theories**: symmetries, separation of scales

   $\hookrightarrow$ ChPT, ChEFT, $\not\pi$EFT, HπEFT, NREFT, ...



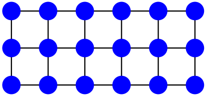
2. **Dispersion relations**: analyticity ($\simeq$ causality), unitarity ($\simeq$ probability conservation), crossing symmetry

   $\hookrightarrow$ Cauchy's theorem, analytic structure



3. **Lattice**: Monte-Carlo simulation

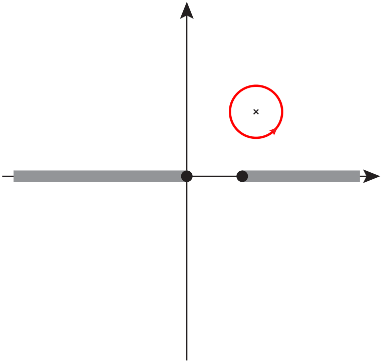
   $\hookrightarrow$ solve discretized version of QCD numerically



From Cauchy's theorem to dispersion relations

- **Cauchy's theorem**

$$f(s) = \frac{1}{2\pi i} \int_{\partial\Omega} \frac{\mathrm{d}s'\, f(s')}{s' - s}$$



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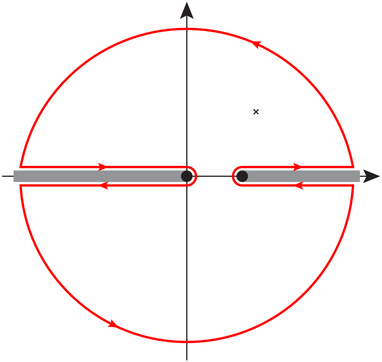
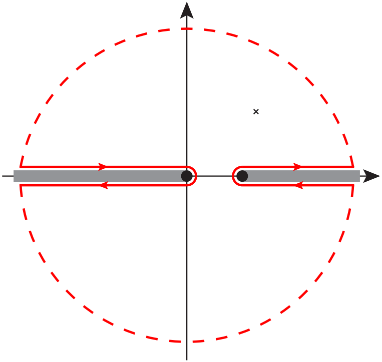


From Cauchy's theorem to dispersion relations

- **Dispersion relation**

$$f(s) = \frac{1}{\pi} \int_{\text{cuts}} \frac{\mathrm{d}s' \, \mathrm{Im}\, f(s')}{s' - s}$$

$\hookrightarrow$ **analyticity**



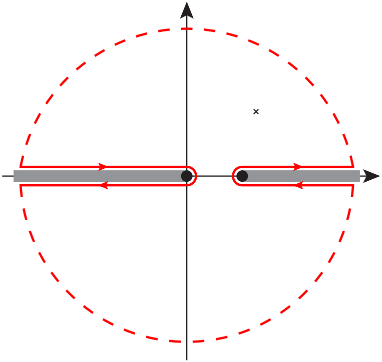
- **Dispersion relation**

$$f(s) = \frac{1}{\pi}\int_{\text{cuts}} \frac{ds' \,\text{Im}\, f(s')}{s'-s}$$

$\hookrightarrow$ **analyticity**

- Subtractions

$$f(s) = f(0) + \frac{s}{\pi}\int_{\text{cuts}} \frac{ds' \,\text{Im}\, f(s')}{s'(s'-s)}$$



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$\hookrightarrow$ **analyticity**

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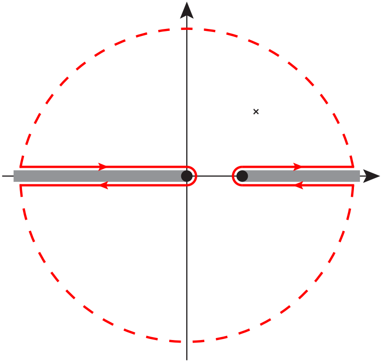
$$f(s) = f(0) + \frac{s}{\pi}\int_{\text{cuts}} \frac{ds'\,\text{Im}\,f(s')}{s'(s'-s)}$$

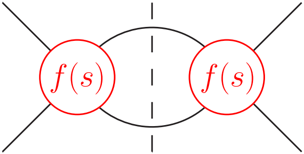
- Imaginary part from **Cutkosky rules**

$\hookrightarrow$ forward direction: **optical theorem**

- **Unitarity** for partial waves

$$\text{Im}\,f(s) = \rho(s)|f(s)|^2$$





$\pi\pi$ Roy equations

Roy equations = Dispersion relations + partial-wave expansion

+ crossing symmetry + unitarity

- **Coupled system of integral equations** for partial waves $t_J^I(s)$ Roy 1971

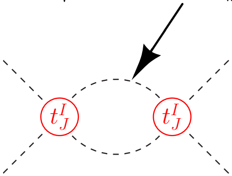
$$t_J^I(s) = k_J^I(s) + \sum_{I'=0}^{2} \sum_{J'=0}^{\infty} \int_{4M_\pi^2}^{\infty} ds' K_{JJ'}^{II'}(s,s') \text{Im}\, t_{J'}^{I'}(s')$$

Roy equations $=$ Dispersion relations $+$ partial-wave expansion

$+$ crossing symmetry $+$ unitarity

- **Coupled system of integral equations** for partial waves $t_J^I(s)$ Roy 1971

$$\sigma_\pi(s) = \sqrt{1-\frac{4M_\pi^2}{s}} \qquad \underbrace{t_J^I(s)}_{\frac{e^{2i\delta_J^I(s)}-1}{2i\sigma_\pi(s)}} = k_J^I(s) + \sum_{I'=0}^{2}\sum_{J'=0}^{\infty}\int_{4M_\pi^2}^{\infty} ds'\, K_{JJ'}^{II'}(s,s')\ \underbrace{\mathrm{Im}\ t_{J'}^{I'}(s')}_{\frac{1}{\sigma_\pi(s)}\sin^2\delta_{J'}^{I'}(s')}$$



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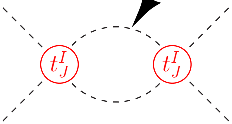


free parameters a_0^0, a_0^2

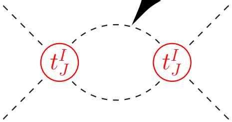
Roy equations = **Dispersion relations** + **partial-wave expansion**

+ **crossing symmetry** + **unitarity**

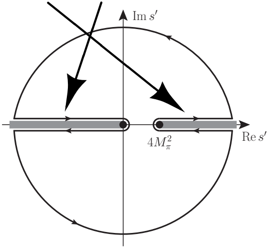
- **Coupled system of integral equations** for partial waves $t^I_J(s)$ Roy 1971

$$\underbrace{t^I_J(s)}_{} = \underbrace{k^I_J(s)}_{\delta^I_0 a^I_0 + \cdots} + \sum_{I'=0}^{2} \sum_{J'=0}^{\infty} \int_{4M_\pi^2}^{\infty} ds' \underbrace{K^{II'}_{JJ'}(s,s')}_{\frac{1}{\pi}\frac{\delta_{JJ'}\delta_{II'}}{s'-s-i\epsilon} + \bar{K}^{II'}_{JJ'}(s,s')} \underbrace{\text{Im}\, t^{I'}_{J'}(s')}_{\frac{1}{\sigma_\pi(s)}\sin^2\delta^{I'}_{J'}(s')}$$

$$\sigma_\pi(s) = \sqrt{1 - \frac{4M_\pi^2}{s}} \qquad t^I_J(s) = \frac{e^{2i\delta^I_J(s)} - 1}{2i\sigma_\pi(s)}$$



free parameters a^0_0, a^2_0



$\hookrightarrow$ **Self-consistency condition** for phase shifts

Matching to ChPT

- Roy equations: $\pi\pi$ phase shifts in terms of a_0^0, a_0^2 Ananthanarayan et al. 2001
- Matching of two-loop ChPT and Roy equations Colangelo, Gasser, Leutwyler 2001
  - Match low-energy polynomials $\Rightarrow \bar{l}_1, \bar{l}_2$ as by-product
  - Scattering lengths in terms of quark-mass LECs $\bar{l}_3, \bar{l}_4$

$$a_0^0 = 0.198 \pm 0.001 + 0.0443\,\text{fm}^{-2}\langle r^2\rangle_\pi^S - 0.0017\,\bar{l}_3 = 0.220 \pm 0.005$$

$$a_0^2 = -0.0392 \pm 0.0003 - 0.0066\,\text{fm}^{-2}\langle r^2\rangle_\pi^S - 0.0004\,\bar{l}_3 = -0.0444 \pm 0.0010$$

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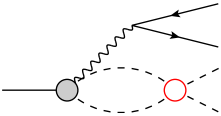
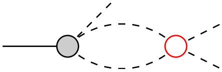
- Prediction tested in K_{e4} and $K \to 3\pi$ decays NA48/2 2010

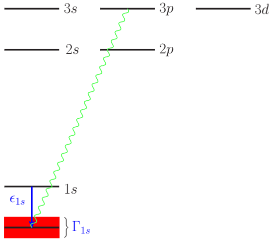
$$a_0^0 = 0.2210 \pm 0.0047_{\text{stat}} \pm 0.0040_{\text{syst}}$$

$$a_0^2 = -0.0429 \pm 0.0044_{\text{stat}} \pm 0.0028_{\text{syst}}$$





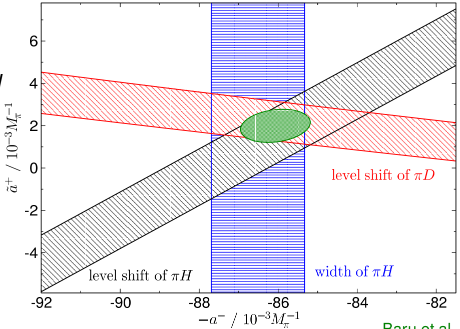
$$\tilde{a}^+ = a^+ + \frac{1}{4\pi(1+M_\pi/m_p)}\left\{\frac{4(M_\pi^2 - M_{\pi^0}^2)}{F_\pi^2}c_1 - 2e^2 f_1\right\}$$



- $\pi H/\pi D$: bound state of π^- and p/d, spectrum sensitive to threshold πN amplitude
  - πH level shift $\Rightarrow \pi^- p \to \pi^- p$
  - πD level shift $\Rightarrow$ isoscalar $\pi^- N \to \pi^- N$
  - πH width $\Rightarrow \pi^- p \to \pi^0 n$
- **Combined analysis** of πH and πD

$$a^+ \equiv a^+_{0+} = (7.5 \pm 3.1) \times 10^{-3} M_\pi^{-1}$$

$$a^- \equiv a^-_{0+} = (86.0 \pm 0.9) \times 10^{-3} M_\pi^{-1}$$



Baru et al. 2011

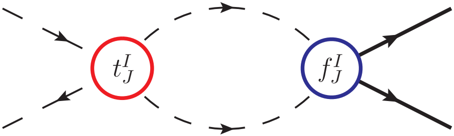
Roy–Steiner equations for πN: differences to $\pi\pi$ Roy equations

Key differences compared to $\pi\pi$ Roy equations

- Crossing: coupling between $\pi N \to \pi N$ (s-channel) and $\pi\pi \to \bar{N}N$ (t -channel)
  $\Rightarrow$ need a different kind of dispersion relations [Hite, Steiner 1973], [Büttiker et al. 2004]

- Unitarity in t-channel, e.g. in single-channel approximation

$$\mathrm{Im} f_\pm^J(t) = \sigma_t^\pi f_\pm^J(t) t_J^I(t)^*$$



$\Rightarrow$ **Watson's theorem**: phase of $f_\pm^J(t)$ equals δ_{IJ} [Watson 1954]
$\hookrightarrow$ solution in terms of Omnès function [Muskhelishvili 1953, Omnès 1958]

- Large pseudo-physical region in t -channel
  $\hookrightarrow$ $\bar{K}K$ intermediate states for s-wave in the region of the $f_0(980)$

Roy–Steiner equations for πN: flow of information

Limited range of validity

$$\sqrt{s} \leq \sqrt{s_m} = 1.38\,\text{GeV} \qquad \sqrt{t} \leq \sqrt{t_m} = 2.00\,\text{GeV}$$

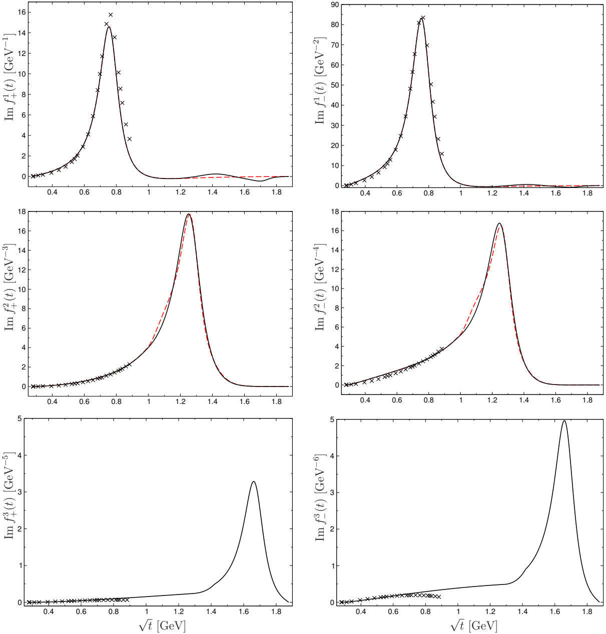
Input/Constraints

- S- and P-waves above matching point $s > s_m$ $(t > t_m)$
- Inelasticities
- Higher waves (D-, F-, $\cdots$)
- Scattering lengths from hadronic atoms

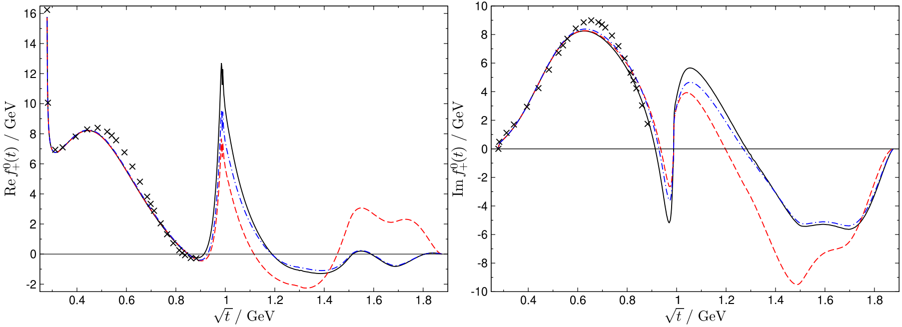
Baru et al. 2011

Output

- S- and P-wave phase-shifts at low energies $s < s_m$ $(t < t_m)$
- Subthreshold parameters
  - ▷ Pion-nucleon σ-term
  - ▷ Nucleon form factor spectral functions
  - ▷ ChPT LECs



Im $f^1_+(t)$ [GeV^{-1}]
Im $f^1_-(t)$ [GeV^{-2}]
Im $f^2_+(t)$ [GeV^{-3}]
Im $f^2_-(t)$ [GeV^{-4}]
Im $f^3_+(t)$ [GeV^{-5}]
Im $f^3_-(t)$ [GeV^{-6}]
$\sqrt{t}$ [GeV]
$\sqrt{t}$ [GeV]



RS solution in general consistent with KH80 results

Solving s-channel: strategy

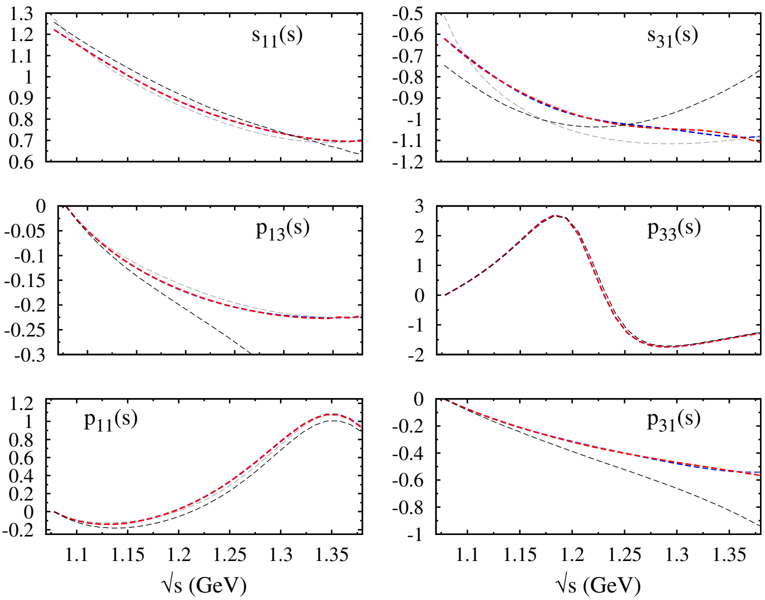
- Parameterize S and P waves up to $W < W_m$
  - Using SAID partial waves as starting point
- Impose as **constraints** the hadronic atom scattering lengths
- Introduce as many subtractions as necessary to match d.o.f [Gasser, Wanders 1999]
- Minimize difference between LHS and the RHS on a grid of points W_j

$$\chi^2 = \sum_{l,I_s,\pm} \sum_{j=1}^{N} \frac{\Big(\text{Re}\, f_{l\pm}^{I_s}(W_j) - F[f_{l\pm}^{I_s}](W_j)\Big)^2}{\text{Re}\, f_{l\pm}^{I_s}(W_j)}$$

$F[f_{l\pm}^{I_s}](W_j) \equiv$ right hand side of RS-equations

- Parametrization and subthreshold parameters are the fitting parameters

Solving s-channel: results

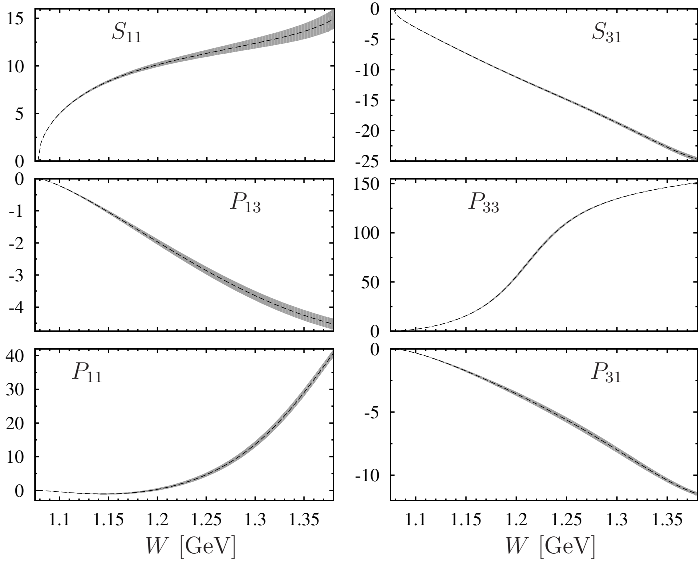


blue/red
⇕
LHS/RHS
after the fit

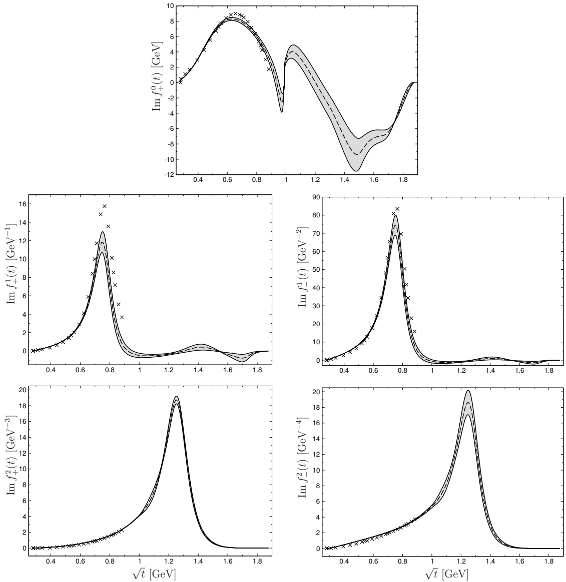
gray/black
⇕
LHS/RHS
before the fit

Notation: $L_{2I_s 2J}$

Uncertainties: s-channel partial waves



Uncertainties: imaginary part t-channel partial waves



Threshold parameters

- Threshold parameters defined as: Re $f^I_{l\pm}(s) = q^{2l}\{a^I_{l\pm} + b^I_{l\pm} q^2 + \cdots\}$
- Extracted from hyperbolic sum rules

	RS	KH80
a^+_{0+} $[10^{-3}M_\pi^{-1}]$	-0.9 ± 1.4	-9.7 ± 1.7
a^-_{0+} $[10^{-3}M_\pi^{-1}]$	85.4 ± 0.9	91.3 ± 1.7
a^+_{1+} $[10^{-3}M_\pi^{-3}]$	131.2 ± 1.7	132.7 ± 1.3
a^-_{1+} $[10^{-3}M_\pi^{-3}]$	-80.3 ± 1.1	-81.3 ± 1.0
a^+_{1-} $[10^{-3}M_\pi^{-3}]$	-50.9 ± 1.9	-56.7 ± 1.3
a^-_{1-} $[10^{-3}M_\pi^{-3}]$	-9.9 ± 1.2	-11.7 ± 1.0
b^+_{0+} $[10^{-3}M_\pi^{-3}]$	-45.0 ± 1.0	-44.3 ± 6.7
b^-_{0+} $[10^{-3}M_\pi^{-3}]$	4.9 ± 0.8	13.3 ± 6.0

- Reasonable agreement with KH80 but for the scattering lengths
- Disagreement in the scattering lengths in $\sim 4\sigma$

Results for the sigma-term

$$\sigma_{\pi N} = F_\pi^2 \left(d_{00}^+ + 2M_\pi^2 d_{01}^+ \right) + \Delta_D - \Delta_\sigma - \Delta_R$$

- subthreshold parameters output of the Roy–Steiner equations

$$d_{00}^+ = -1.36(3) M_\pi^{-1} \qquad [\text{KH: } -1.46(10) M_\pi^{-1}]$$

$$d_{01}^+ = 1.16(2) M_\pi^{-3} \qquad [\text{KH: } 1.14(2) M_\pi^{-3}]$$

- $\Delta_D - \Delta_\sigma = -(1.8 \pm 0.2)$ MeV [MH at al. 2012], $|\Delta_R| \lesssim 2$ MeV [Bernard, Kaiser, Meißner 1996]
- Isospin breaking in the CD theorem shifts $\sigma_{\pi N}$ by $+3.0$ MeV
- Final results:

$$\sigma_{\pi N} = (59.1 \pm 1.9_{\text{RS}} \pm 3.0_{\text{LET}}) \text{ MeV} = (59.1 \pm 3.5) \text{ MeV}$$

[MH, JRE, Kubis, Meißner]

- $\sigma_{\pi N}$ depends linearly on the scattering lengths

$$\sigma_{\pi N} = 59.1 + \sum_{I_s} c_{I_s} \Delta a_{0+}^{I_s}$$

- KH input $\Rightarrow \sigma_{\pi N} = 46$ MeV

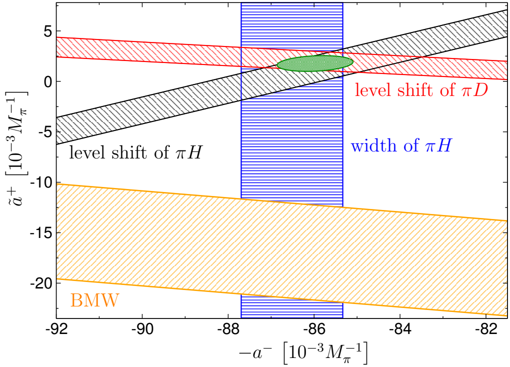
  $\hookrightarrow$ to be compared with $\sigma_{\pi N} = 45$ MeV [Gasser, Leutwyler, Socher, Sainio 1988]
- compare also $\sigma_{\pi N} \sim (64 \pm 8)$ MeV [Pavan et al. 2002]

Comparison with lattice $\sigma_{\pi N}$ results

- Recent lattice determination of $\sigma_{\pi N}$ from the BMW collaboration

  $\sigma_{\pi N} = 38(3)(3)\text{MeV}$ [Durr. et al. 2015]

- The linear dependence of $\sigma_{\pi N}$ on the scattering lengths introduces an additional constraint



- Fully inconsistent with the hadronic atom phenomenology

Matching to Chiral Perturbation Theory

Matching to ChPT at the subthreshold point:

- Chiral expansion expected to work best at subthreshold point
  - ▷ Maximal distance from threshold singularities
  - ▷ πN amplitude can be expanded as polynomial
- Preferred choice for *NN* scattering due to proximity of relevant kinematic regions

Express the subthreshold parameters in terms of the LECs to $\mathcal{O}(p^4)$

$$d_{00}^{+} = -\frac{2M_\pi^2(2\tilde{c}_1 - \tilde{c}_3)}{F_\pi^2} + \frac{g_a^2(3+8g_a^2)M_\pi^3}{64\pi F_\pi^4} + M_\pi^4\left\{\frac{16\bar{e}_{14}}{F_\pi^2} - \frac{2c_1 - c_3}{16\pi^2 F_\pi^4}\right\}$$

- Chiral πN amplitude to $\mathcal{O}(p^4)$ depends on 13 low-energy constants
- Roy–Steiner system contains 10 subtraction constants
  - ▷ Calculate remaining 3 from sum rules
  - ▷ Invert the system to solve for LECs

Chiral low-energy constants

	NLO	N^2LO	N^3LO
c_1 [GeV^{-1}]	-0.74 ± 0.02	-1.07 ± 0.02	-1.11 ± 0.03
c_2 [GeV^{-1}]	1.81 ± 0.03	3.20 ± 0.03	3.13 ± 0.03
c_3 [GeV^{-1}]	-3.61 ± 0.05	-5.32 ± 0.05	-5.61 ± 0.06
c_4 [GeV^{-1}]	2.17 ± 0.03	3.56 ± 0.03	4.26 ± 0.04
$\bar{d}_1 + \bar{d}_2$ [GeV^{-2}]	—	1.04 ± 0.06	7.42 ± 0.08
$\bar{d}_3$ [GeV^{-2}]	—	-0.48 ± 0.02	-10.46 ± 0.10
$\bar{d}_5$ [GeV^{-2}]	—	0.14 ± 0.05	0.59 ± 0.05
$\bar{d}_{14} - \bar{d}_{15}$ [GeV^{-2}]	—	-1.90 ± 0.06	-12.18 ± 0.12
$\bar{e}_{14}$ [GeV^{-3}]	—	—	0.89 ± 0.04
$\bar{e}_{15}$ [GeV^{-3}]	—	—	-0.97 ± 0.06
$\bar{e}_{16}$ [GeV^{-3}]	—	—	-2.61 ± 0.03
$\bar{e}_{17}$ [GeV^{-3}]	—	—	0.01 ± 0.06
$\bar{e}_{18}$ [GeV^{-3}]	—	—	-4.20 ± 0.05

- Subthreshold errors tiny, chiral expansion dominates uncertainty
- $\bar{d}_i$ at N^3LO increase by an order of magnitude
  - $\hookrightarrow$ due to terms proportional to $g_A^2(c_3 - c_4) = -16\ \text{GeV}^{-1}$
  - $\hookrightarrow$ mimic loop diagrams with Δ degrees of freedom
- What's going on with chiral convergence?
  - $\hookrightarrow$ look at convergence of threshold parameters with LECs fixed at subthreshold point

Convergence of the chiral series

	NLO	N^2LO	N^3LO	RS
a_{0+}^{+} $[10^{-3}M_\pi^{-1}]$	-23.8	0.2	-7.9	-0.9 ± 1.4
a_{0+}^{-} $[10^{-3}M_\pi^{-1}]$	79.4	92.9	59.4	85.4 ± 0.9
a_{1+}^{+} $[10^{-3}M_\pi^{-3}]$	102.6	121.2	131.8	131.2 ± 1.7
a_{1+}^{-} $[10^{-3}M_\pi^{-3}]$	-65.2	-75.3	-89.0	-80.3 ± 1.1
a_{1-}^{+} $[10^{-3}M_\pi^{-3}]$	-45.0	-47.0	-72.7	-50.9 ± 1.9
a_{1-}^{-} $[10^{-3}M_\pi^{-3}]$	-11.2	-2.8	-22.6	-9.9 ± 1.2
b_{0+}^{+} $[10^{-3}M_\pi^{-3}]$	-70.4	-23.3	-44.9	-45.0 ± 1.0
b_{0+}^{-} $[10^{-3}M_\pi^{-3}]$	20.6	23.3	-64.7	4.9 ± 0.8

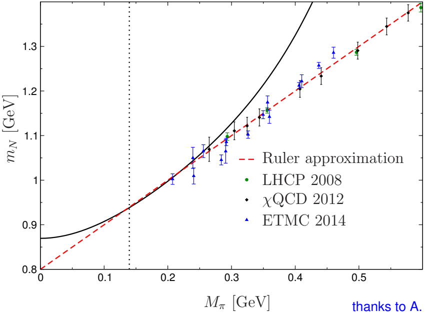
- N^3LO results bad due to large Delta loops
- Conclusion: lessons for few-nucleon applications
  1. either include the Δ to reduce the size of the loop corrections or use LECs from subthreshold kinematics — work in progress
  2. error estimates: consider chiral convergence of a given observable, difficult to assign a global chiral error to LECs

The "ruler plot" vs. ChPT

Lattice QCD simulations can be performed at different quark/pion masses

Pion mass dependence of m_N up to NNNLO in ChPT, using

- Input from Roy–Steiner solution



thanks to A. Walker-Loud for providing the lattice data

↪ range of convergence of the chiral expansion is very limited

↪ huge cancellation amongst terms to produce a linear behavior

Summary & Outlook

Summary

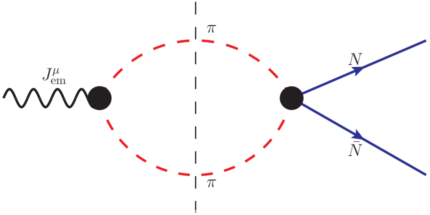
- Derived a closed system of Roy–Steiner equations for πN
- Numerical solution and error analysis of the full system of RS eqs.
- Precise determination of the $\sigma_{\pi N}$
  - ▷ Roy–Steiner formalism reproduces KH80 result with KH80 input
  - ▷ With modern input for scattering lengths and coupling constant $\sigma_{\pi N}$ increases
- Precise determination of threshold parameters
- Extraction of the ChPT LECs
- Study of the chiral convergence

Outlook

- Dispersive determination of Δ pole parameters
- Matching to ChPT with explicit Δ's
  - ▷ LECs extraction and study of the chiral convergence
  - ▷ Large-N_c constraints on Δ LECs

- Proton Radius Puzzle

  $\hookrightarrow$ strong constraints from analyticity and unitarity

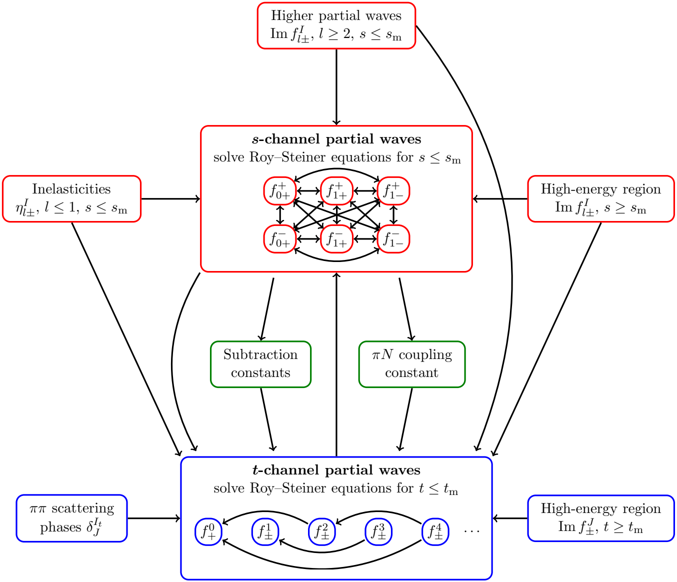


- first inelastic correction $\hookrightarrow$ $\pi\pi$ continuum, rigorous constraint fixed from:
  - ▷ RS t-channel partial waves
  - ▷ pion form factor
- update of Höhler spectral functions, including also isospin breaking

criticism by Lee et al. 2015

Spare slides

Roy-Steiner equations for πN: flow of information



Uncertainties

- Statistical errors (at intermediate energies)
  - ▷ important correlations between subthreshold parameters
  - ▷ shallow fit minima
  - ⇒ Sum rules for subthreshold parameters become essential to reduce the errors
- Input variation (small)
  - ▷ small effect for considering s-channel KH80 input
  - ▷ very small effects from $L > 5$ s-channel PWs
  - ▷ small effect from the different S-wave extrapolation for $t > 1.3$ GeV
  - ▷ negligible effect of ρ' and ρ''
  - ▷ very significant effects of the D-waves ($f_2(1275)$)
  - ▷ F-waves shown to be negligible
- matching conditions (close to W_m)
- scattering length (SL) errors (on S-waves and subthreshold parameters)
  - ▷ very important for the $\sigma_{\pi\mathbf{N}}$