

The Nucleon Axial Coupling from QCD

PRD 96 054513 1701.07559

1704.01114

Nature 558 91-94 (2018) 1805.12130

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JÜLICH

Forschungszentrum

20 November 2018

Dr. Klaus Erkelenz Kolloquium



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Plymouth
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NERSC
Thorsten Kurth



UNC
Amy Nicholson,
Henry Monge Camacho



nVidia
Kate Clark



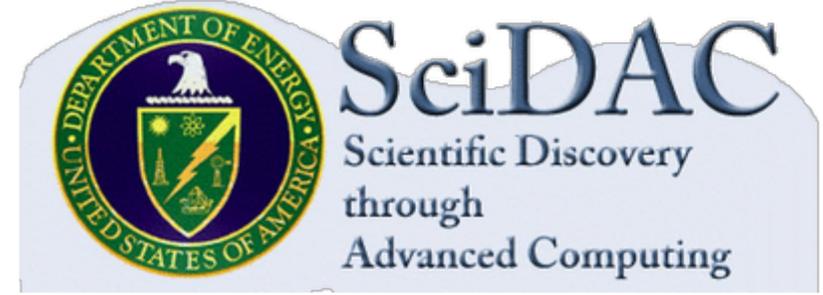
Glasgow
Chris Bouchard

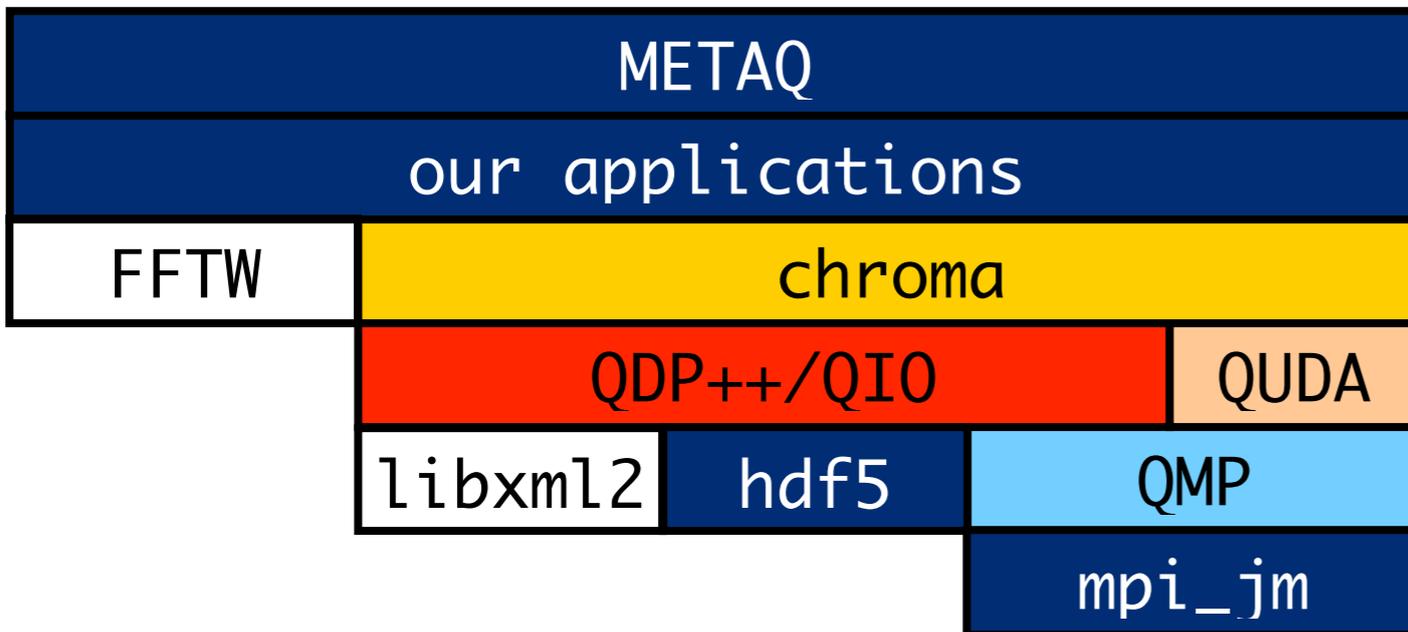


INT
Chris Monahan



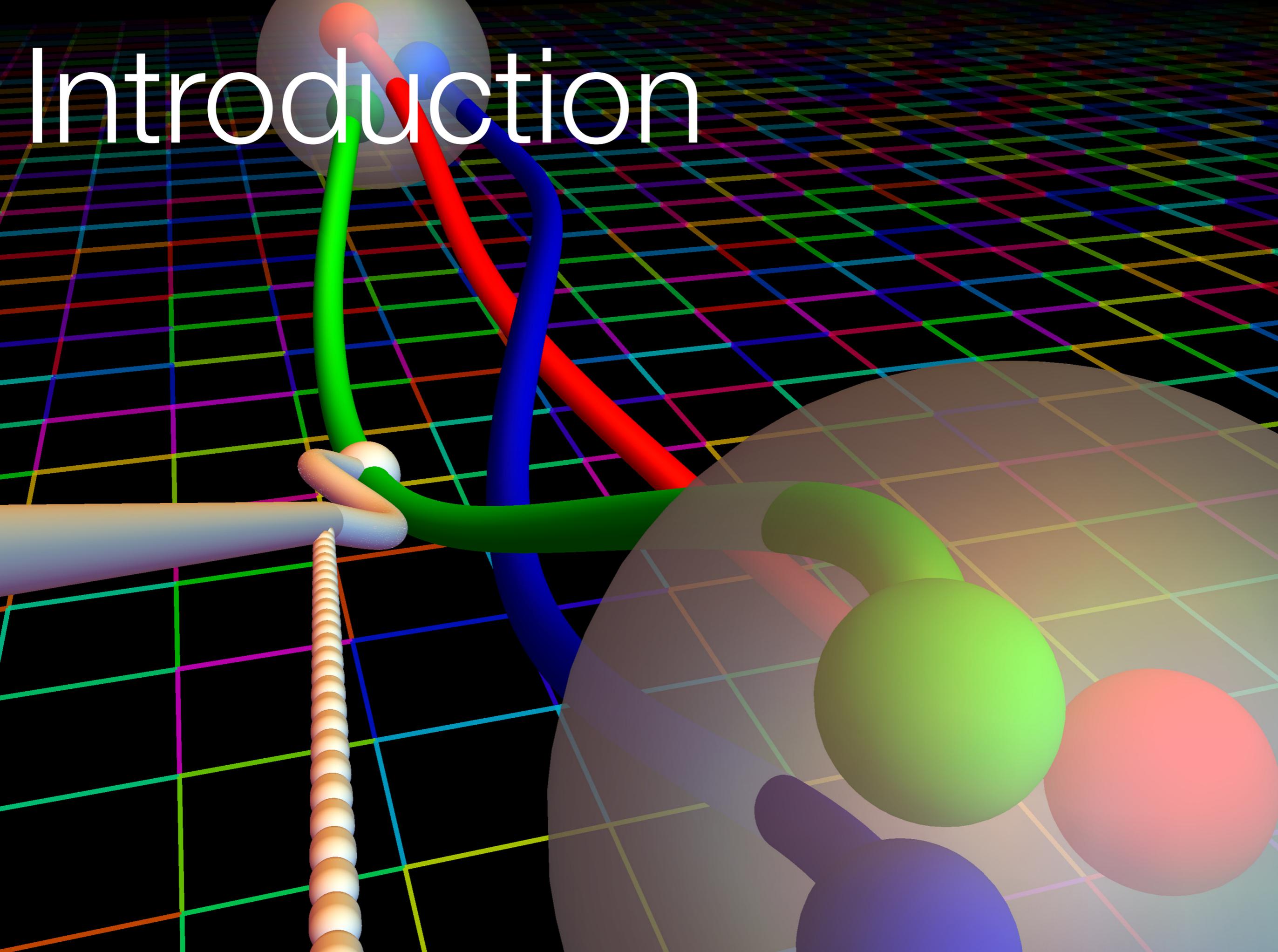
William &
Mary
Kostas Orginos





Software	References
METAQ	Berkowitz arXiv:1702.06122 github.com/evanberkowitz/metaq Berkowitz et al. EPJ (LATTICE2017) 175 09007 (2018)
chroma QDP++	Edwards and Joo (SciDAC, LHPC and UKQCD Collaborations) Nucl. Phys. Proc. Suppl 140, 832 (2005)
QUDA	Clark et al. Comput. Phys. Commun. 181 1517 (2010) Babich et al. Supercomputing 11, 70
hdf5 in QDP++	Kurth et al PoS LATTICE2014 045 (2015)
qmp	Chen, Edwards, and Watson et al. https://github.com/usqcd-software/qmp
mpi_jm	Berkowitz et al. EPJ (LATTICE2017) 175 09007 (2018) McElvain et al. https://github.com/kenmcelvain/mpi_jm/

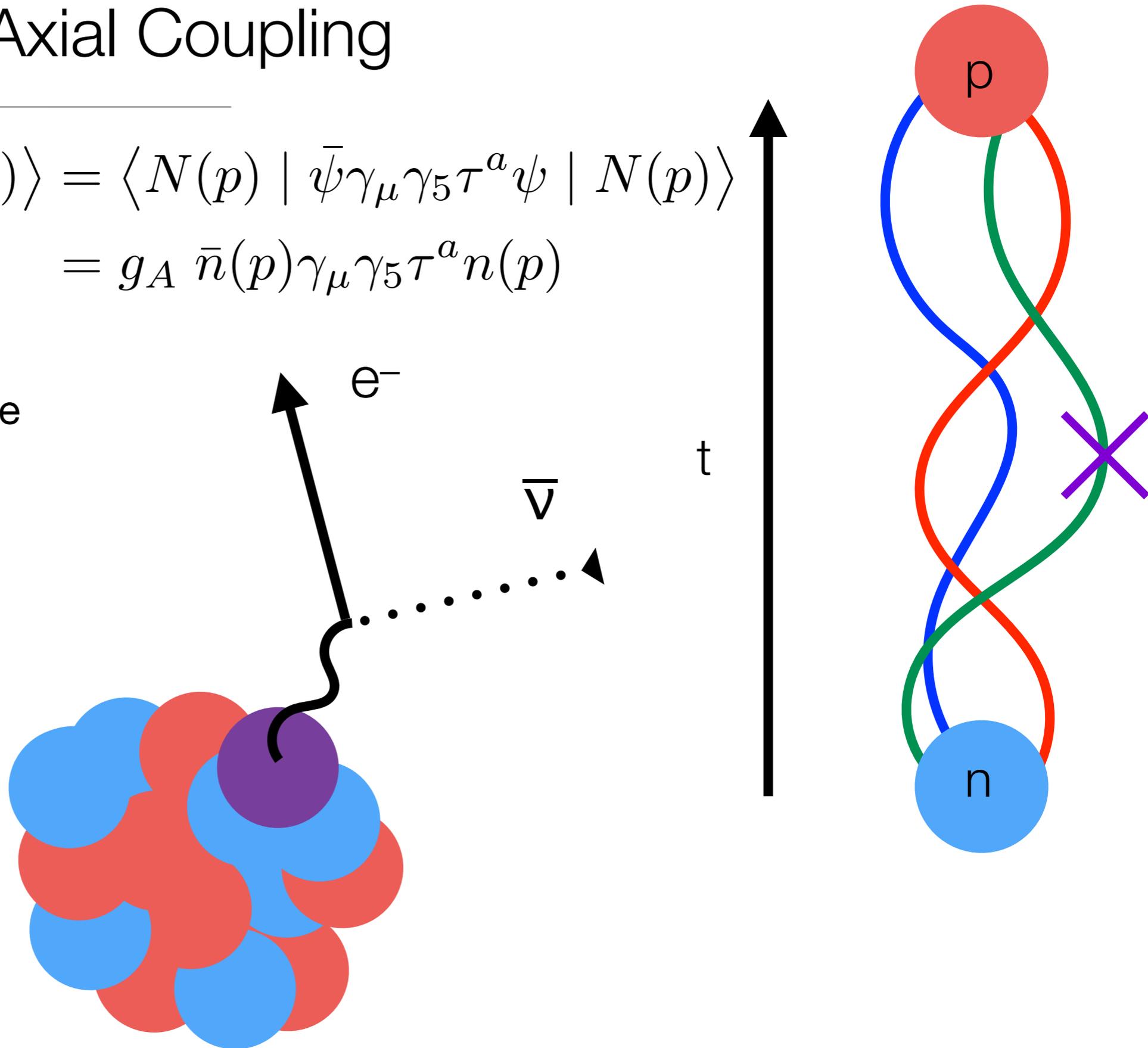
Introduction



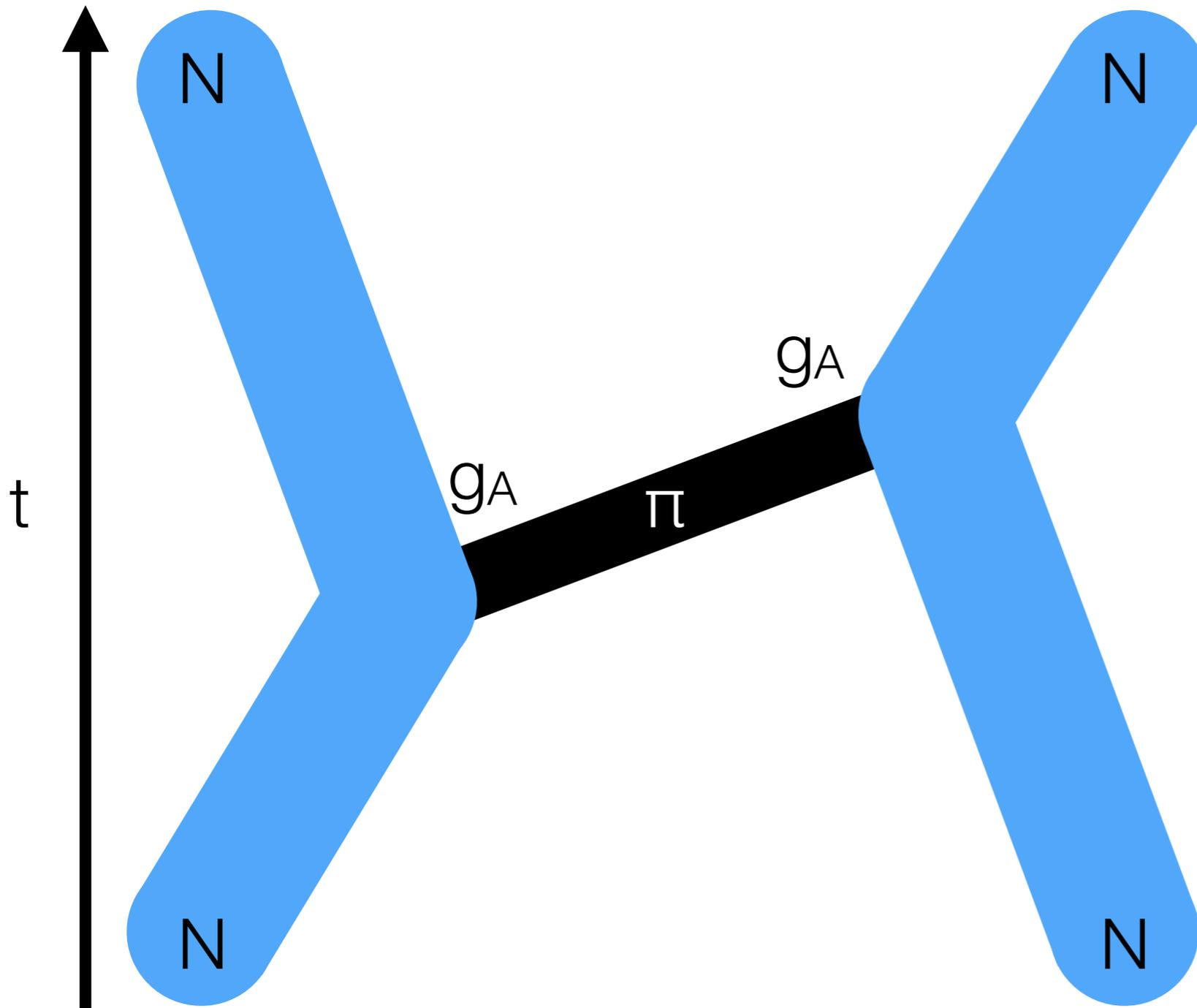
The Nucleon Axial Coupling

$$\begin{aligned}\langle N(p) | A_\mu^a | N(p) \rangle &= \langle N(p) | \bar{\psi} \gamma_\mu \gamma_5 \tau^a \psi | N(p) \rangle \\ &= g_A \bar{n}(p) \gamma_\mu \gamma_5 \tau^a n(p)\end{aligned}$$

- Free neutron lifetime
- Nuclear β decay
- Nuclear force



The Nucleon Axial Coupling



- Essential in modern nuclear interactions and chiral EFT forces

Epelbaum, Hammer, Meißner

Rev. Mod. Phys 81 (2009) 1773-1825

- Appears in all meson exchange models

K. Erkelenz, K. Holinde, K. Bleuler

Nucl. Phys. A139 (1969) 308-328

Nucl. Phys. A161 (1971) 155-176

Nucl. Phys. A194 (1972) 161-176

K. Holinde, **K. Erkelenz**, R. Alzetta

Nucl. Phys. A198 (1972) 598-608

K. Bleuler, **K. Erkelenz**, K. Holinde, R. Machleidt

Nucl. Phys. A205 (1973) 292-298

R. Machleidt, **K. Erkelenz**, K. Holinde

Nucl. Phys. A232 (1974) 398-416

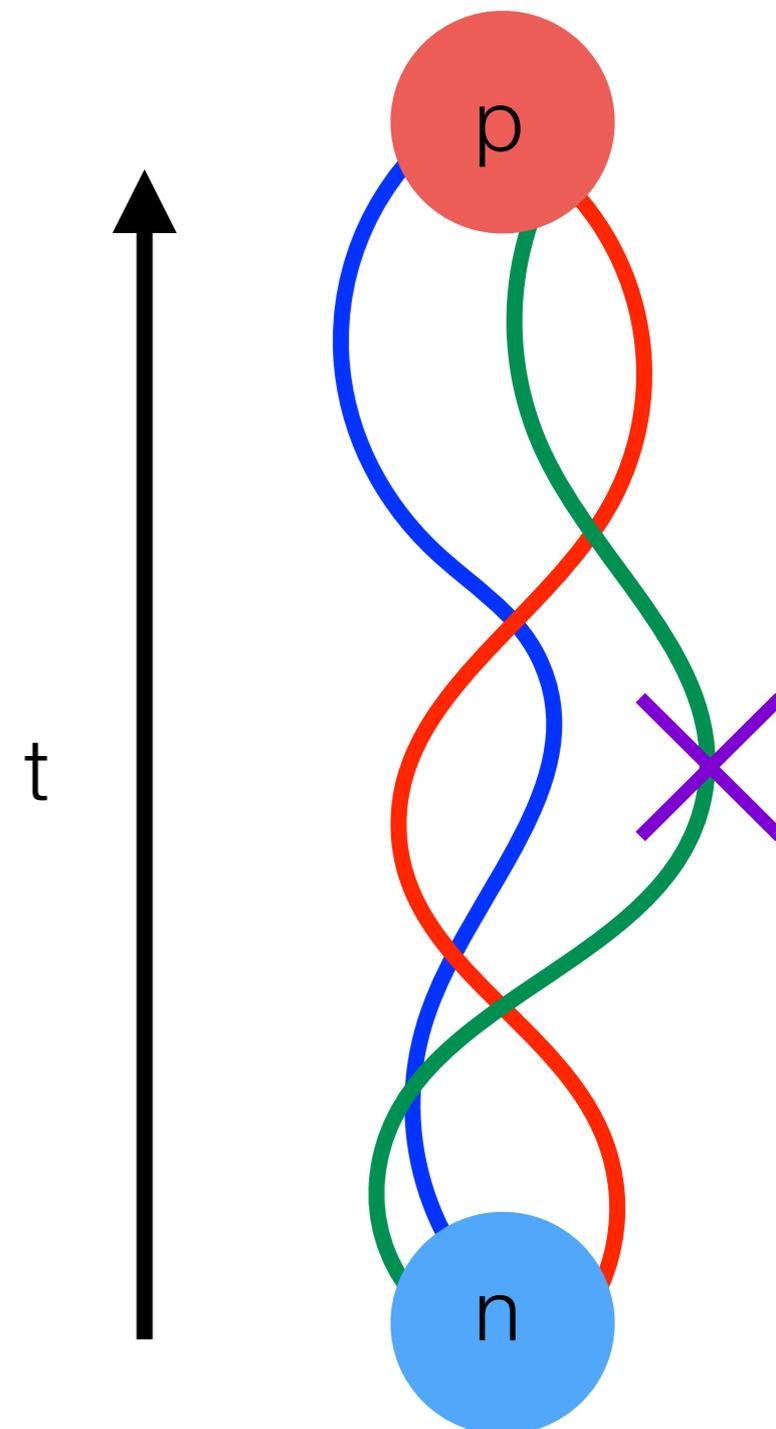
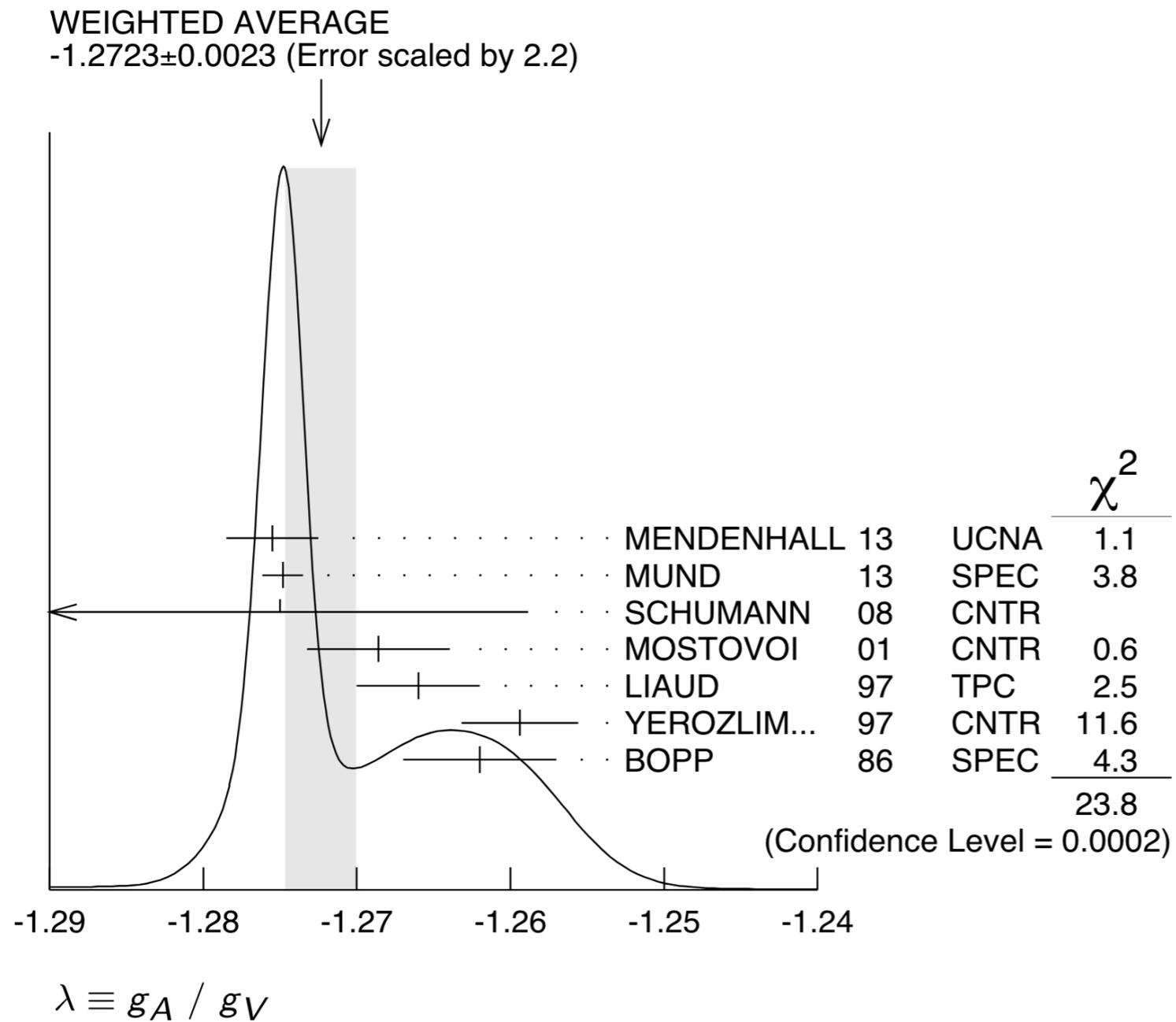
PLB 49 (1974) 209-212

K. Erkelenz, Phys. Rept. 13 (1974) 191

- Notably, the pioneering **Bonn Potential**

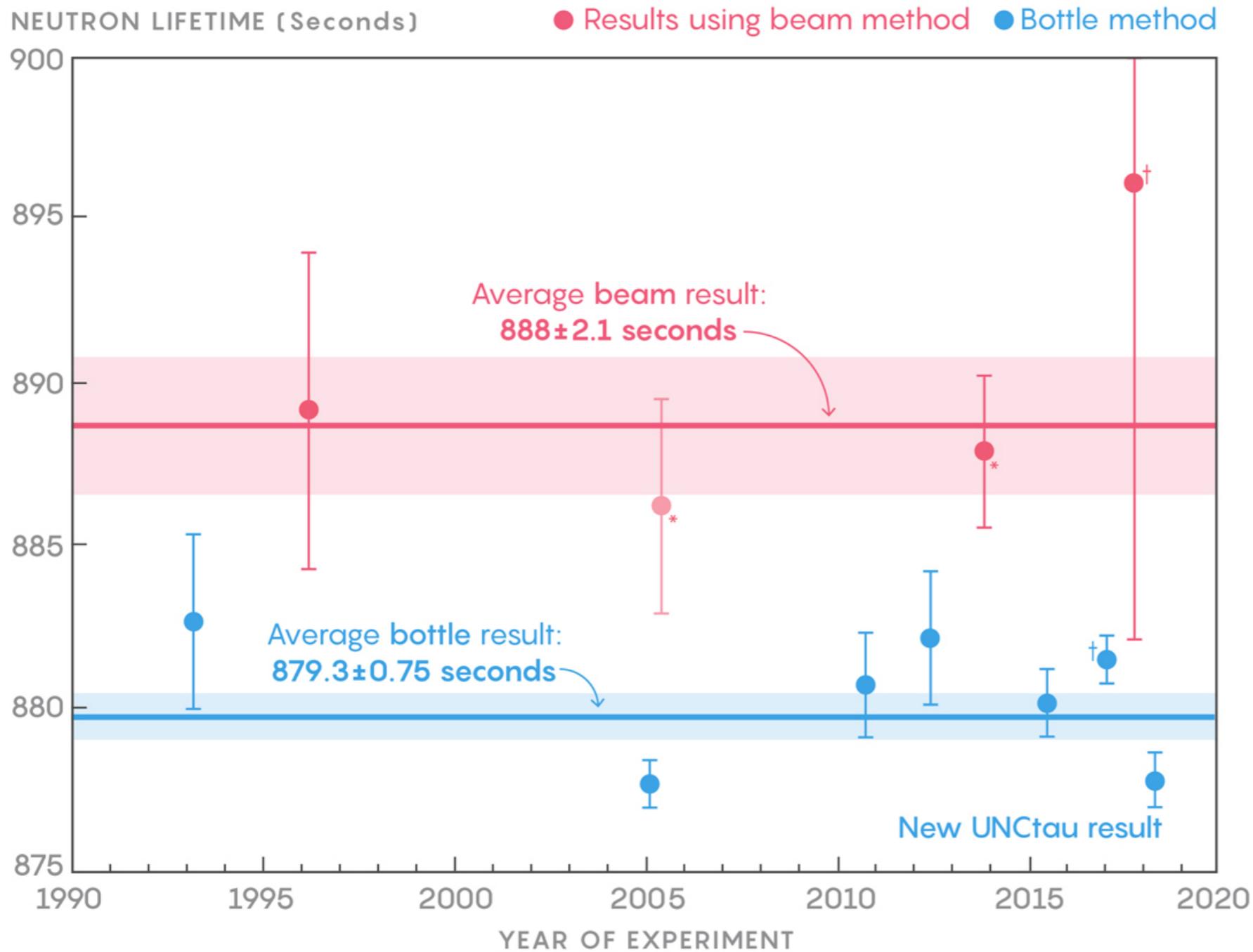
The Nucleon Axial Coupling

PDG 2016



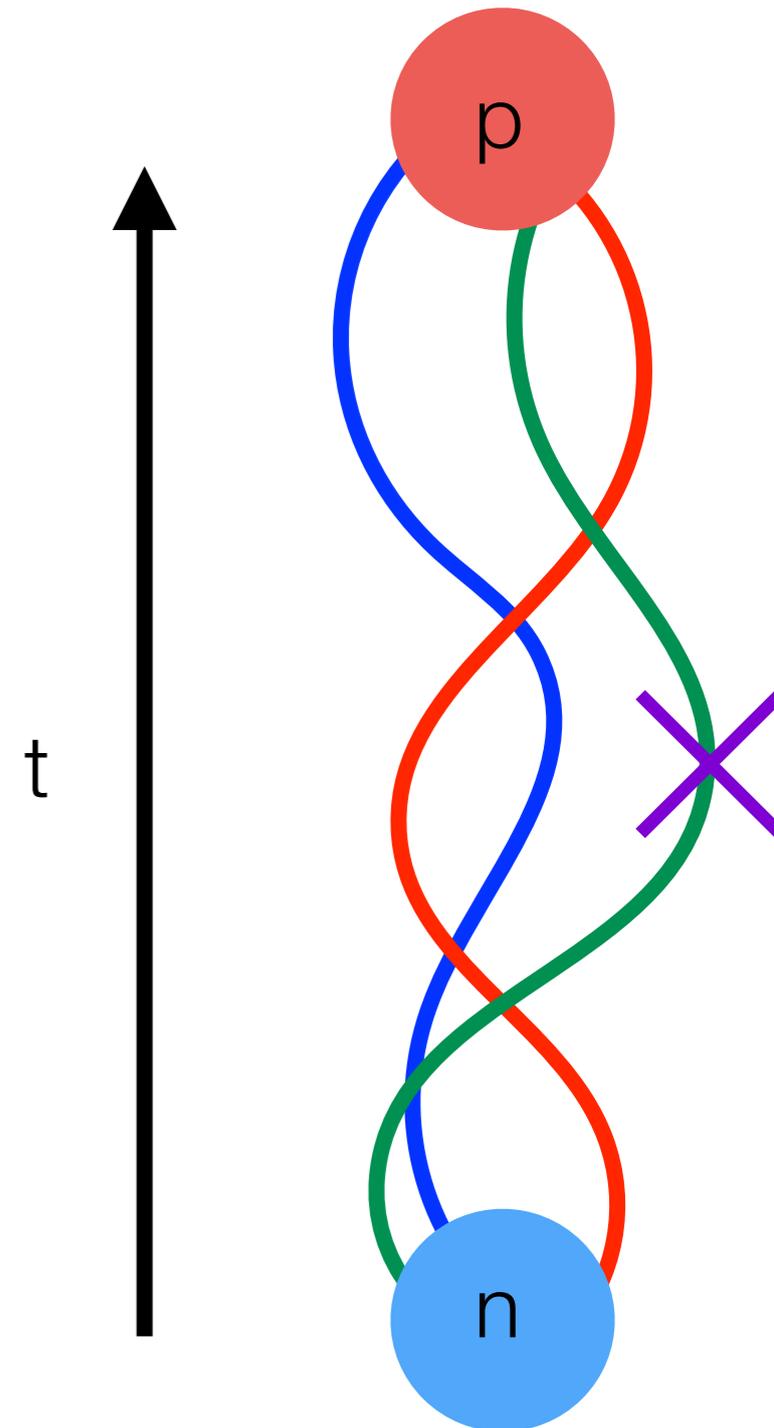
The Nucleon Axial Coupling

PDG 2016

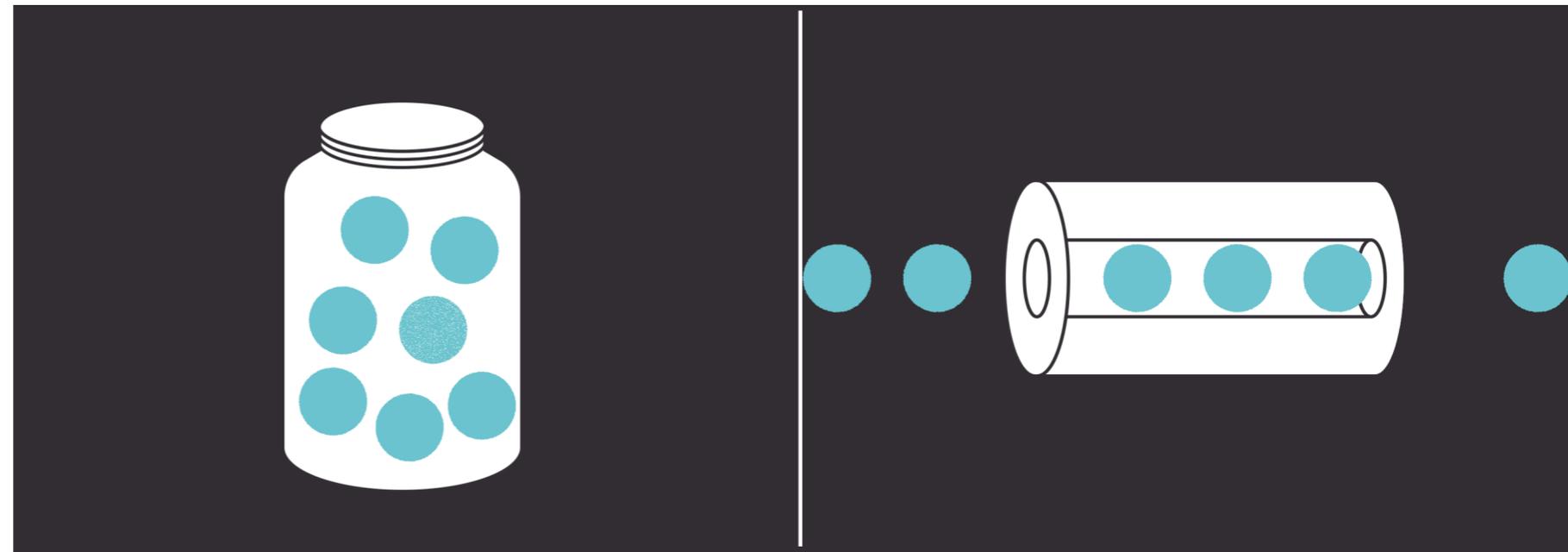
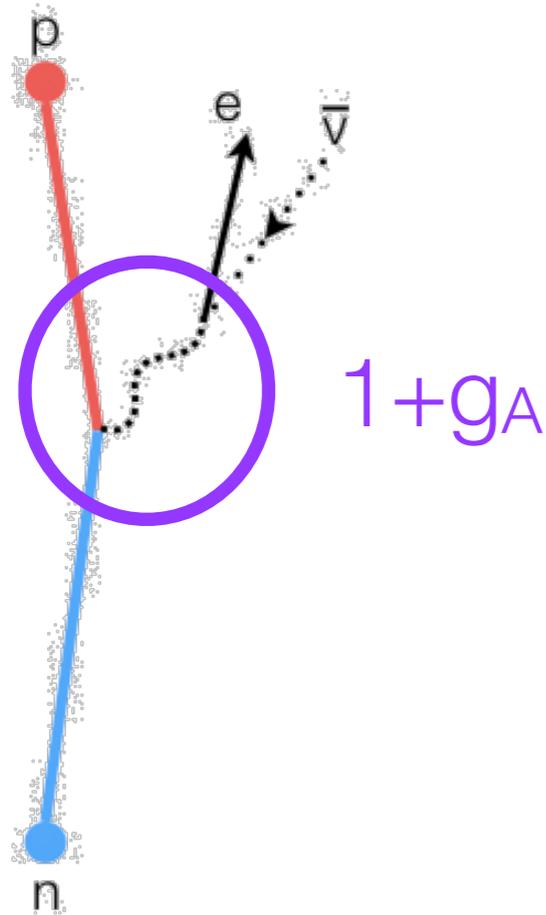


*Nico result (2005) was superseded by an updated and improved result, Yue (2013);

†Preliminary results



Simple Problems Remain Outstanding



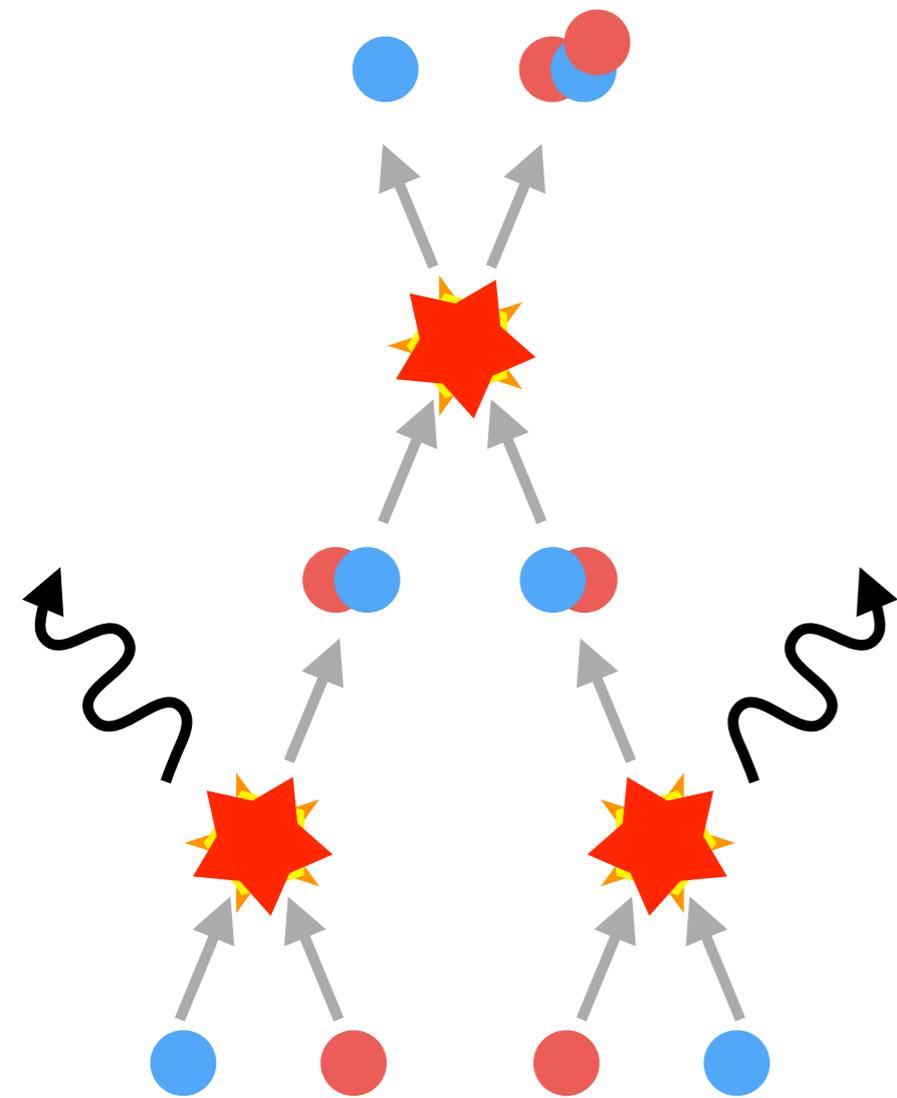
BOTTLE and BEAM

methods of measuring the neutron lifetime disagree at the 99% level

$$\tau_n = \frac{(5172.0 \pm 1.1)\text{seconds}}{1 + 3g_A^2}$$

Can we calculate g_A ?

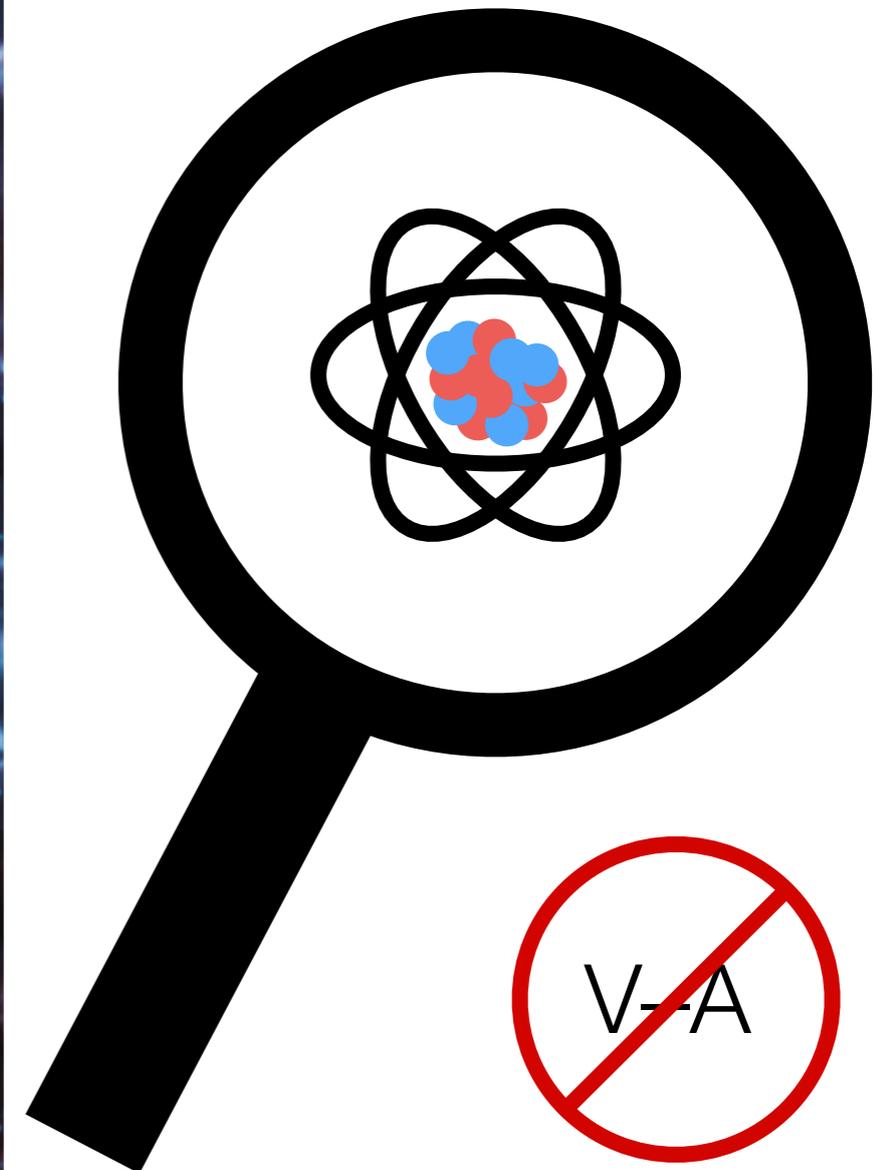
Applications



Big Bang
Nucleosynthesis

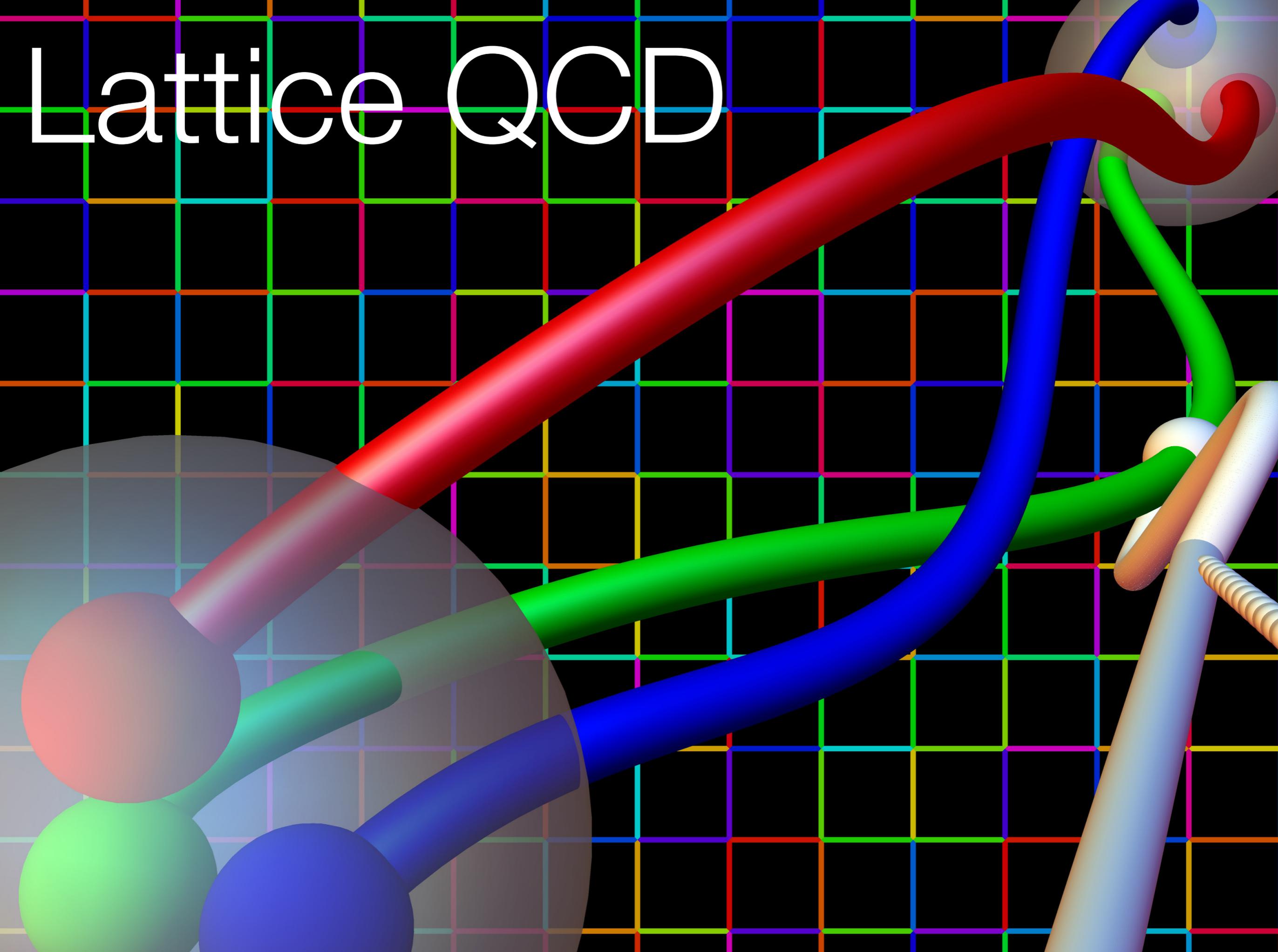


Astrophysics



New Physics
Searches

Lattice QCD



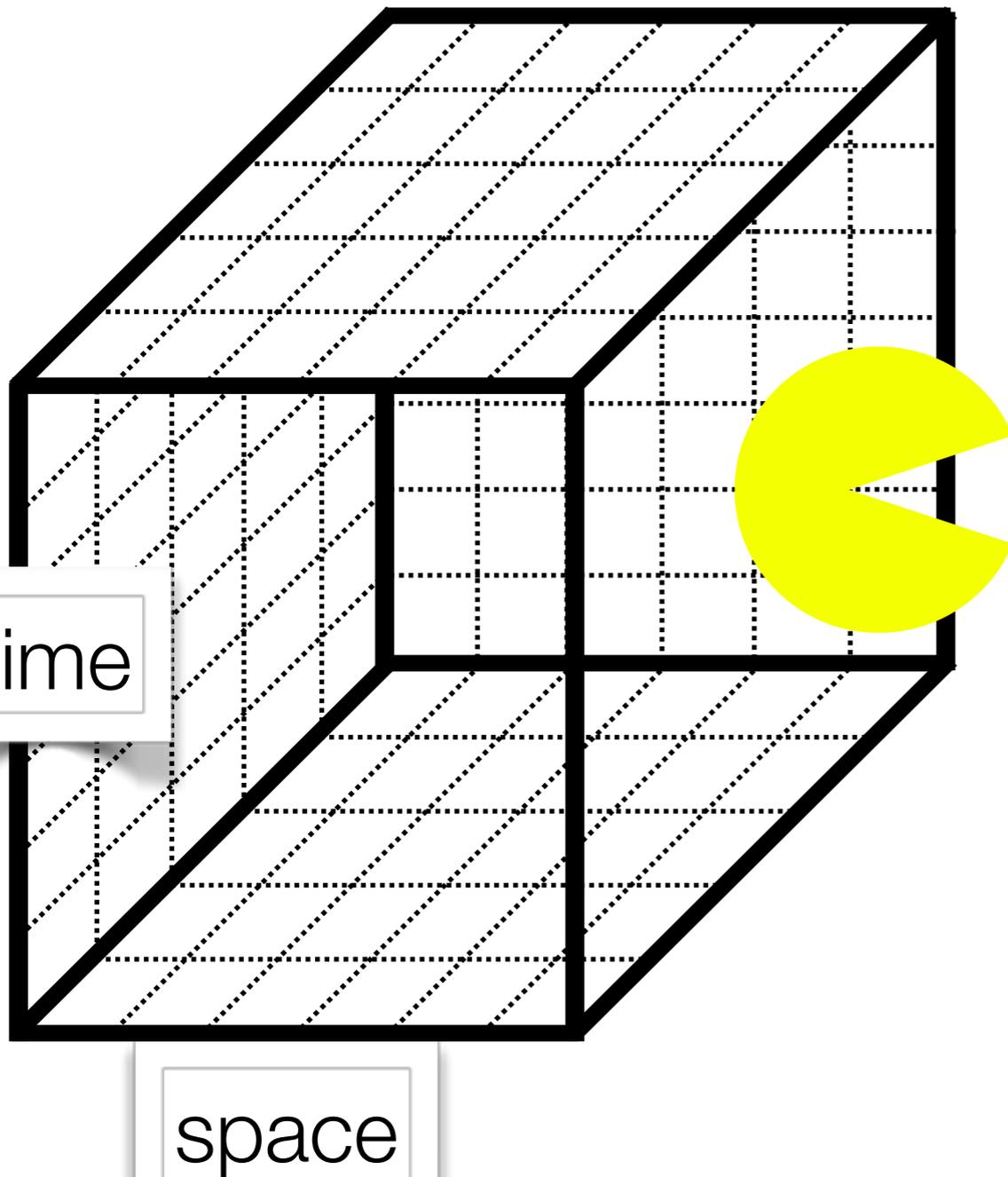
Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} + m)\psi$$

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \mathcal{O}(t)\mathcal{O}^\dagger(0) e^{-S[\bar{\psi},\psi,U]}$$

$$= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \det(\not{D} + M) e^{-S[U]} \mathcal{O}(t)\mathcal{O}^\dagger(0)$$

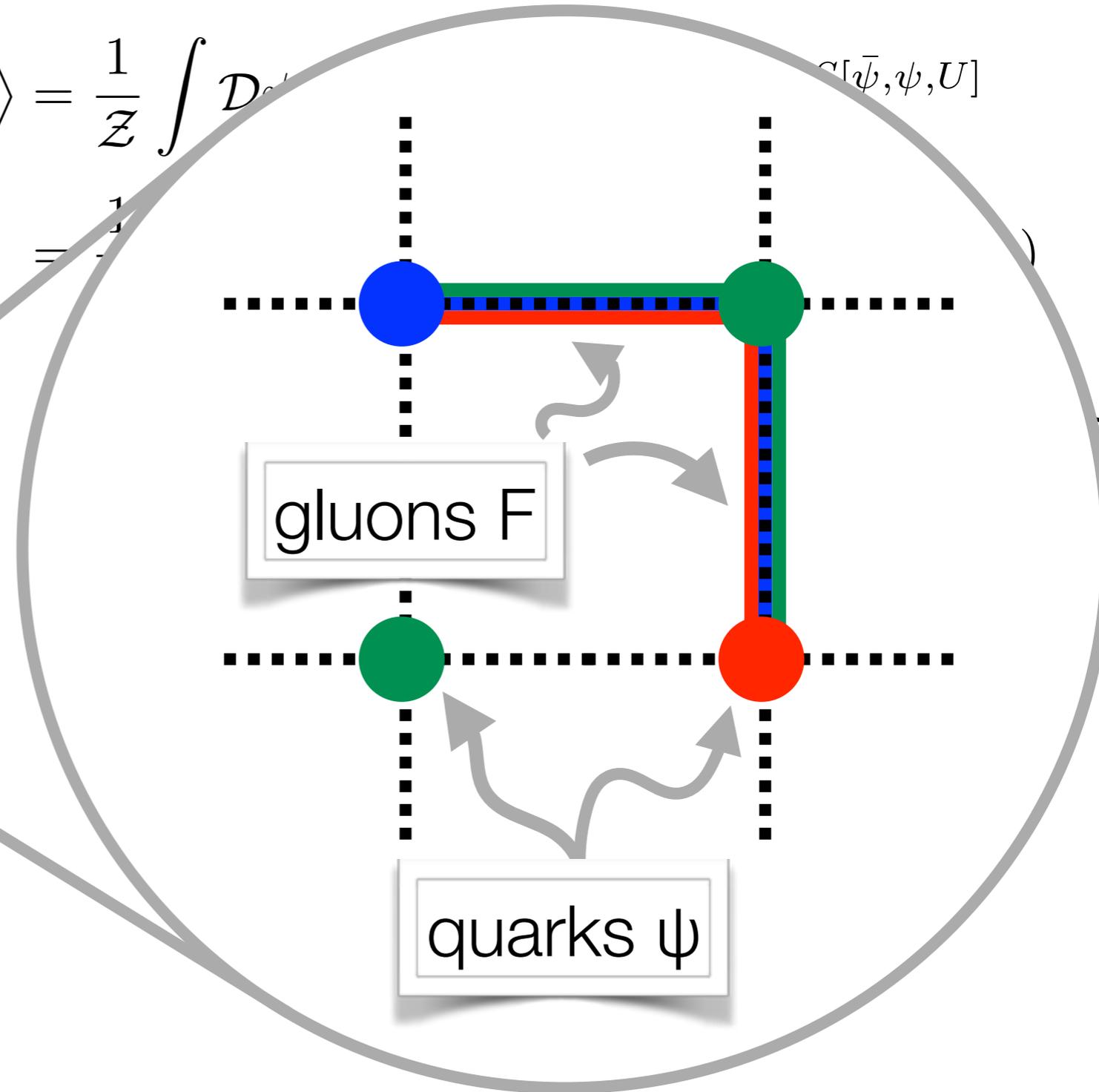
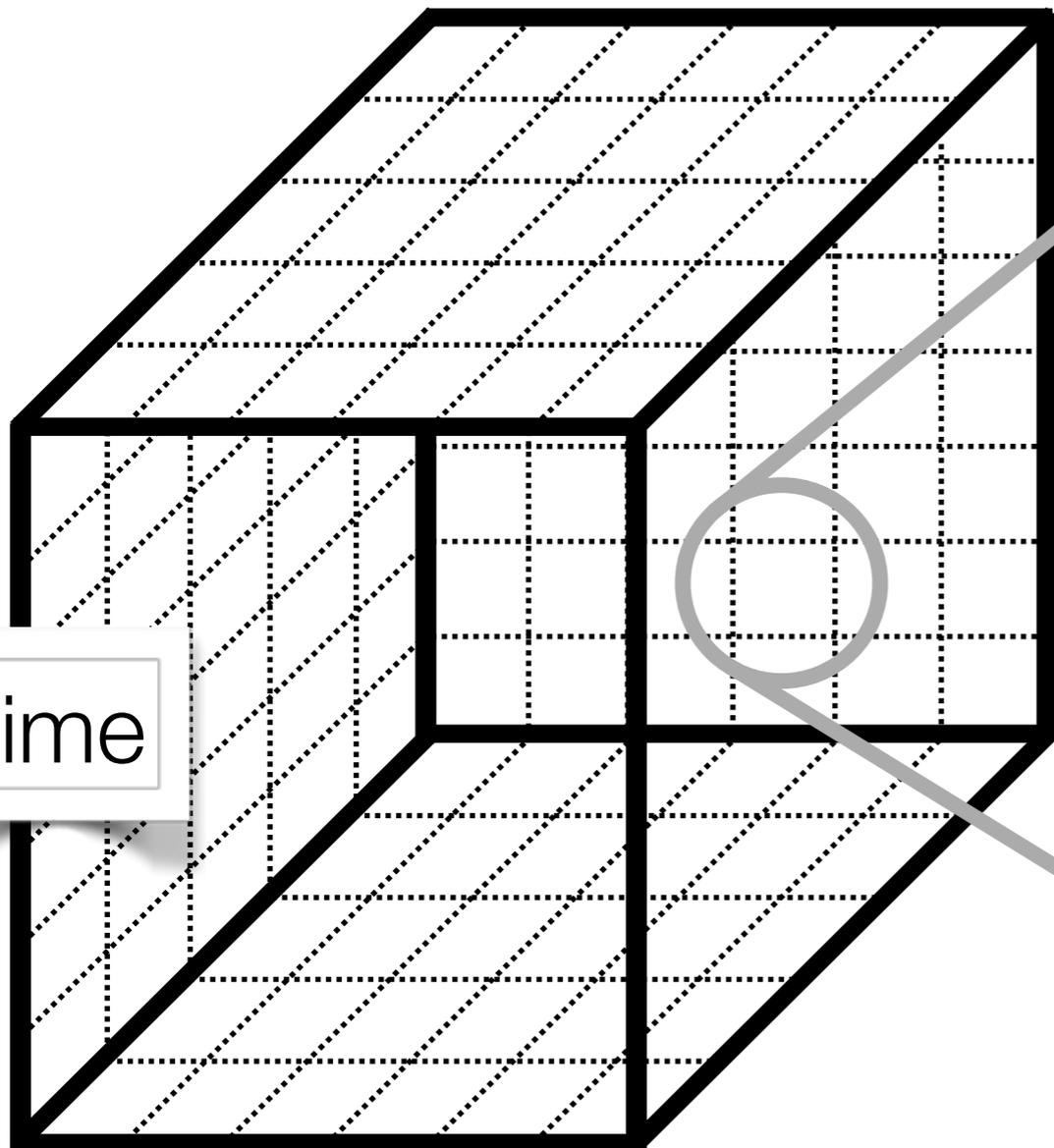
lattice
finite volume



Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} + m)\psi$$

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \frac{1}{Z} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U e^{iS[\bar{\psi}, \psi, U]}$$



Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} + m)\psi$$

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \mathcal{O}(t)\mathcal{O}^\dagger(0) e^{-S[\bar{\psi},\psi,U]}$$

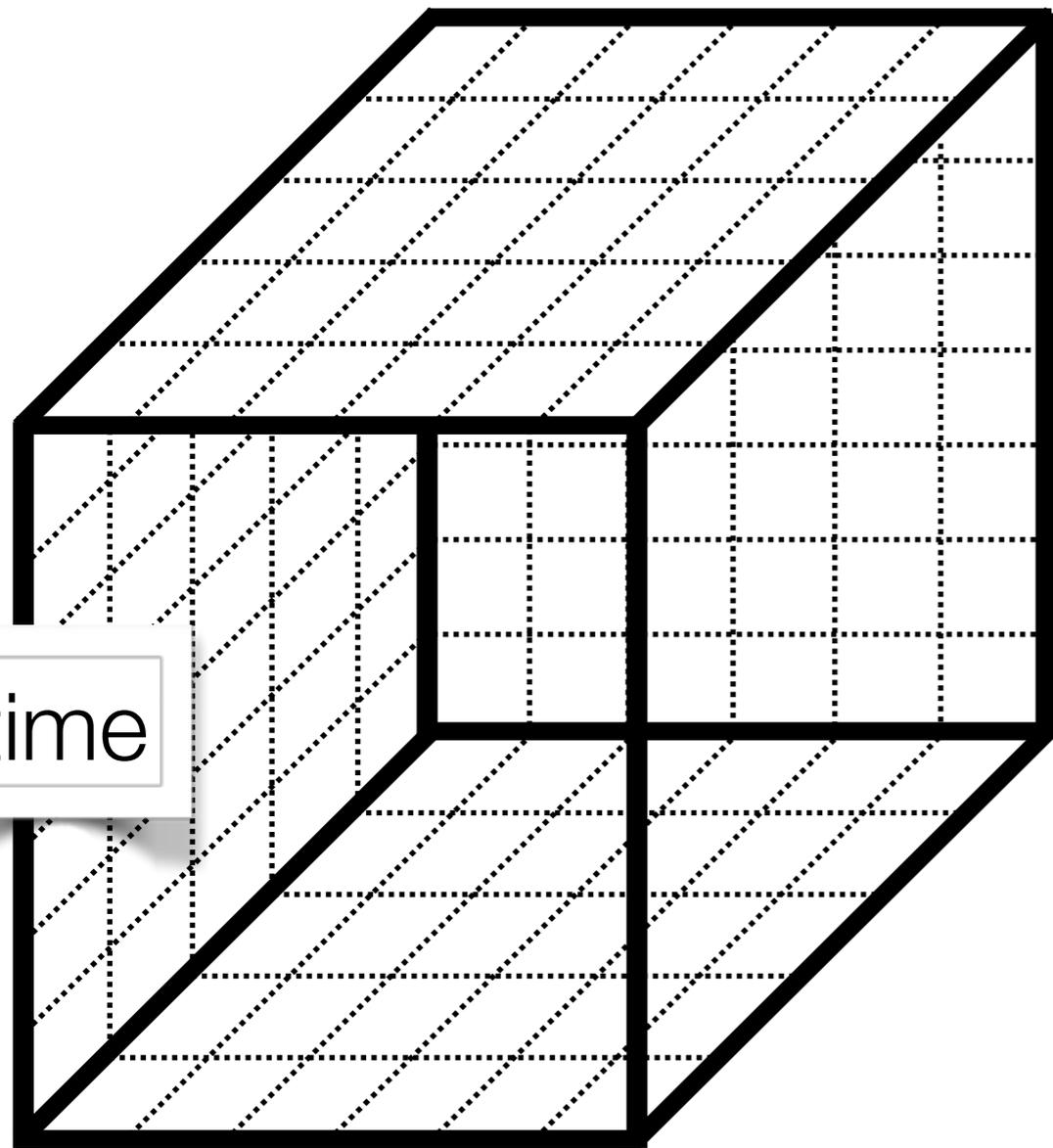
$$= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \underbrace{\det(\not{D} + M) e^{-S[U]}}_{\text{Probability}} \mathcal{O}(t)\mathcal{O}^\dagger(0)$$

Probability

$$\{U_1, U_2, U_3, \dots, U_N\}$$

Markov Chain Monte Carlo

$$\approx \frac{1}{N} \sum_{i=1}^N \mathcal{O}(t)\mathcal{O}^\dagger(0)[U_i] + O\left(\frac{1}{\sqrt{N}}\right)$$



time

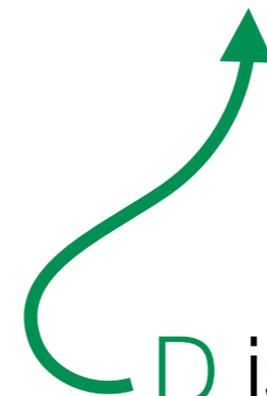
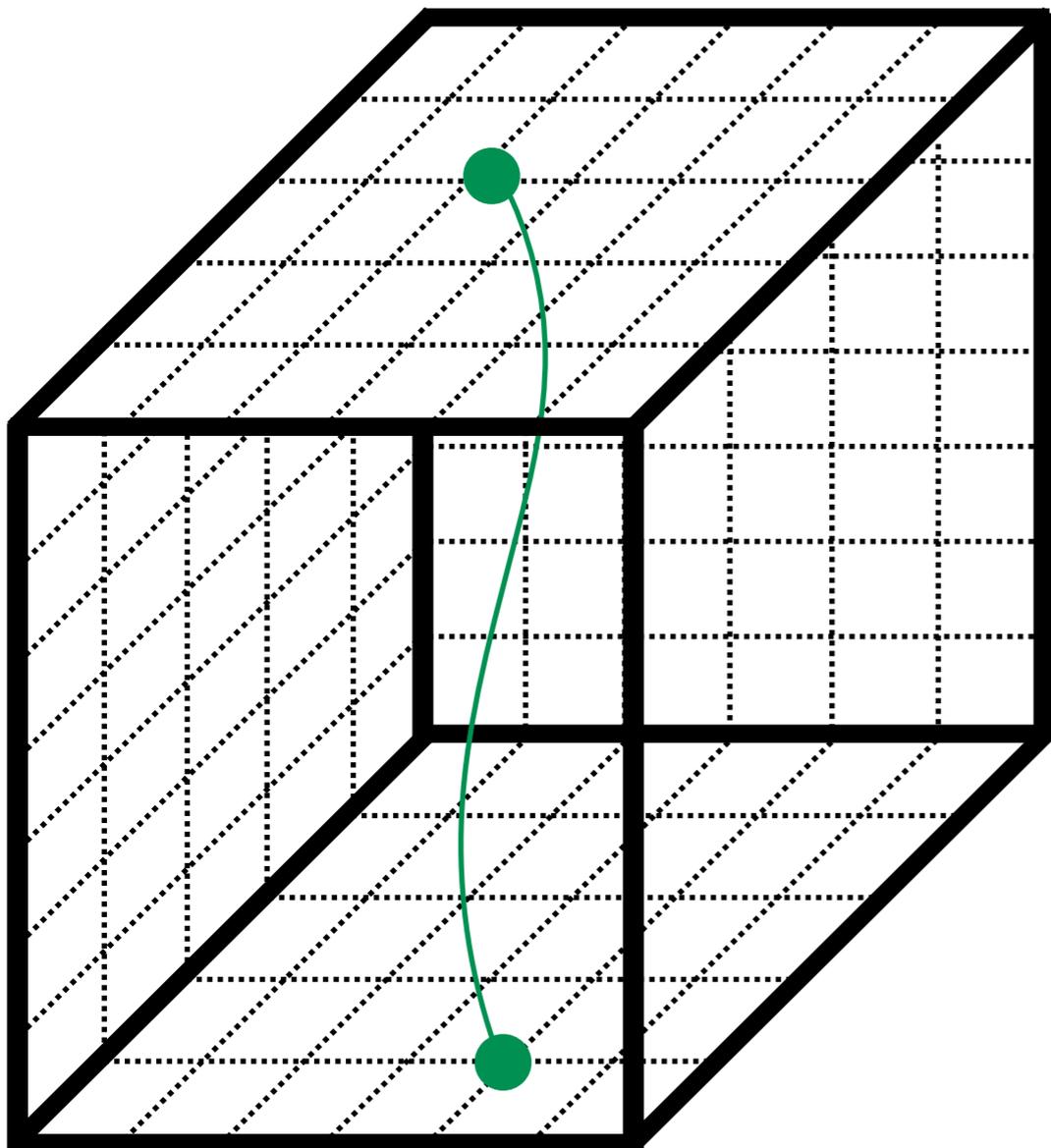
space

Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\mathcal{D} + m)\psi$$

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \mathcal{O}(t)\mathcal{O}^\dagger(0) e^{-S[\bar{\psi},\psi,U]}$$

$$= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \det(\mathcal{D} + M) e^{-S[U]} \mathcal{O}(t)\mathcal{O}^\dagger(0)$$



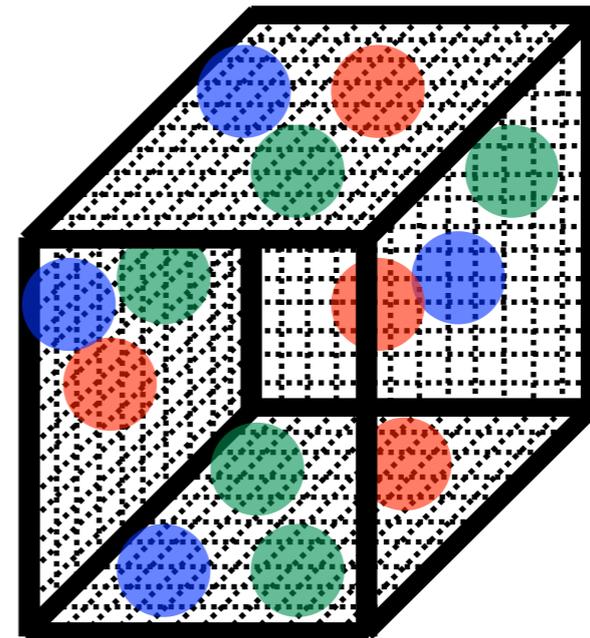
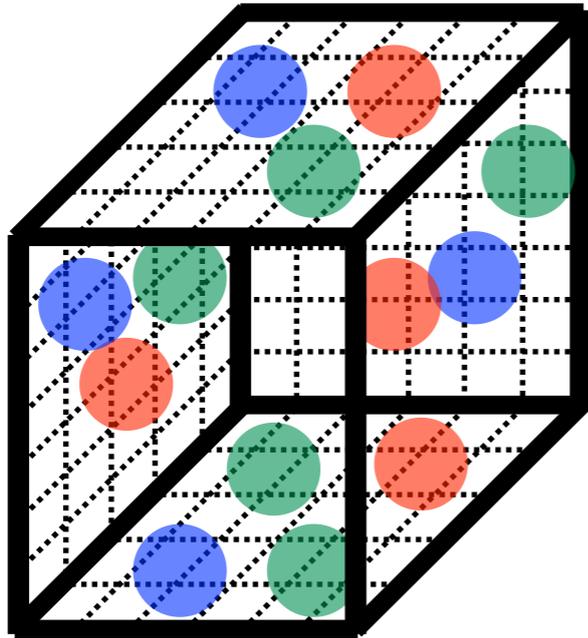
\mathcal{D} is a sparse matrix
U-dependent

$(N_x N_t N_c N_s)$ on a side

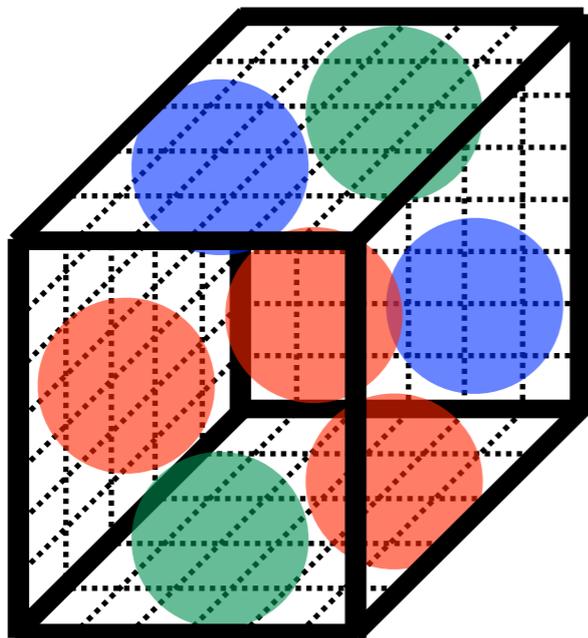
$$(48^3 \times 64 \times 3 \times 4) = 127\,401\,984$$

$$(\mathcal{D}[U] + M)S_F(x \leftarrow y) = i\delta(x - y)$$

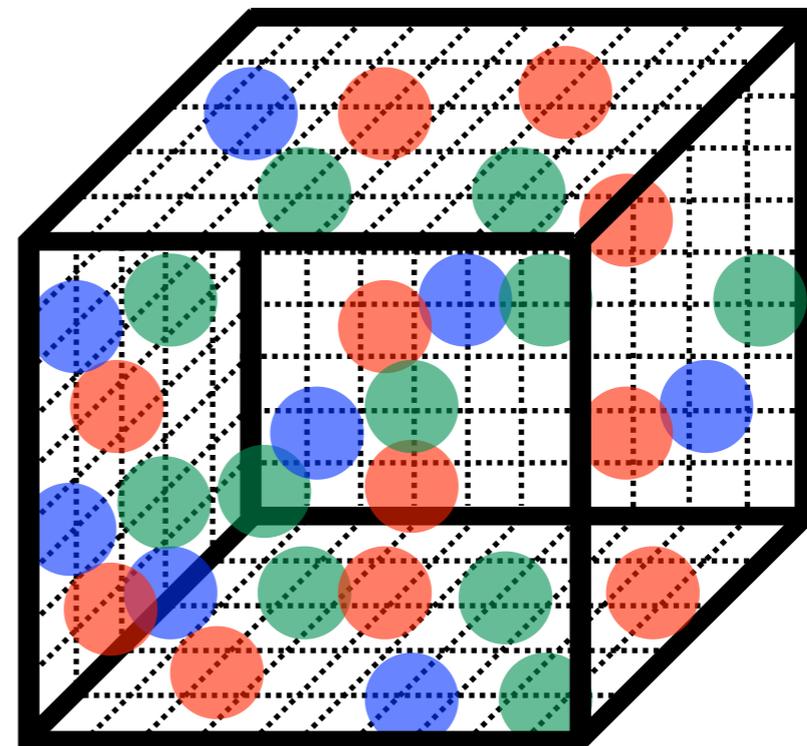
LQCD Systematics



continuum limit



physical quark masses

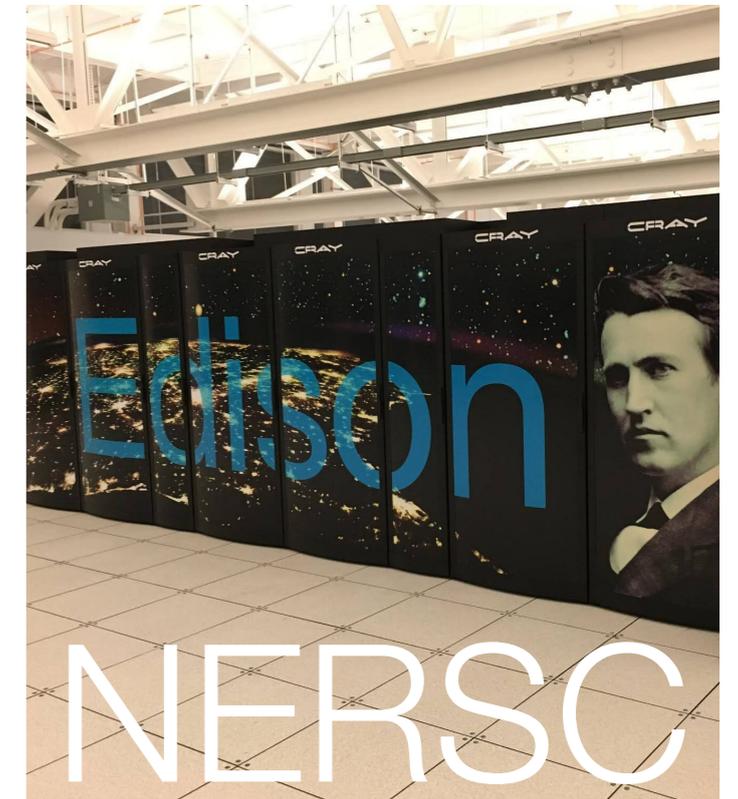


infinite volume limit

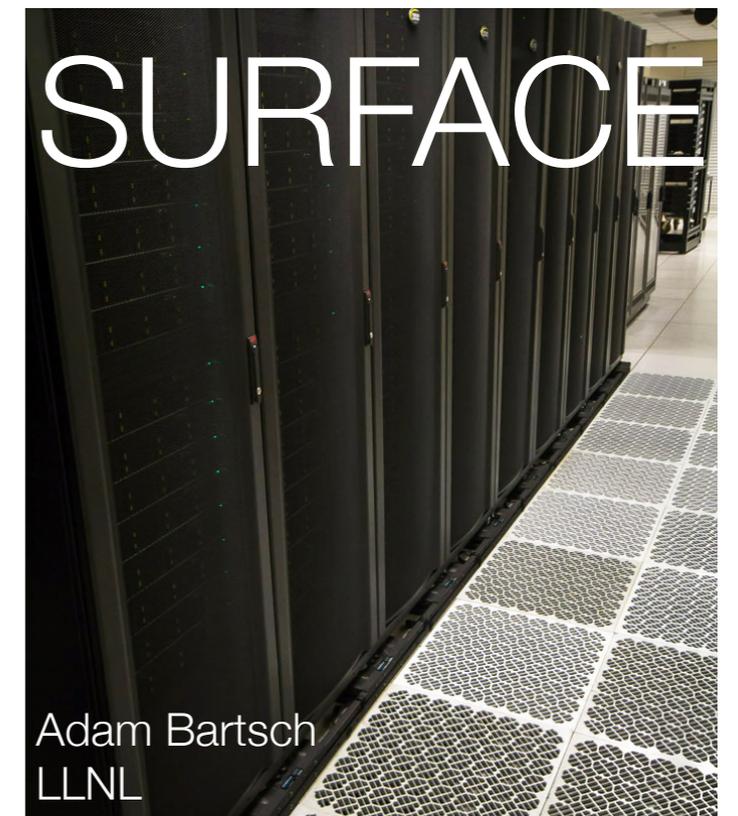


TITAN

OLCF



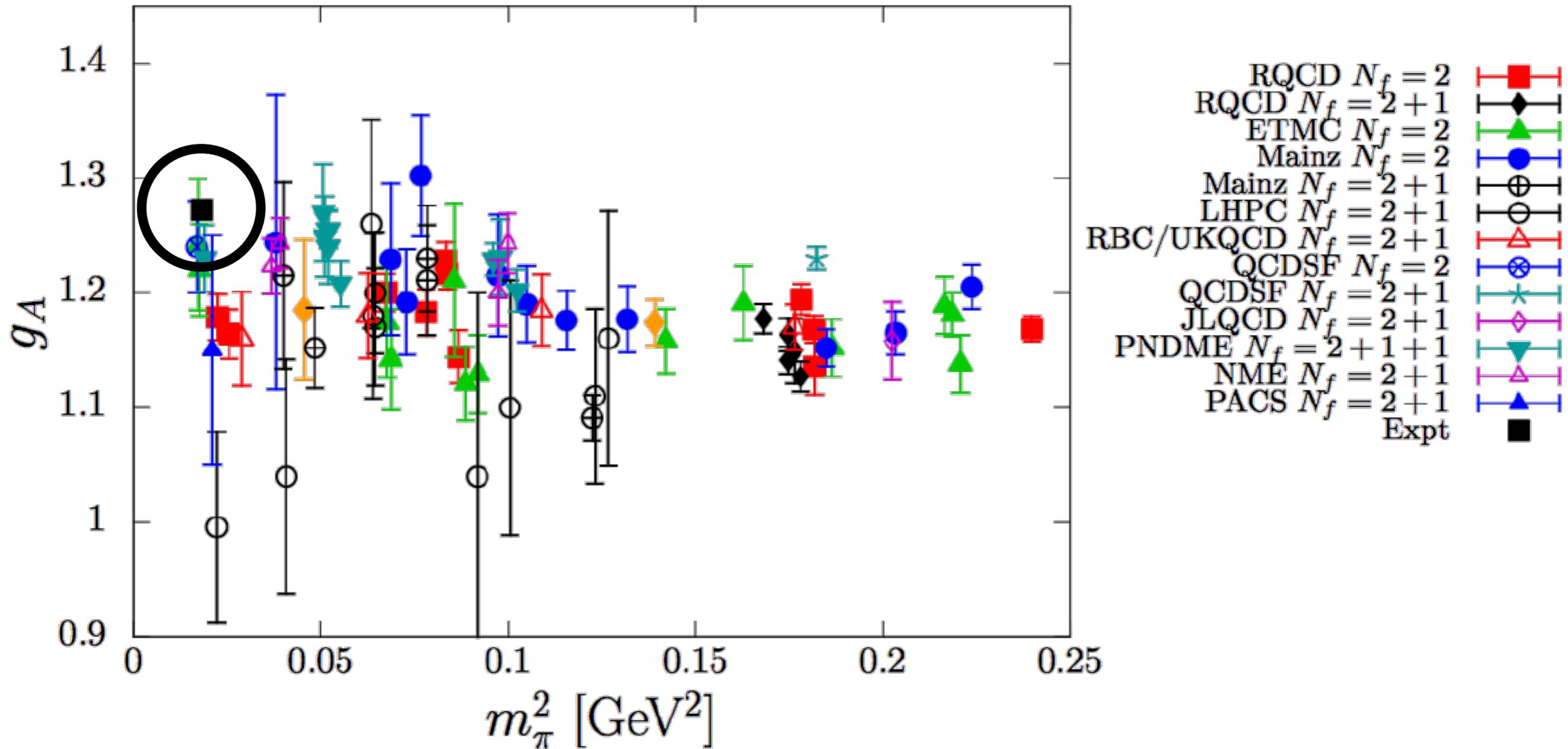
NERSC



Adam Bartsch
LLNL

A long-outstanding problem for LQCD

S. Collins, LATTICE2016 plenary



MILC Ensembles

MILC Collaboration Phys. Rev. D87 (2013) 054505

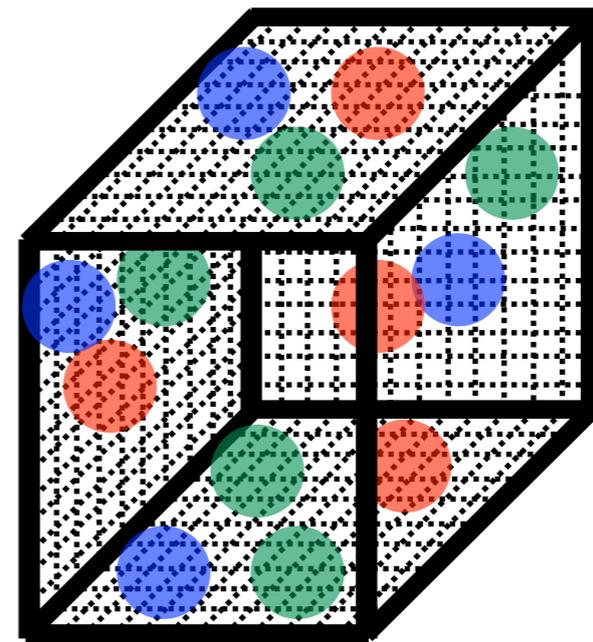
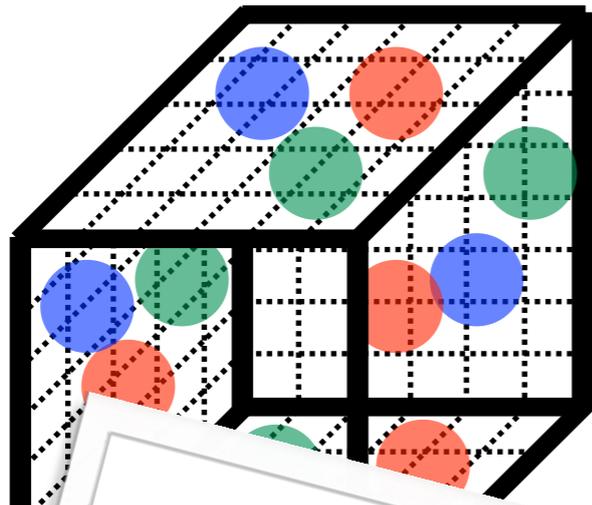
+ additional HISQ ensembles generated at LLNL

HISQ gauge configuration parameters							valence parameters								
abbr.	N_{cfg}	volume	$\sim a$ [fm]	m_l/m_s	$\sim m_{\pi_5}$ [MeV]	$\sim m_{\pi_5}L$	N_{src}	L_5/a	aM_5	b_5	c_5	$am_l^{\text{val.}}$	σ_{smr}	N_{smr}	
coarser	a15m400	1000	$16^3 \times 48$	0.15	0.334	400	4.8	8	12	1.3	1.5	0.5	0.0278	3.0	30
	a15m350	1000	$16^3 \times 48$	0.15	0.255	350	4.2	16	12	1.3	1.5	0.5	0.0206	3.0	30
	a15m310	1960	$16^3 \times 48$	0.15	0.2	310	3.8	24	12	1.3	1.5	0.5	0.01580	4.2	60
	a15m220	1000	$24^3 \times 48$	0.15	0.1	220	4.0	12	16	1.3	1.75	0.75	0.00712	4.5	60
	a15m130	1000	$32^3 \times 48$	0.15	0.036	130	3.2	5	24	1.3	2.25	1.25	0.00216	4.5	60
middle	a12m400	1000	$24^3 \times 64$	0.12	0.334	400	5.8	8	8	1.2	1.25	0.25	0.02190	3.0	30
	a12m350	1000	$24^3 \times 64$	0.12	0.255	350	5.1	8	8	1.2	1.25	0.25	0.01660	3.0	30
	a12m310	1053	$24^3 \times 64$	0.12	0.2	310	4.5	8	8	1.2	1.25	0.25	0.01260	3.0	30
	a12m220S	1000	$24^3 \times 64$	0.12	0.1	220	3.2	4	12	1.2	1.5	0.5	0.00600	6.0	90
	a12m220	1000	$32^3 \times 64$	0.12	0.1	220	4.3	4	12	1.2	1.5	0.5	0.00600	6.0	90
	a12m220L	1000	$40^3 \times 64$	0.12	0.1	220	5.4	4	12	1.2	1.5	0.5	0.00600	6.0	90
finer	a12m130	1000	$48^3 \times 64$	0.12	0.036	130	3.9	3	20	1.2	2.0	1.0	0.00195	7.0	150
	a09m400	1201	$32^3 \times 64$	0.09	0.335	400	5.8	8	6	1.1	1.25	0.25	0.0160	3.5	45
	a09m350	1201	$32^3 \times 64$	0.09	0.255	350	5.1	8	6	1.1	1.25	0.25	0.0121	3.5	45
	a09m310	784	$32^3 \times 96$	0.09	0.2	310	4.5	8	6	1.1	1.25	0.25	0.00951	7.5	167
a09m220	1001	$48^3 \times 96$	0.09	0.1	220	4.7	6	8	1.1	1.25	0.25	0.00449	8.0	150	

MDWF pion mass tuned to taste-5 HISQ
pion mass within 1-2% - ensuring the
unitary limit is recovered in the continuum

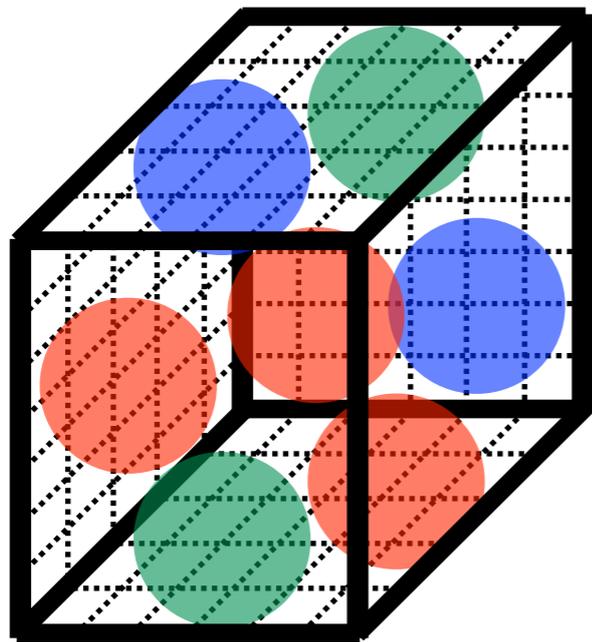
Free to use; large statistics available
Capable of controlling all systematics
We use domain wall valence on the HISQ sea,
 $\mathcal{O}(a^2)$ errors [1701.07559].

LQCD Systematics

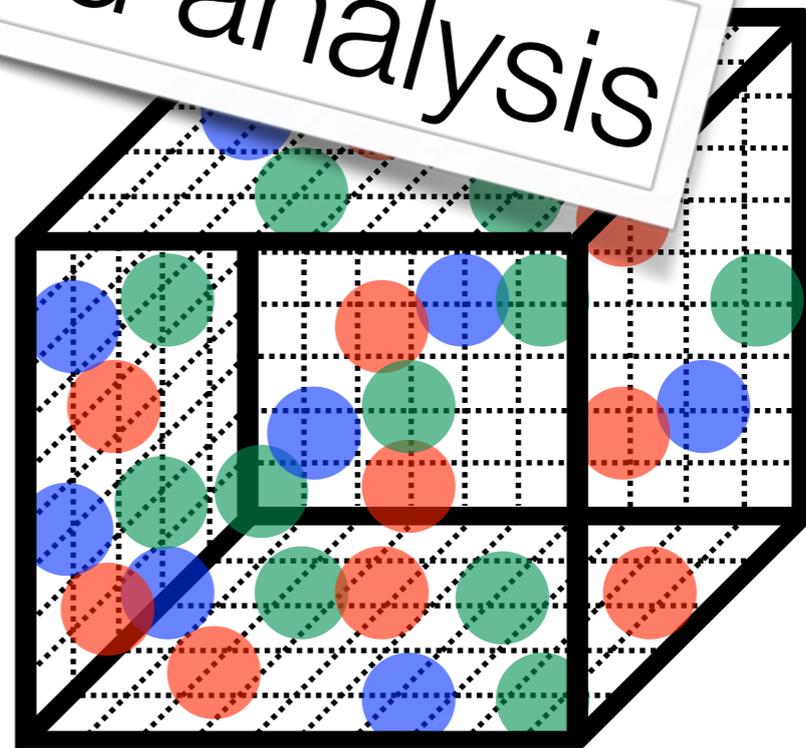


method, fitting, and analysis

continuum limit

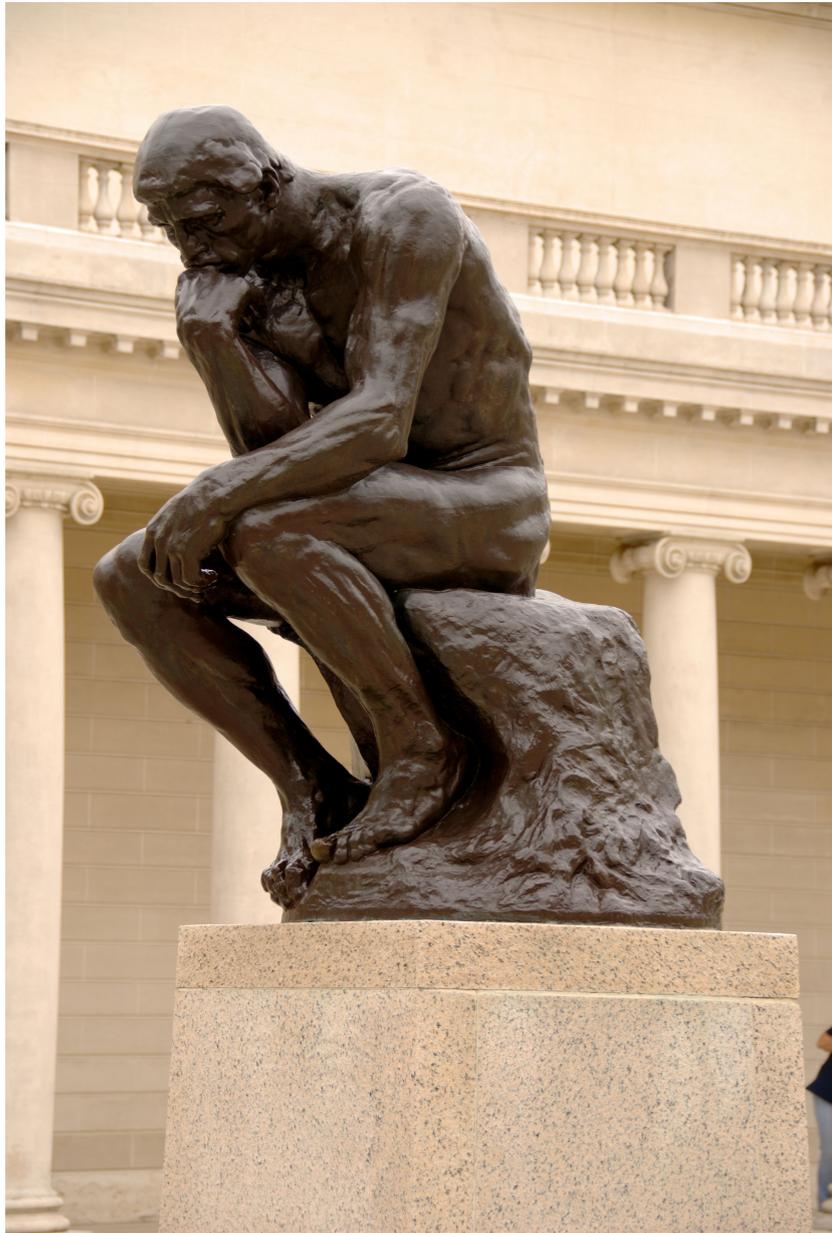


physical quark masses



infinite volume limit

New Methods



New Analytic Tools

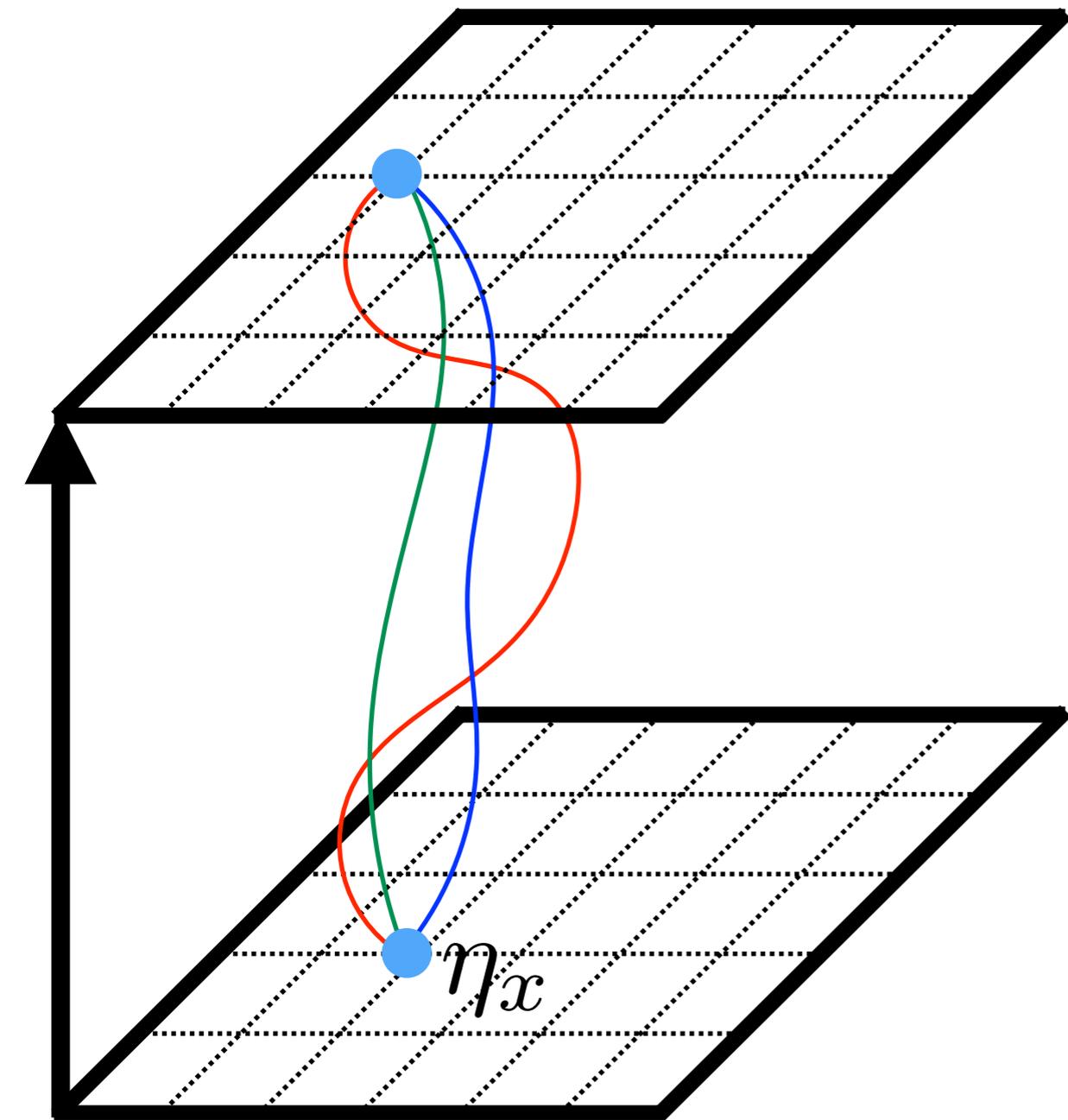


Improved Systematics



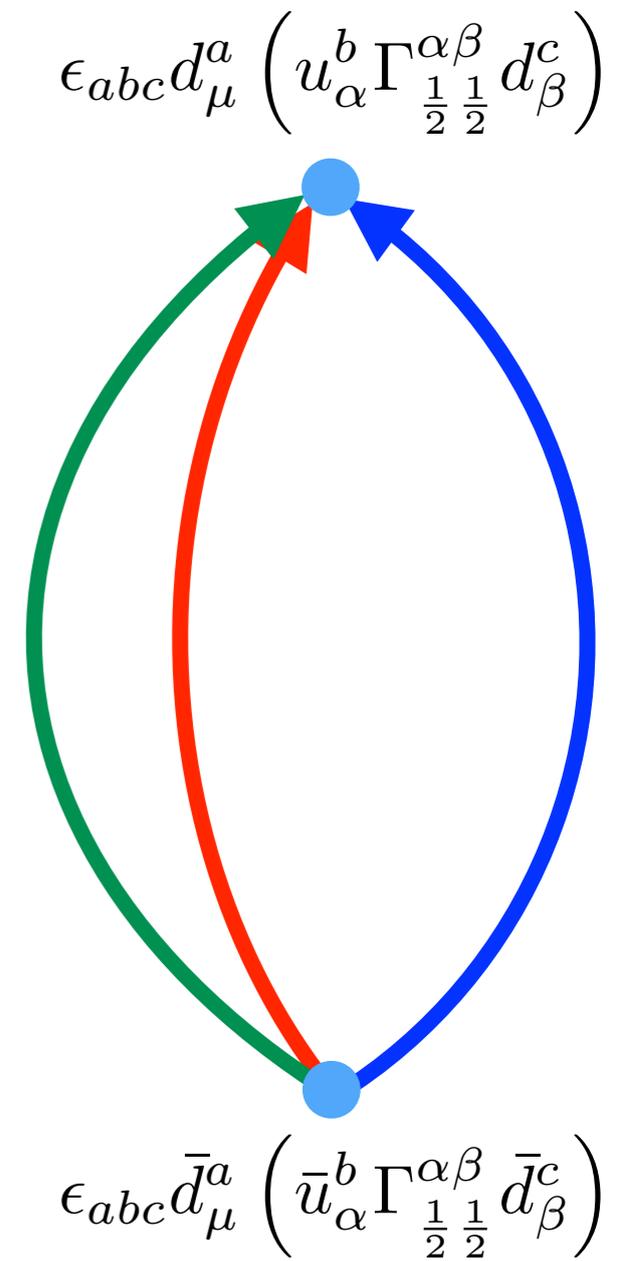
Computationally Affordable

2-Point Standard Method



$$(\mathbb{D}[M] + M)$$

$$\eta x$$



Effective Mass

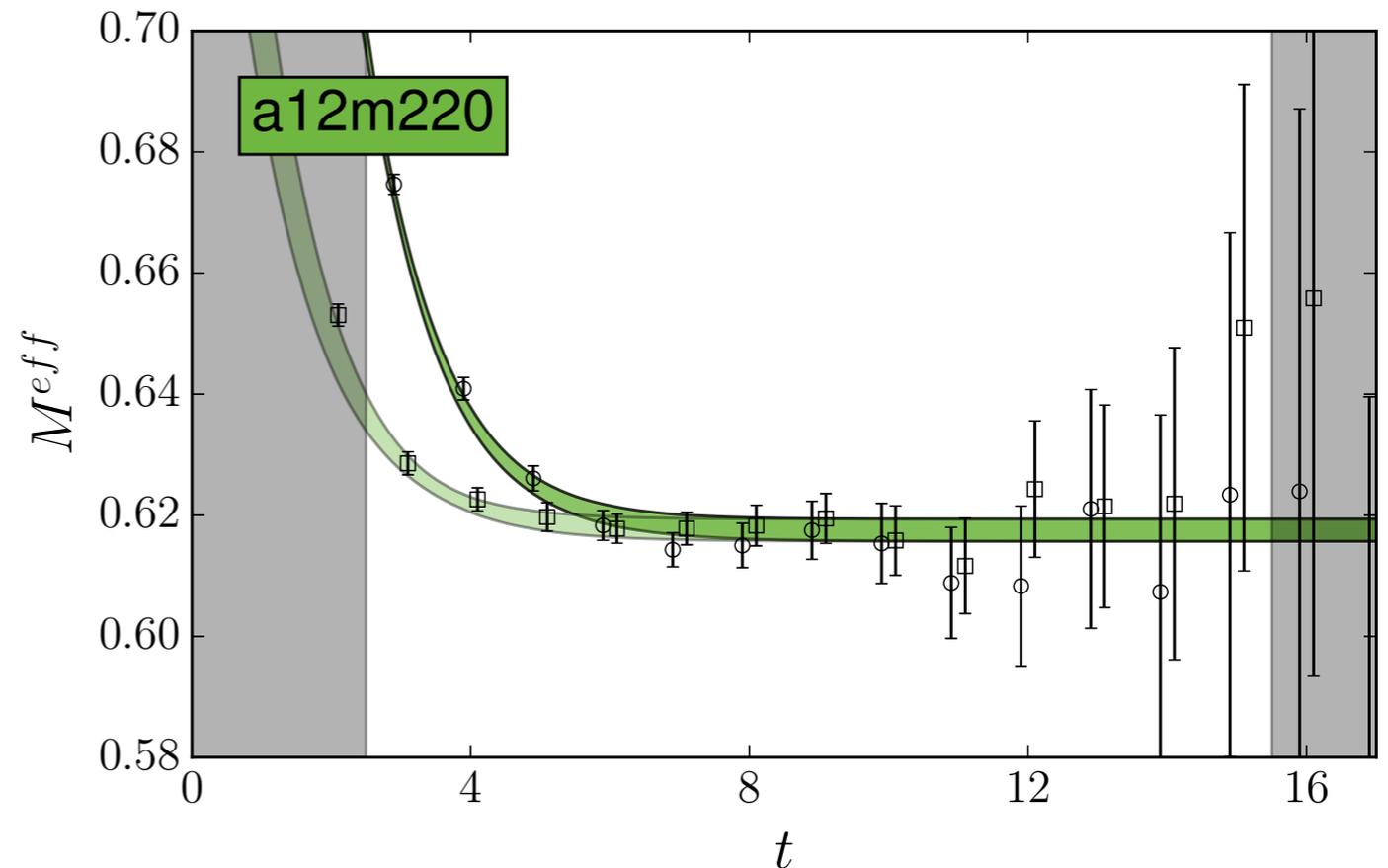
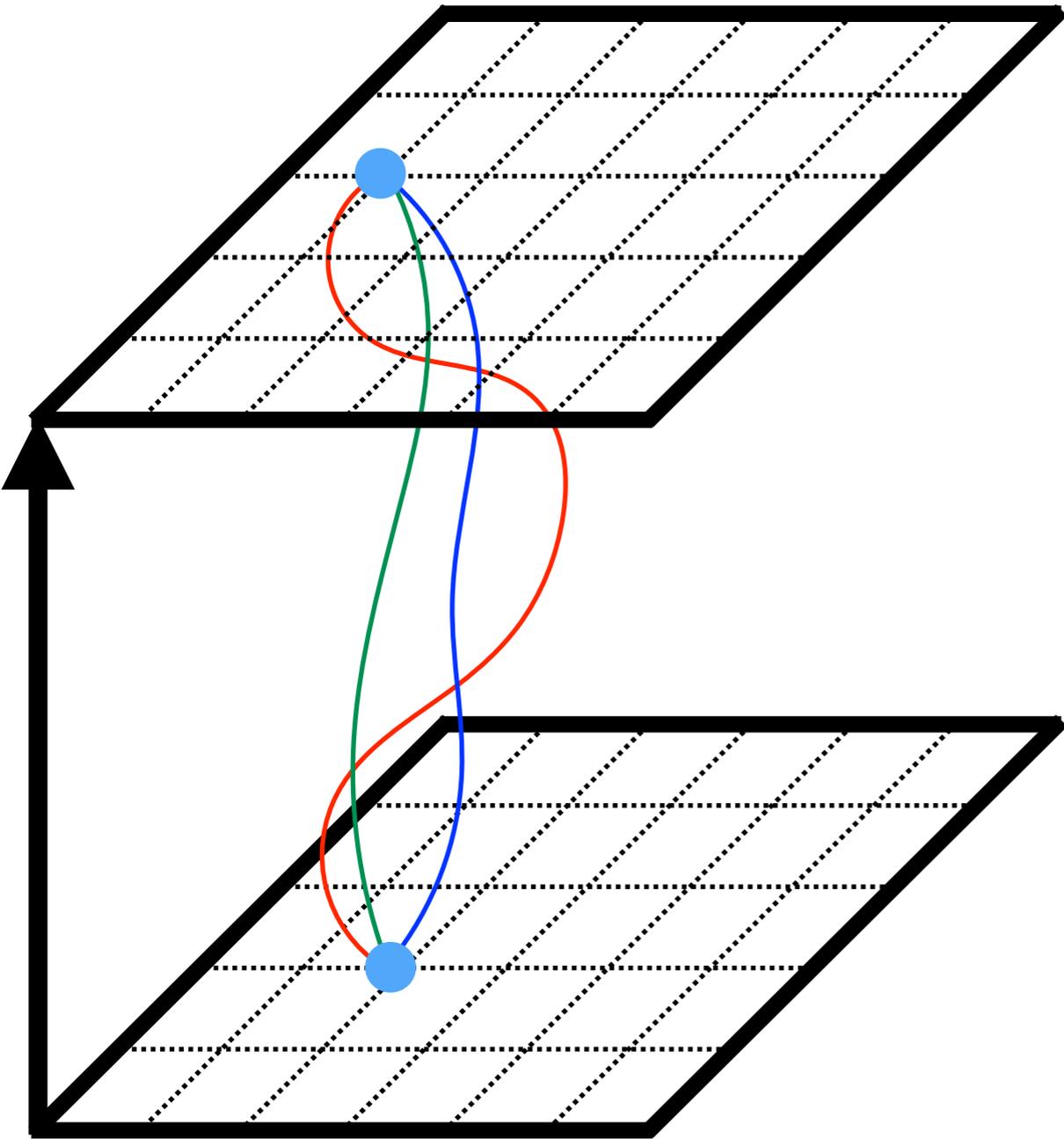
$$C(t) = \langle \Omega | \mathcal{O}(t) \mathcal{O}^\dagger(0) | \Omega \rangle$$

$$= \sum_n \langle \Omega | e^{\hat{H}t} \mathcal{O}(0) e^{-\hat{H}t} \frac{|n\rangle\langle n|}{2E_n} \mathcal{O}^\dagger(0) | \Omega \rangle$$

$$= \sum_n Z_n Z_n^\dagger \frac{e^{-E_n t}}{2E_n}$$

$$M^{eff}(t) = -\partial_t \ln(C(t))$$

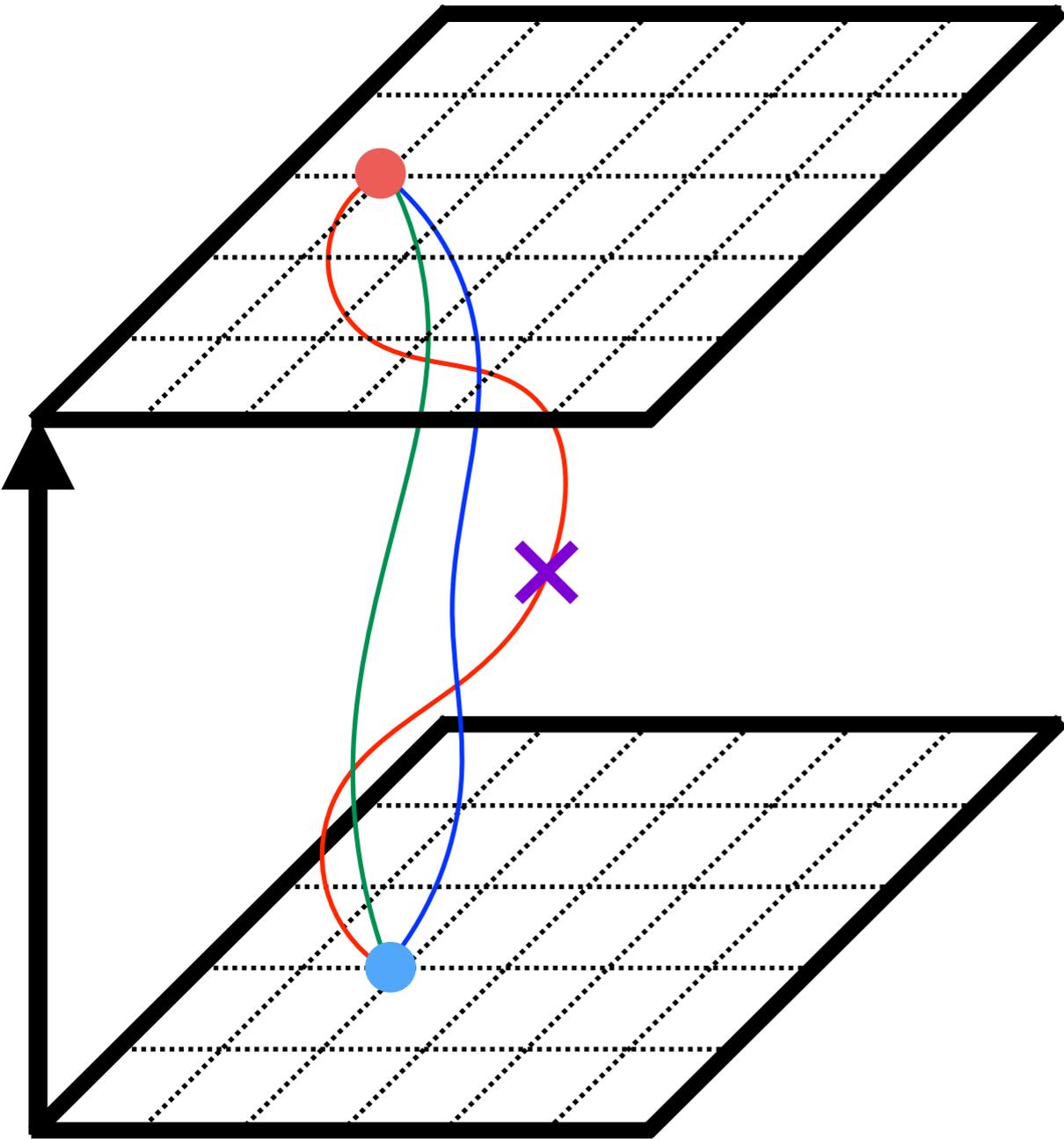
$$\lim_{t \rightarrow \infty} M^{eff}(t) = E_0$$



Matrix Elements

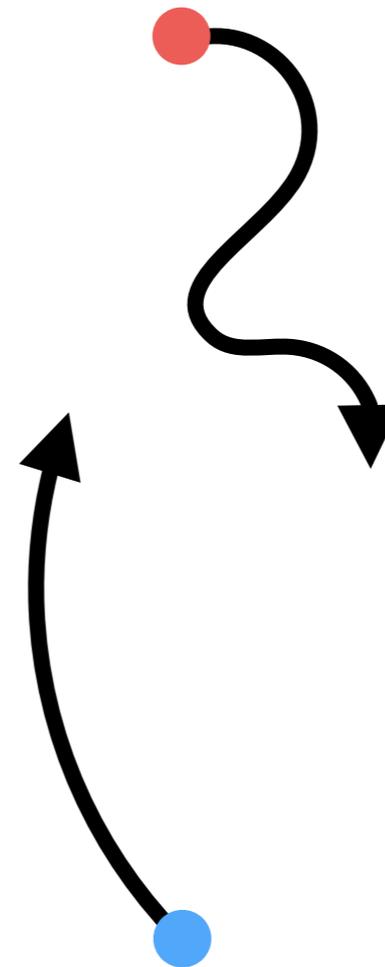
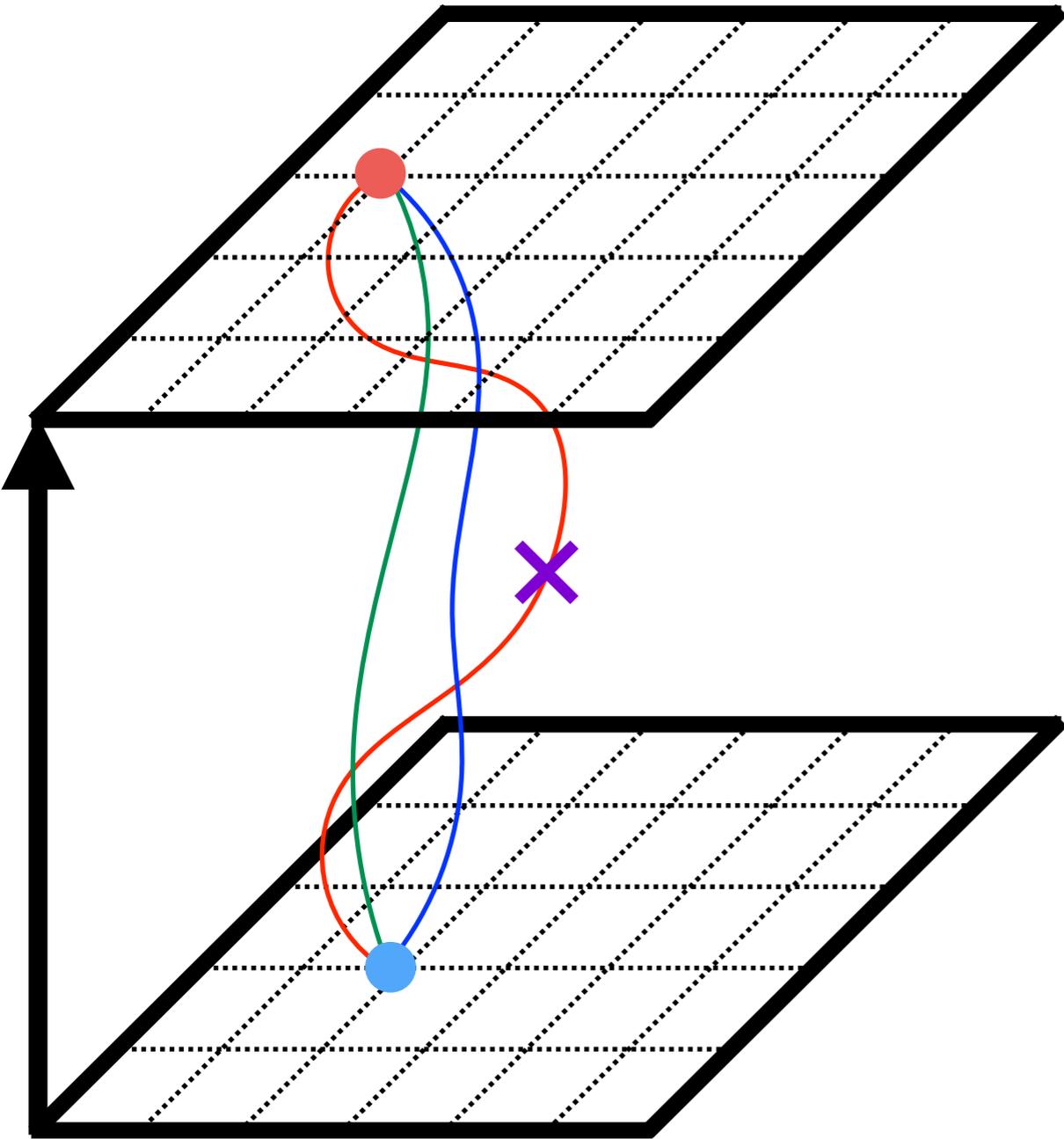
$$\langle \Omega | \mathcal{O}(t) \bar{q} \Gamma q(\tau) \mathcal{O}^\dagger(0) | \Omega \rangle$$

$$g_A : \bar{q} \gamma_\mu \gamma_5 \tau^a q$$

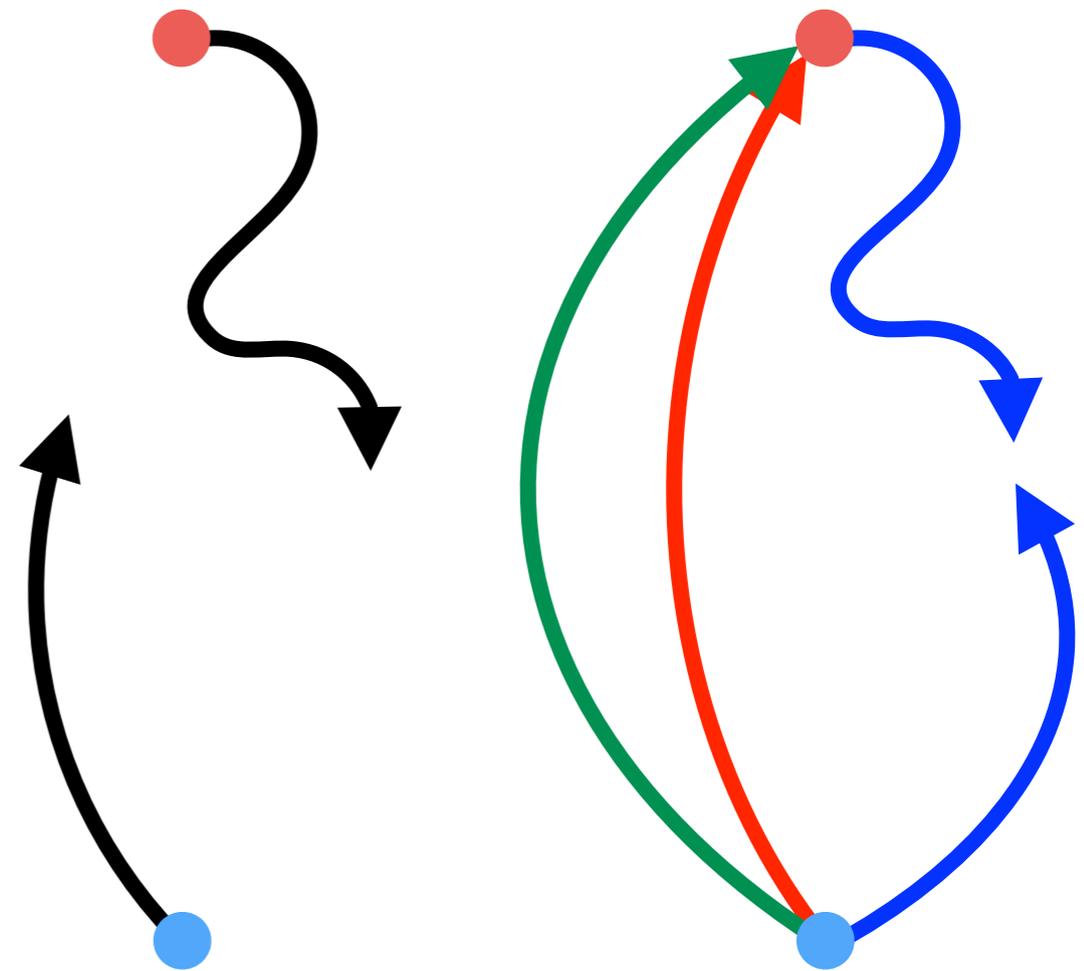
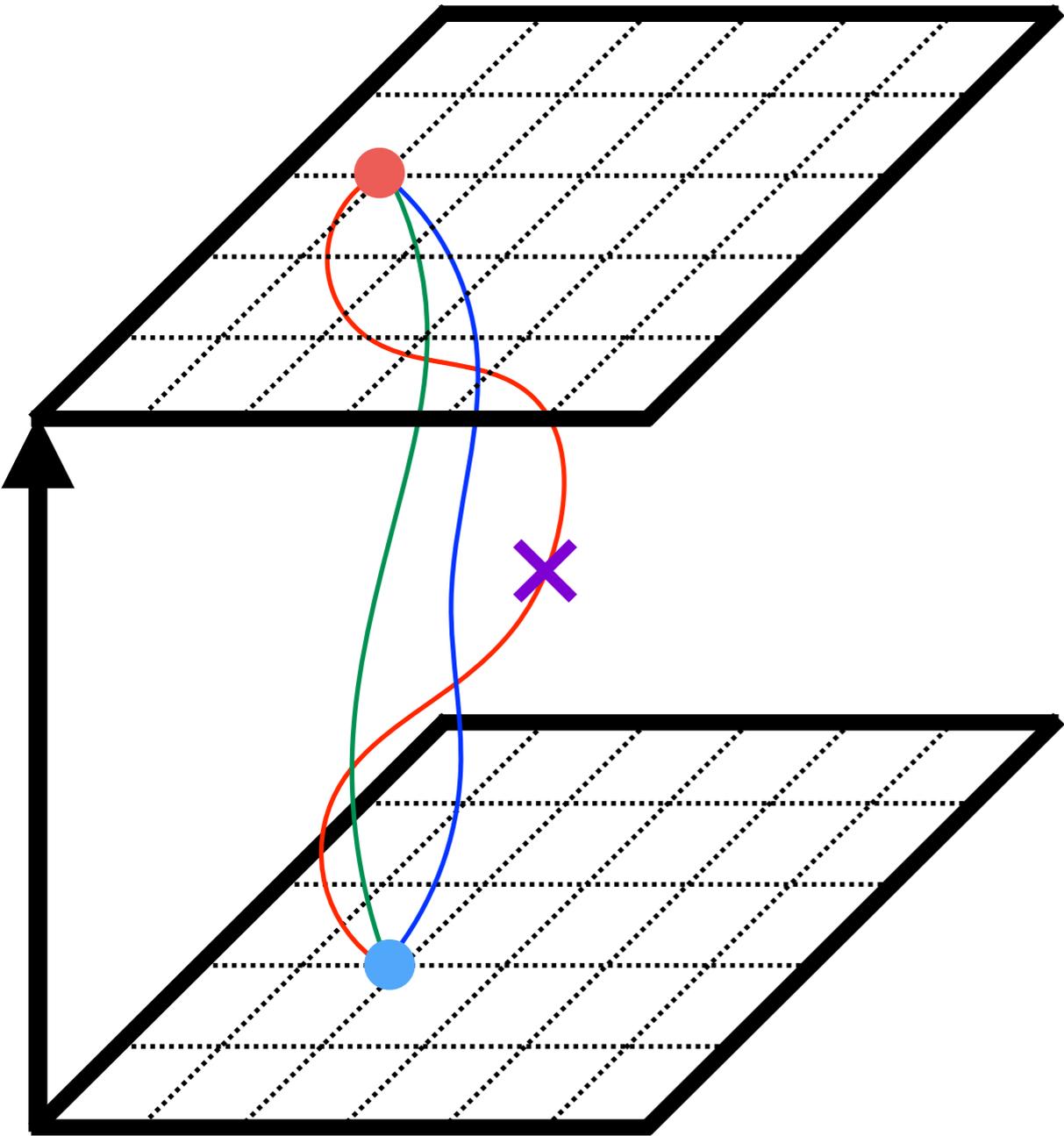


Widely separate
sink, current insertion, and source

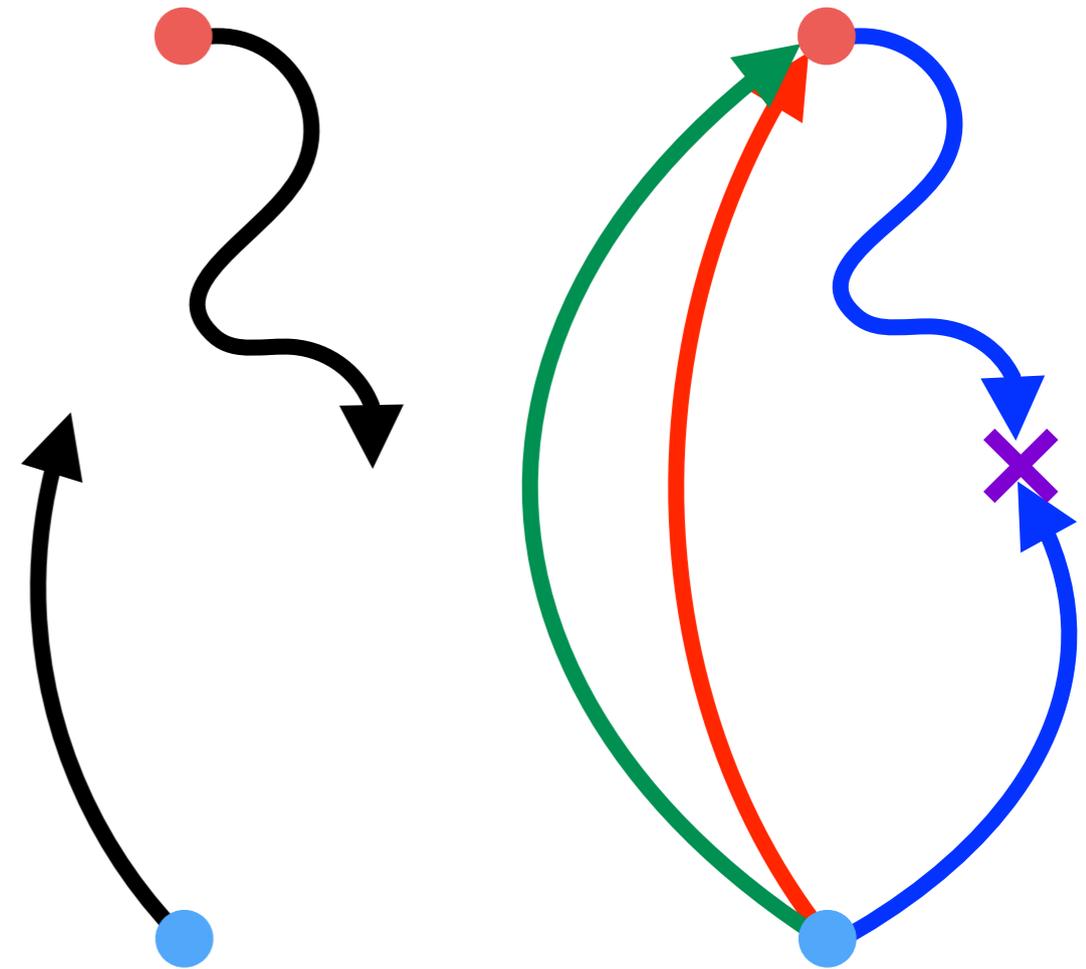
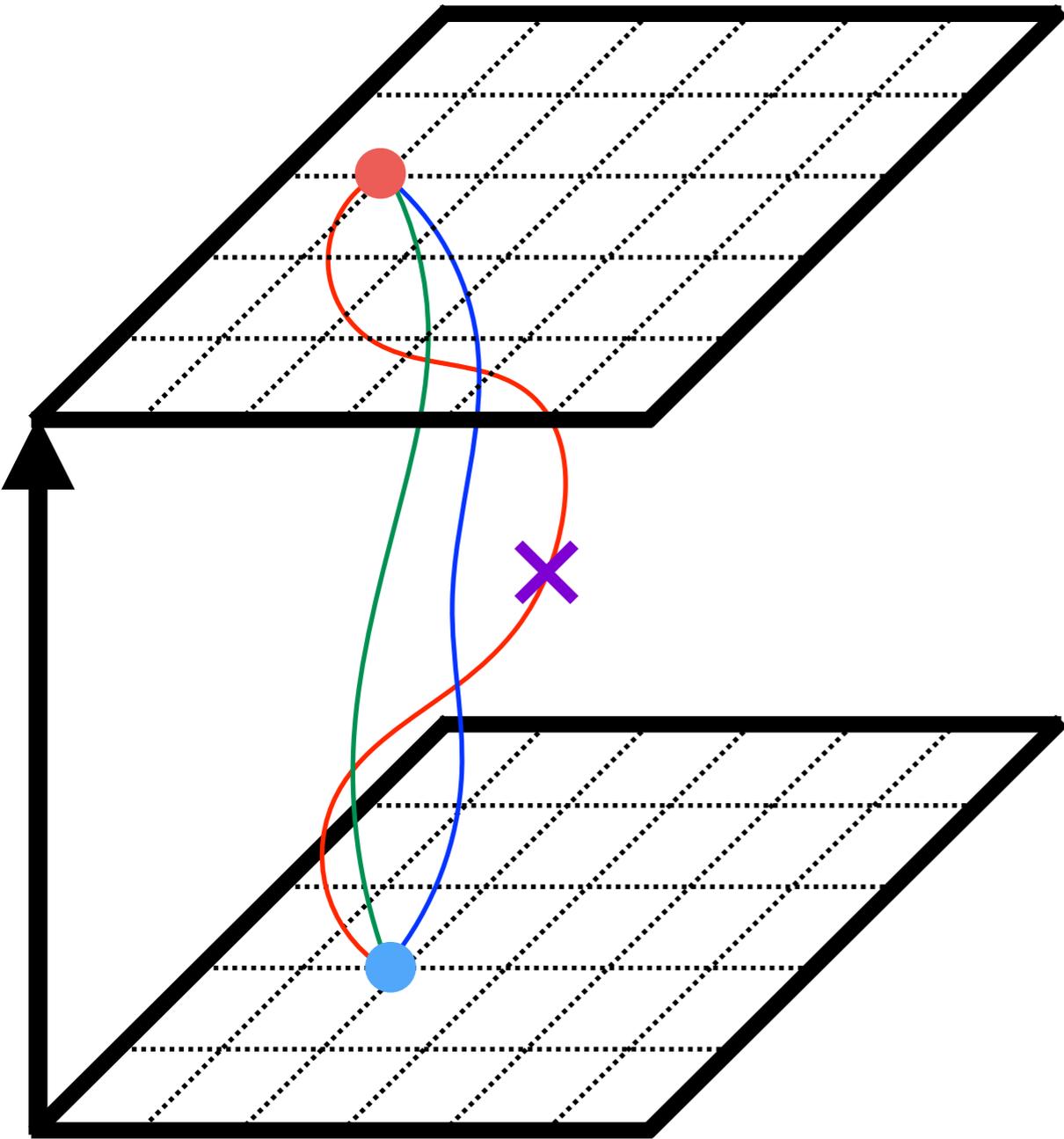
3-Point Standard Method



3-Point Standard Method



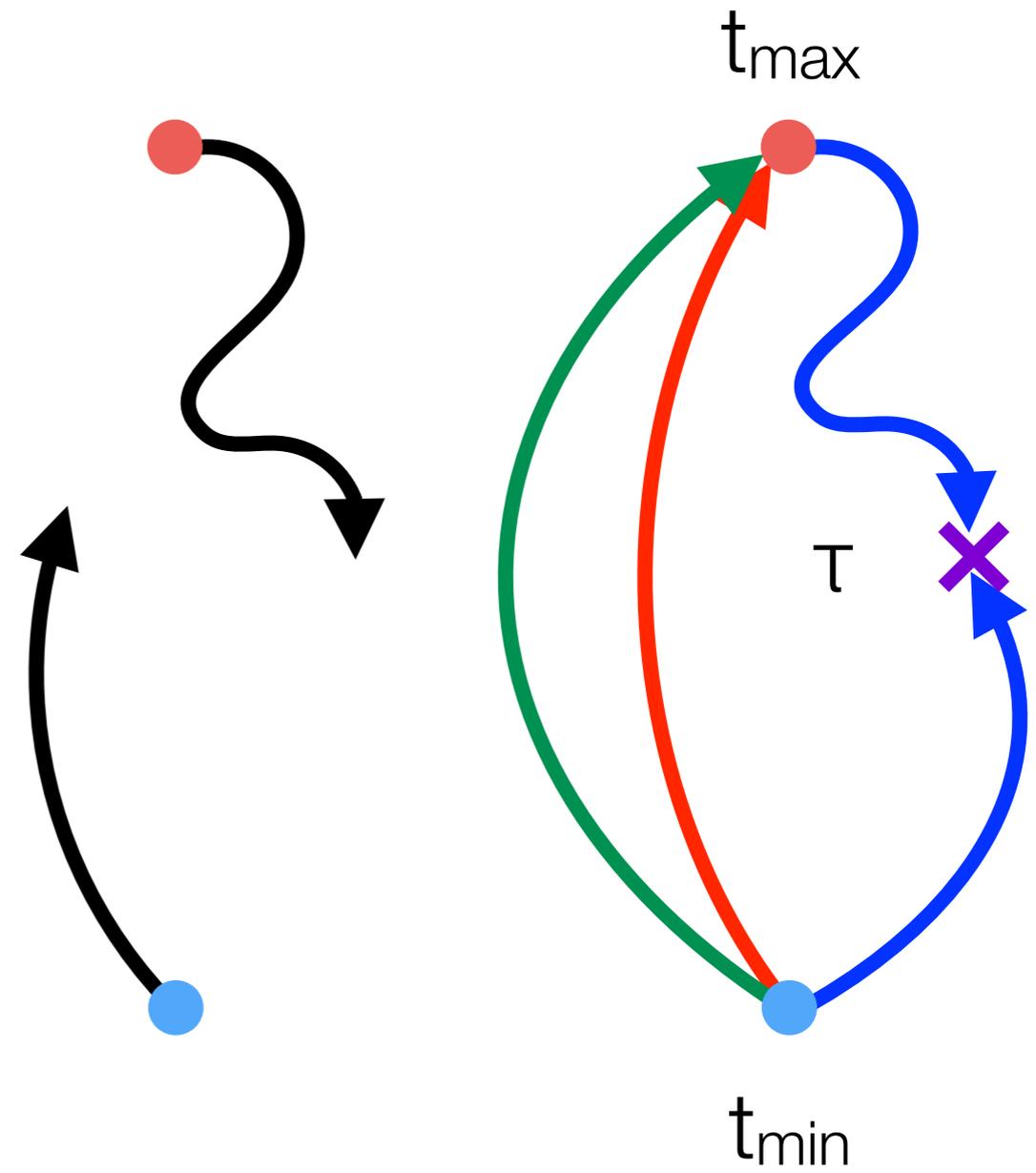
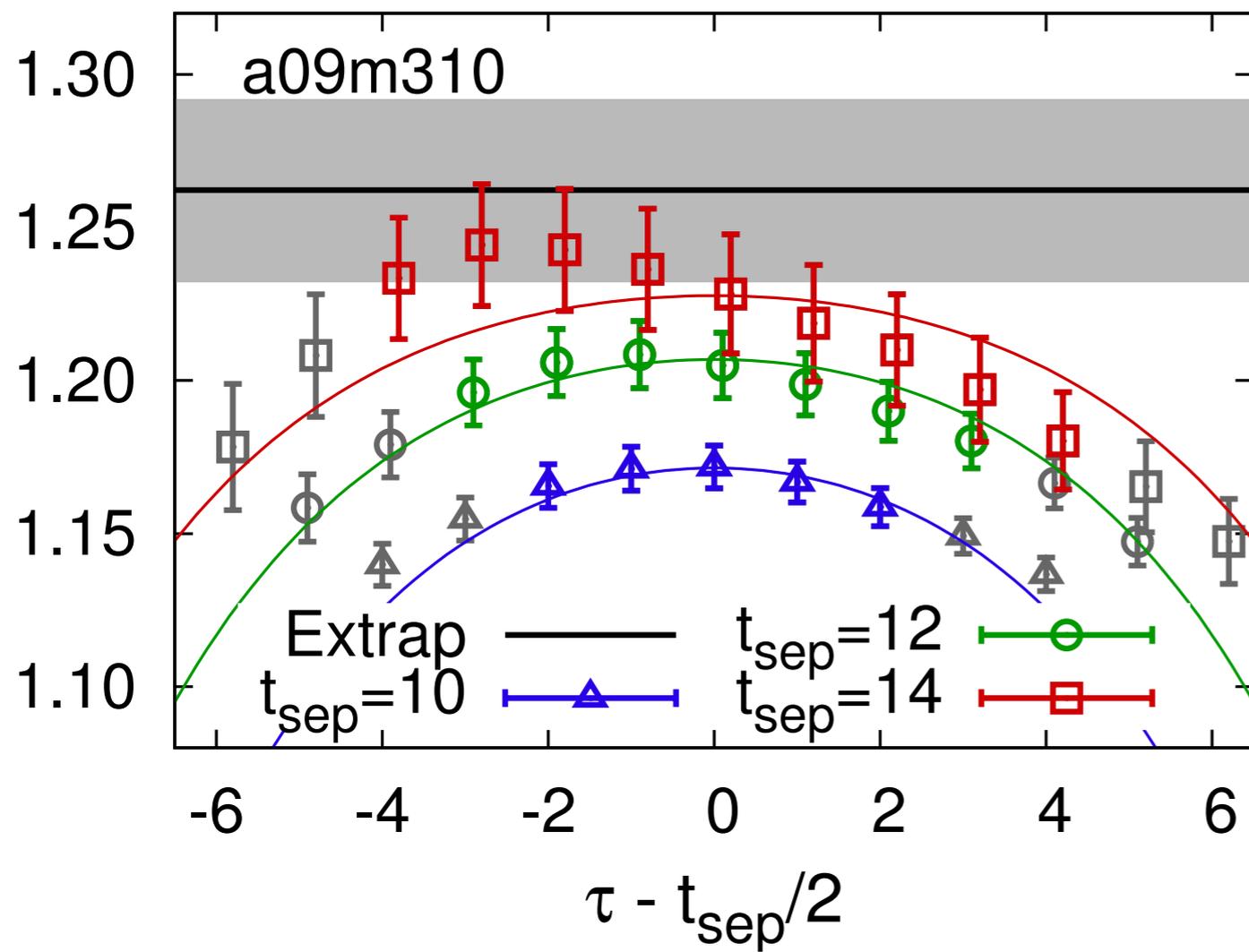
3-Point Standard Method



Two solves: one time separation but all bilinears

Standard Method

PNDME Phys. Rev. D94 (2016) arXiv:1606.07049



Feynman-Hellmann Method

Bouchard, Chang, Kurth, Orginos, and Walker-Loud arXiv:1612.06963

See also Maiani, Martinelli, Paciello and Taglienti Nucl. Phys. B293 (1987)
NPLQCD 1610.04545, 1611.00344, 1701.03456, 1702.02929

similar methods (other FH / GEVP):

J. Bulava et. al. JHEP 01,140 (2012)

F. Bernardoni et. al. Phys. Lett. B740, 278-284 (2015)

A.J. Chambers et. al. Phys. Rev. D 90, 014510

A.J. Chambers et. al. Phys. Rev. D 92, 114517

M.J. Savage et. al. Phys. Rev. Lett. 119, 062002

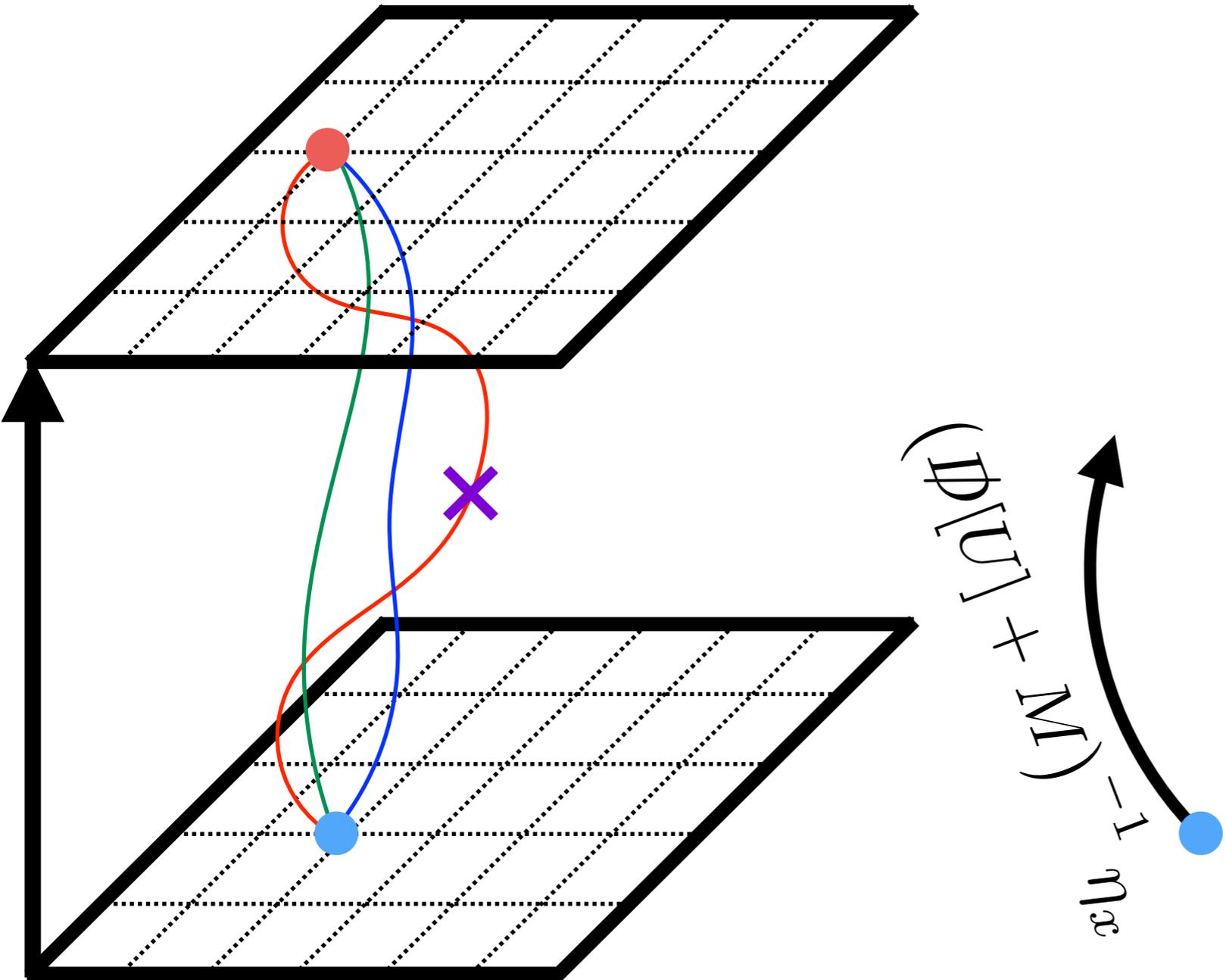
similar fit function:

S. Capitani et. al. Phys. Rev. D 86, 074502

propagator construction:

L. Maiani et. al. Nucl. Phys. B293 (1987)

G.M. de Divitiis et. al. Phys. Lett. B718 (2012))



Feynman-Hellmann Method

Bouchard, Chang, Kurth, Orginos, and Walker-Loud arXiv:1612.06963

See also Maiani, Martinelli, Paciello and Taglienti Nucl. Phys. B293 (1987)
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A.J. Chambers et. al. Phys. Rev. D 90, 014510

A.J. Chambers et. al. Phys. Rev. D 92, 114517

M.J. Savage et. al. Phys. Rev. Lett. 119, 062002

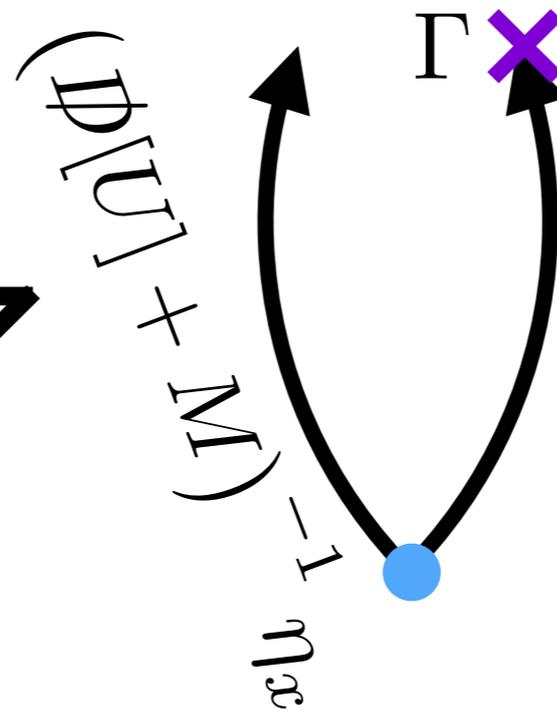
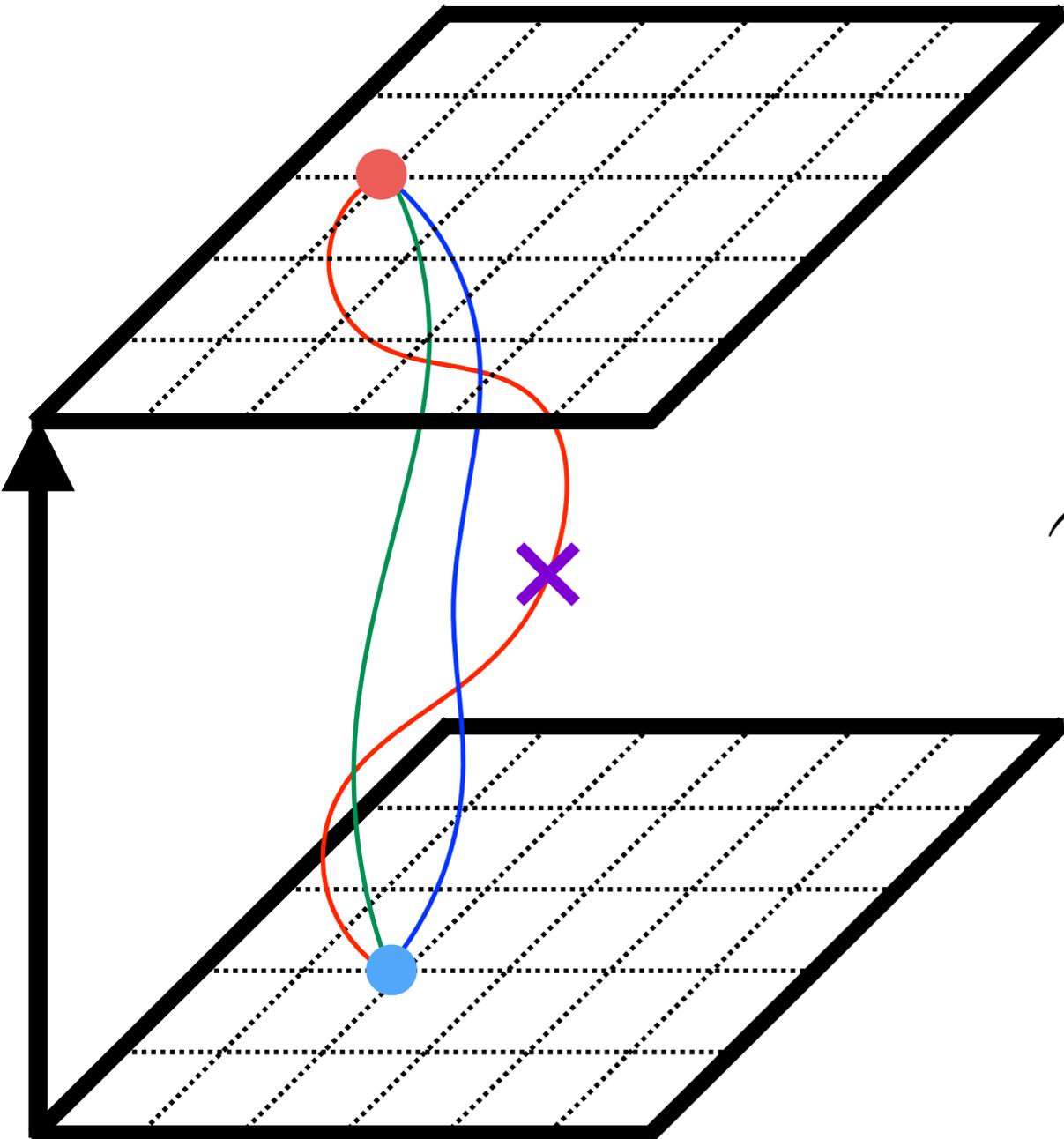
similar fit function:

S. Capitani et. al. Phys. Rev. D 86, 074502

propagator construction:

L. Maiani et. al. Nucl. Phys. B293 (1987)

G.M. de Divitiis et. al. Phys. Lett. B718 (2012)



Feynman-Hellmann Method

Bouchard, Chang, Kurth, Orginos, and Walker-Loud arXiv:1612.06963

See also Maiani, Martinelli, Paciello and Taglienti Nucl. Phys. B293 (1987)
NPLQCD 1610.04545, 1611.00344, 1701.03456, 1702.02929

similar methods (other FH / GEVP):

J. Bulava et. al. JHEP 01,140 (2012)

F. Bernardoni et. al. Phys. Lett. B740, 278-284 (2015)

A.J. Chambers et. al. Phys. Rev. D 90, 014510

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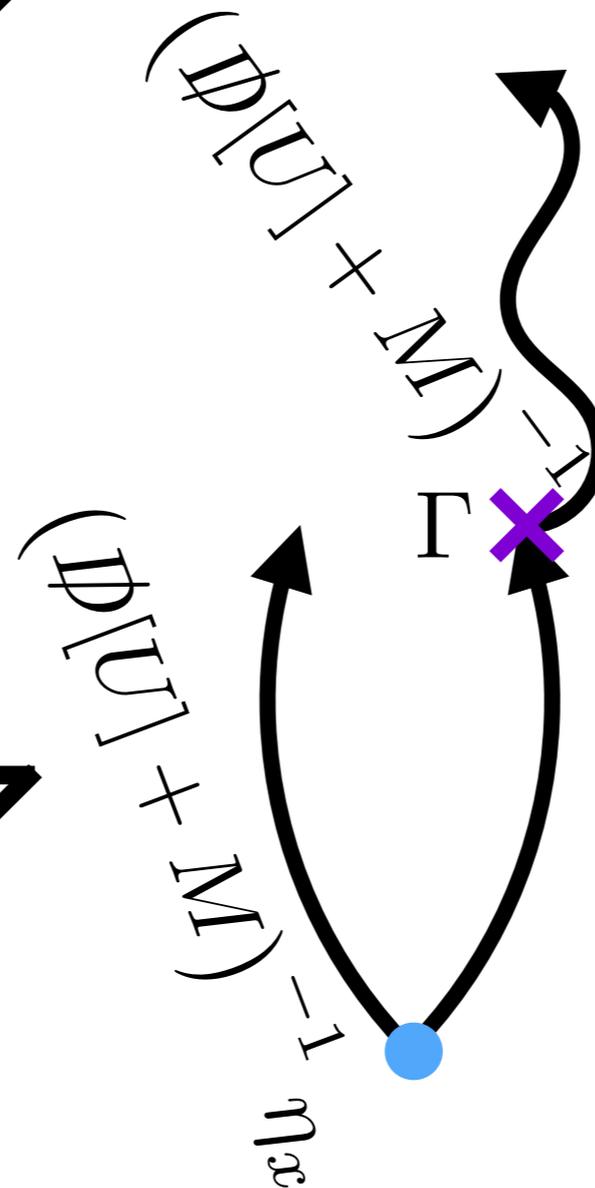
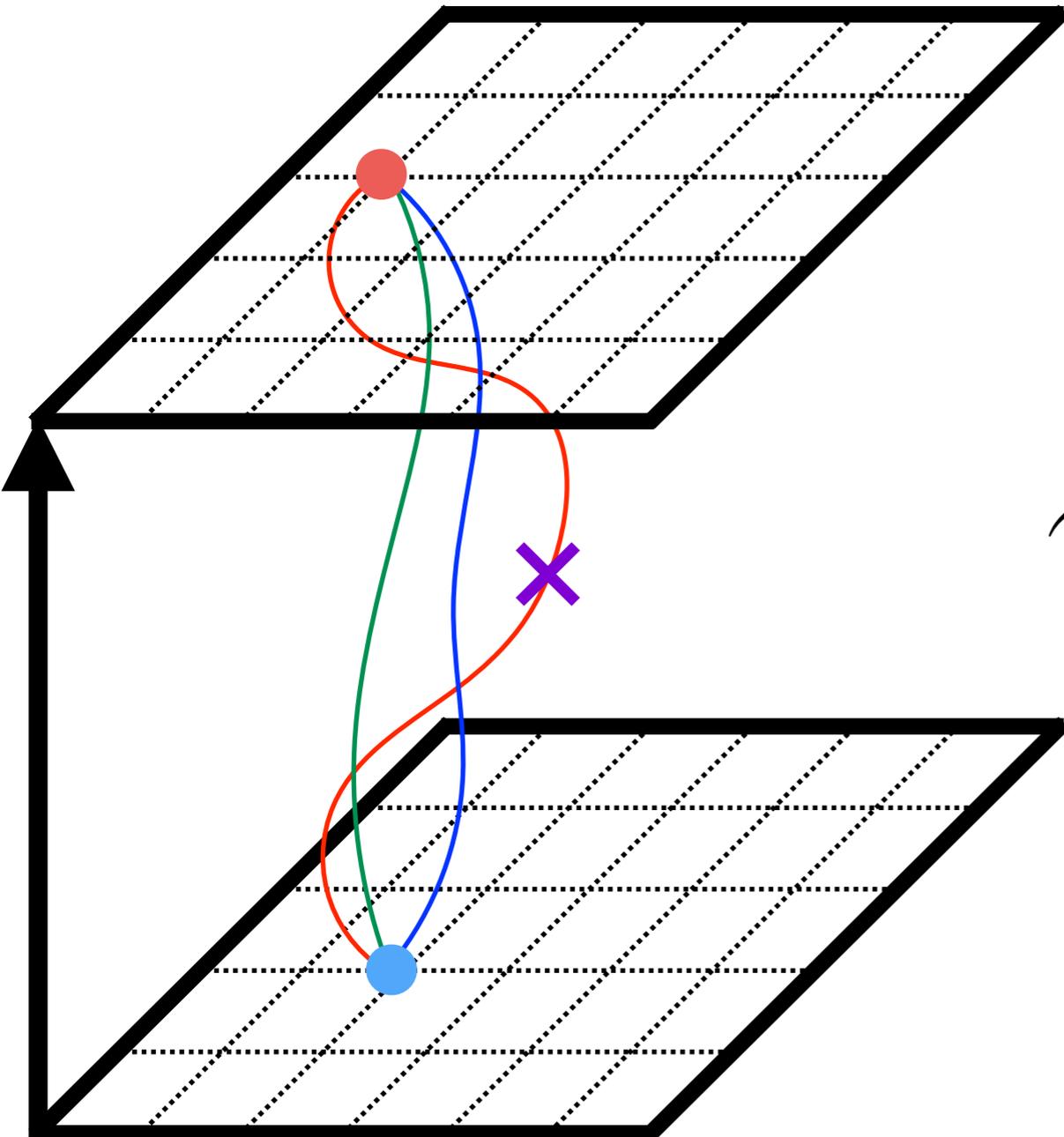
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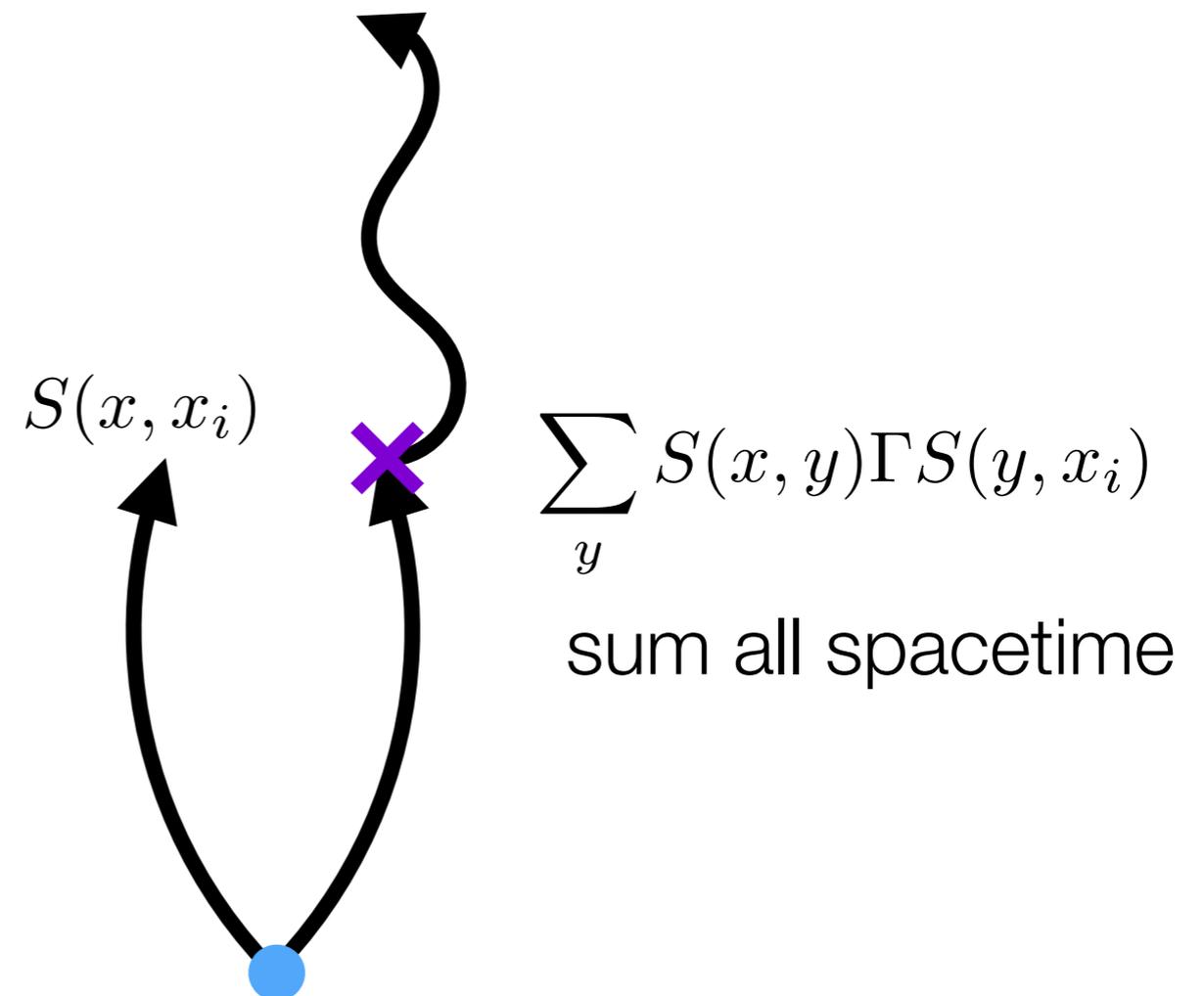
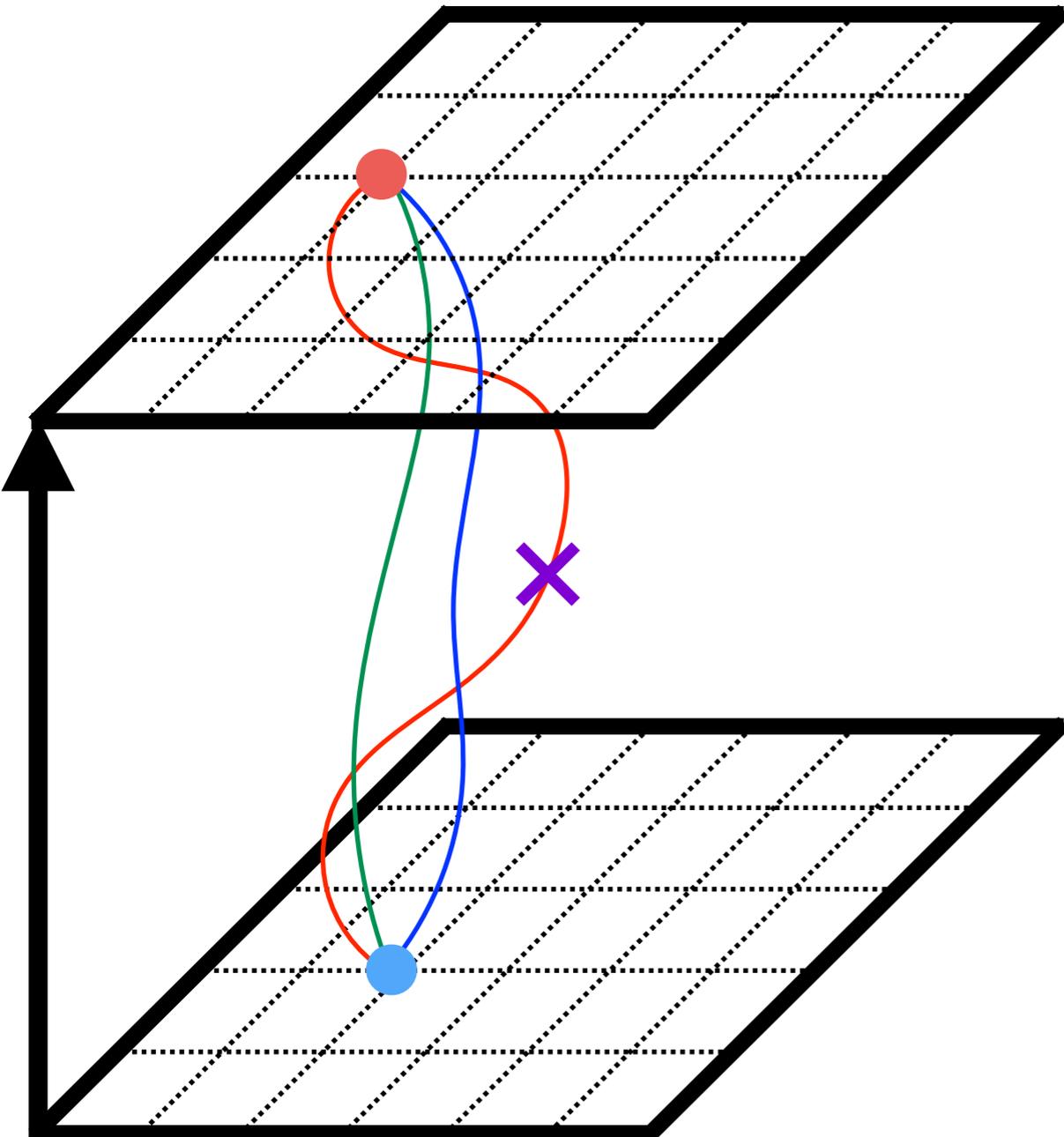
similar fit function:

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NPLQCD 1610.04545, 1611.00344, 1701.03456, 1702.02929

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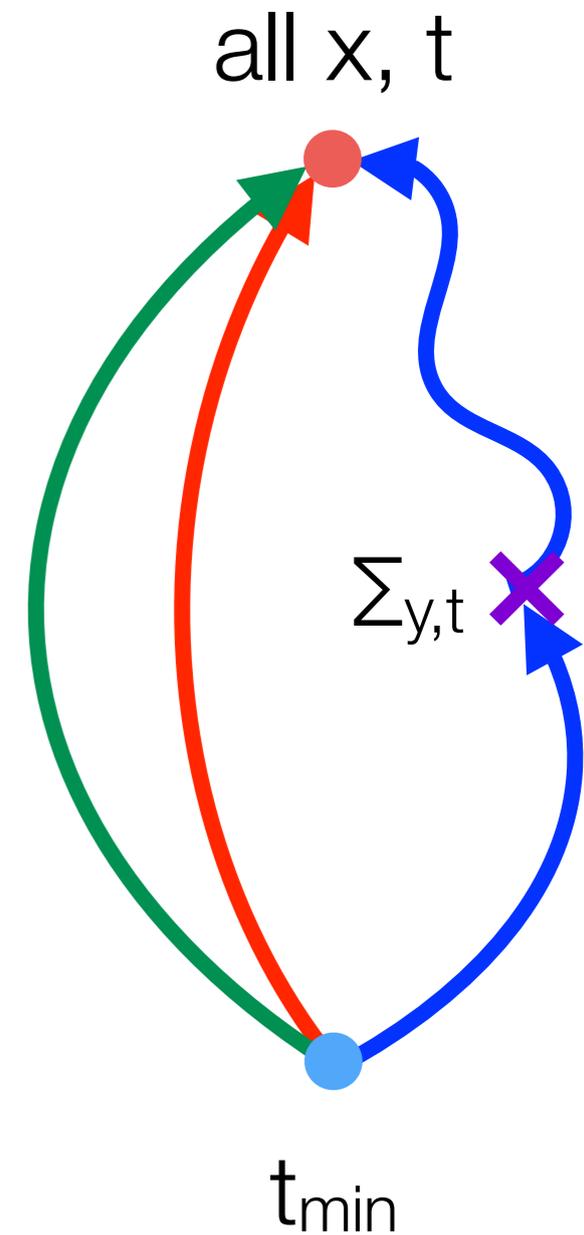
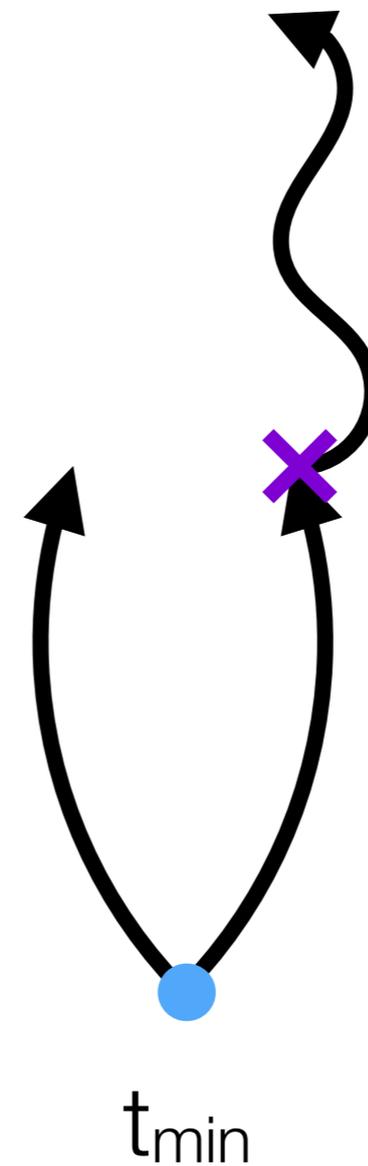
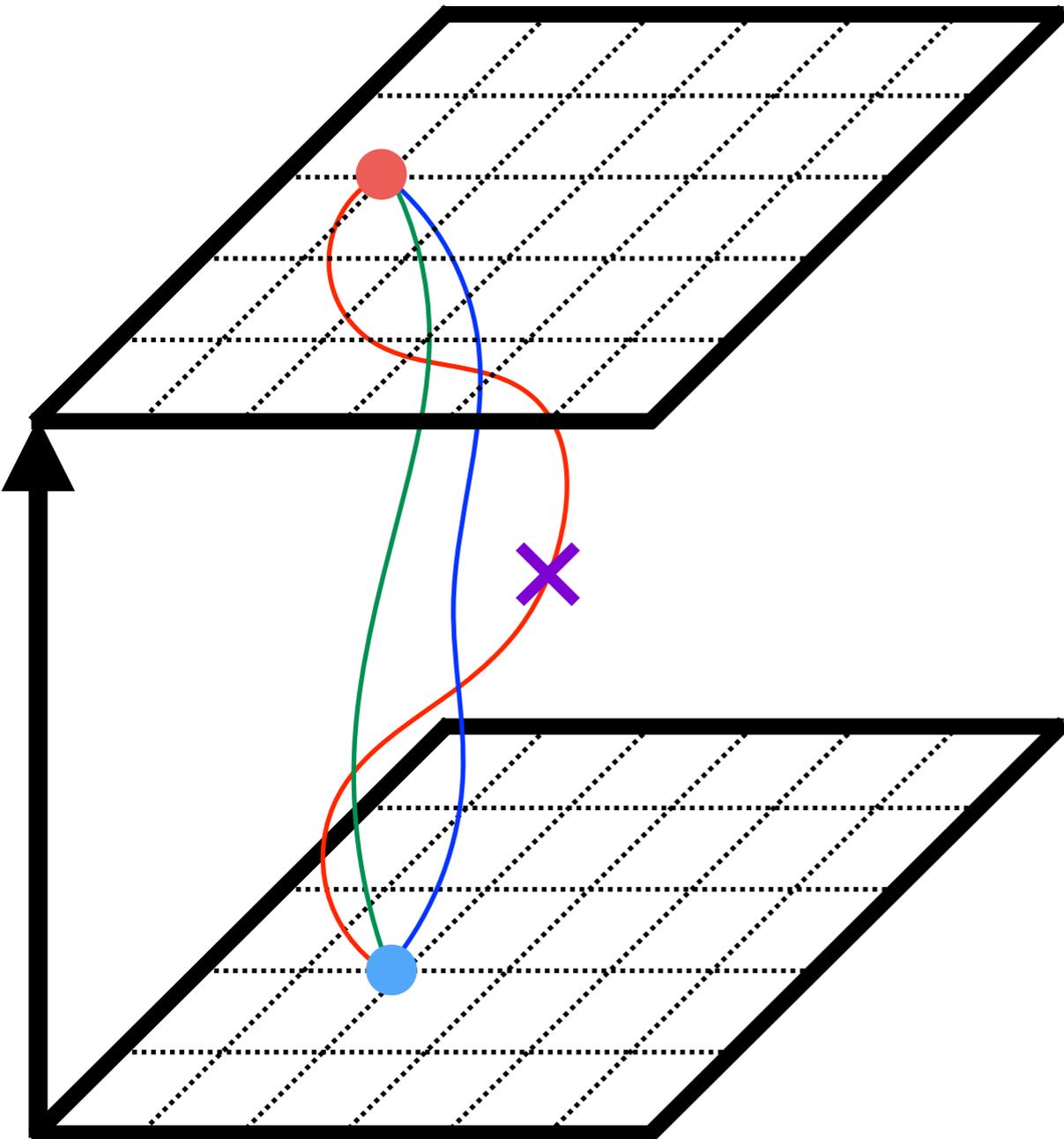
J. Bulava et. al. JHEP 01,140 (2012)
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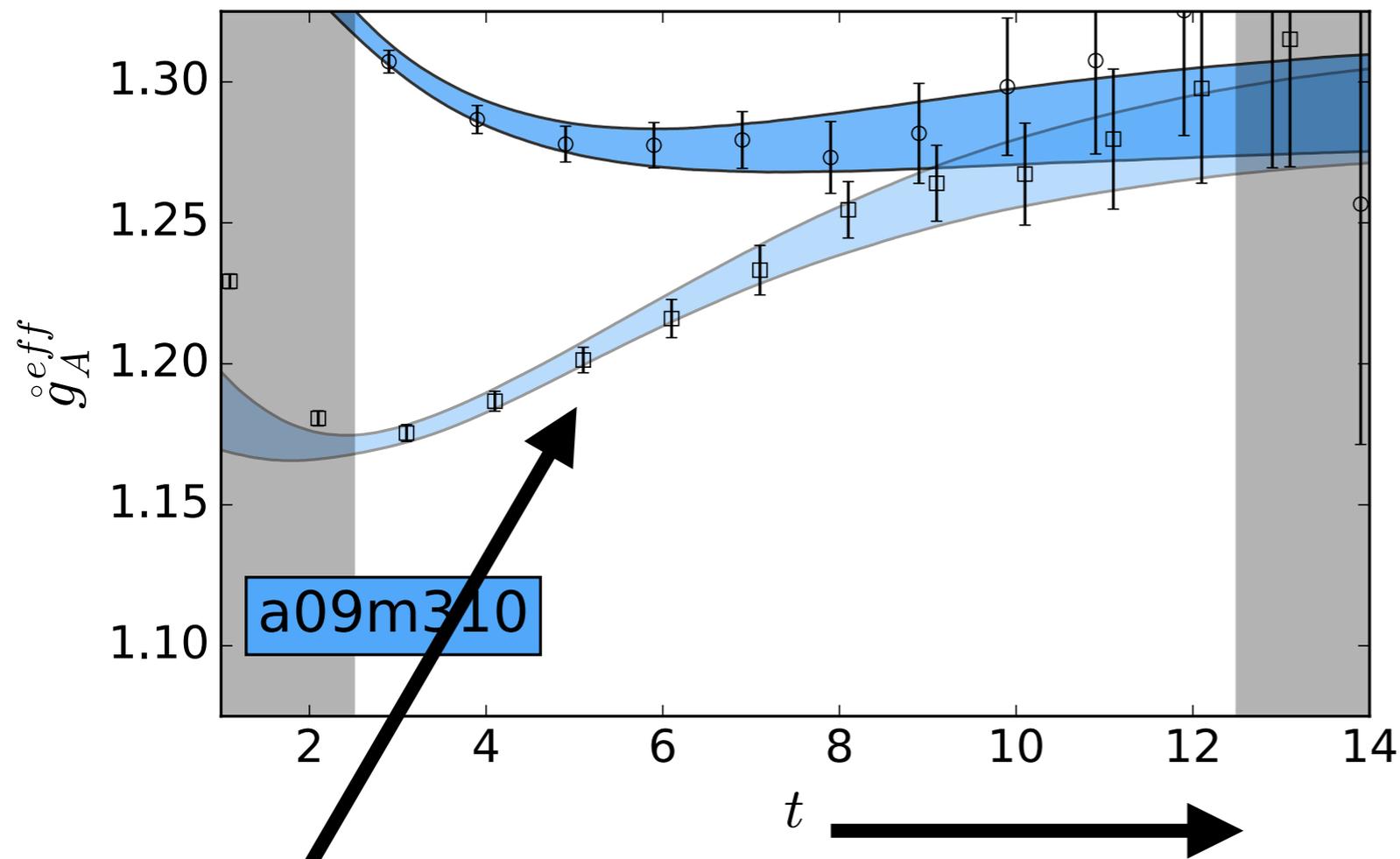
L. Maiani et. al. Nucl. Phys. B293 (1987)
G.M. de Divitiis et. al. Phys. Lett. B718 (2012))



Two solves: all time separations but one bilinear

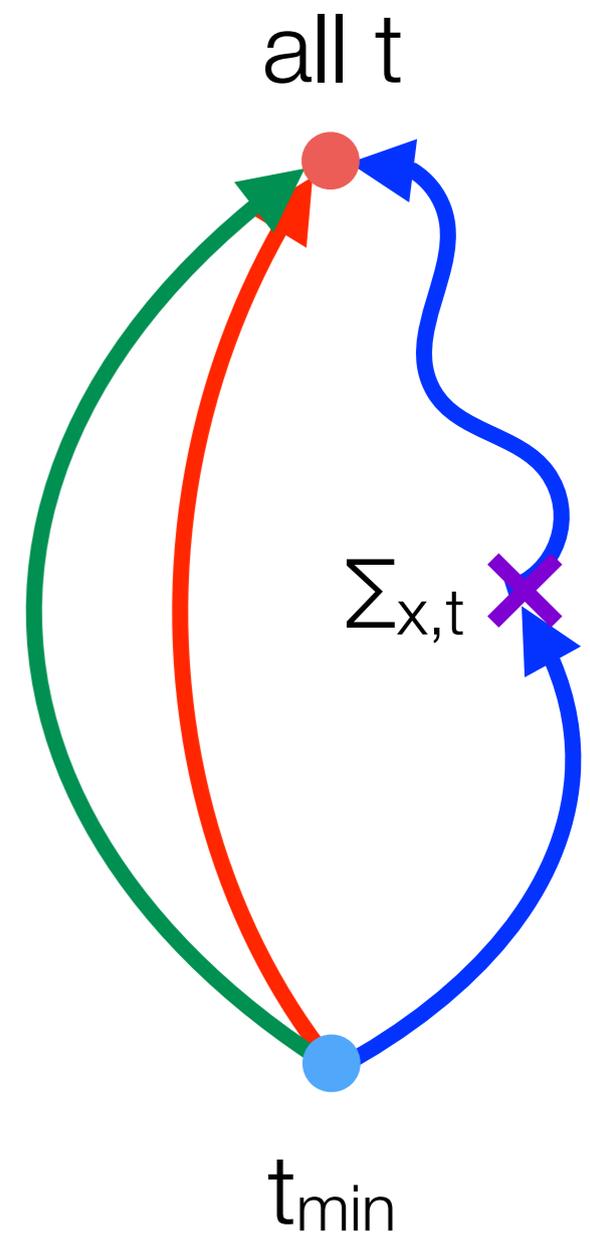
Example Effective Matrix Element

arXiv:1704.01114



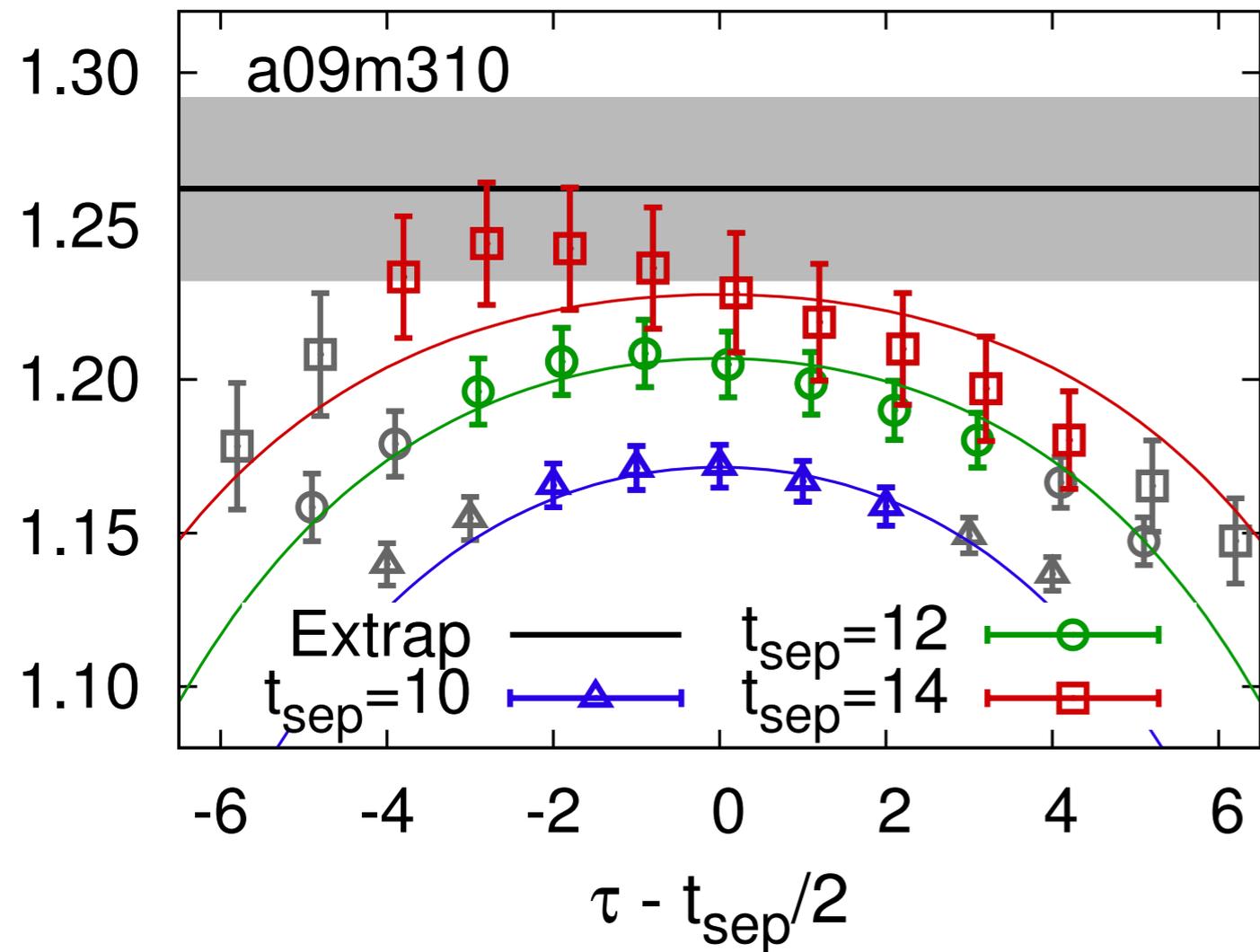
known functional form (spectral decomposition)

asymptotes in just one time variable



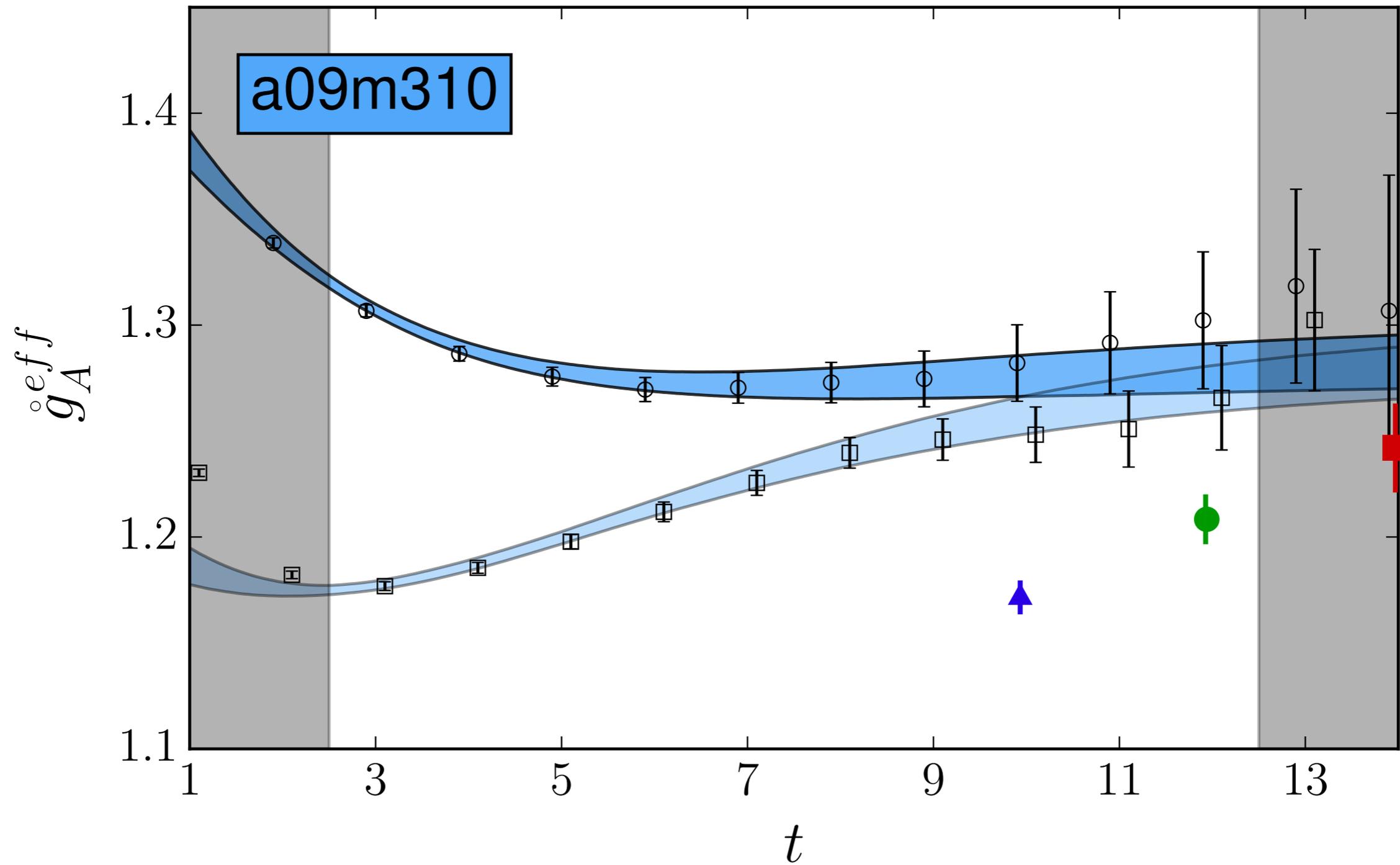
Improved Systematics

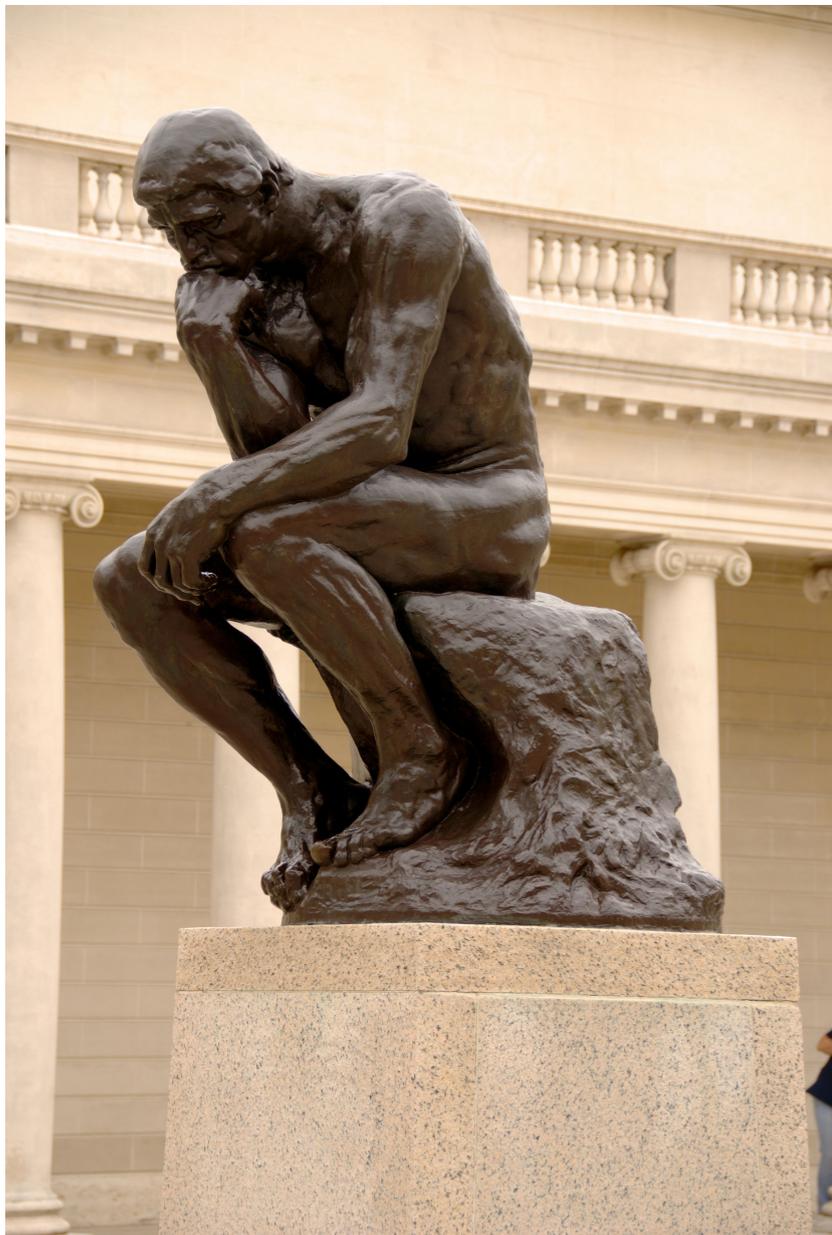
PNDME Phys. Rev. D94 (2016) arXiv:1606.07049



Improved Systematics

arXiv:1704.01114



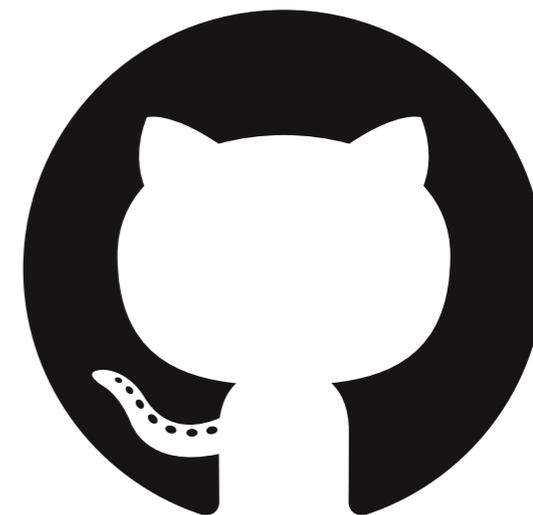


- Not QCD Specific
- Any fermion bilinear matrix element
- 3-point \rightarrow 2-point function: easier fits
- Known spectral decomposition
- Stochastic enhancement
- $3/2$ the cost of one temporal separation

Results

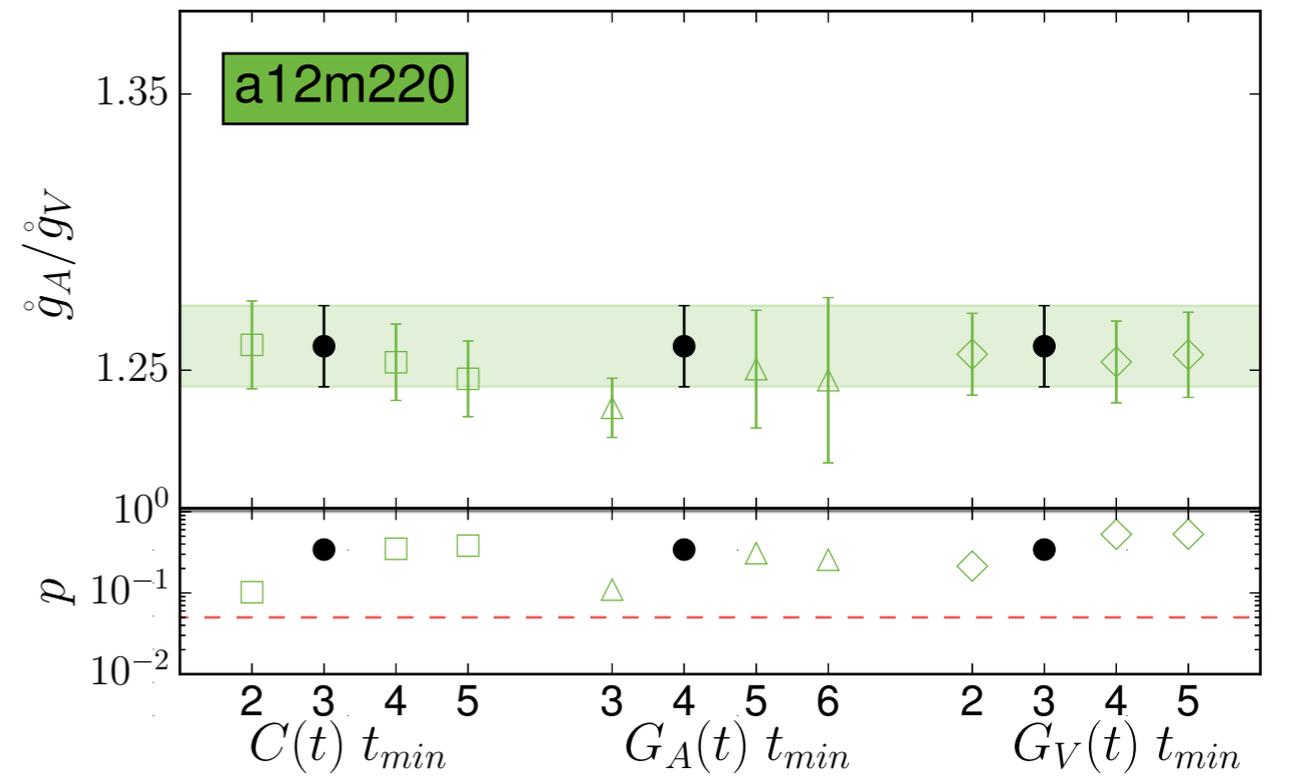
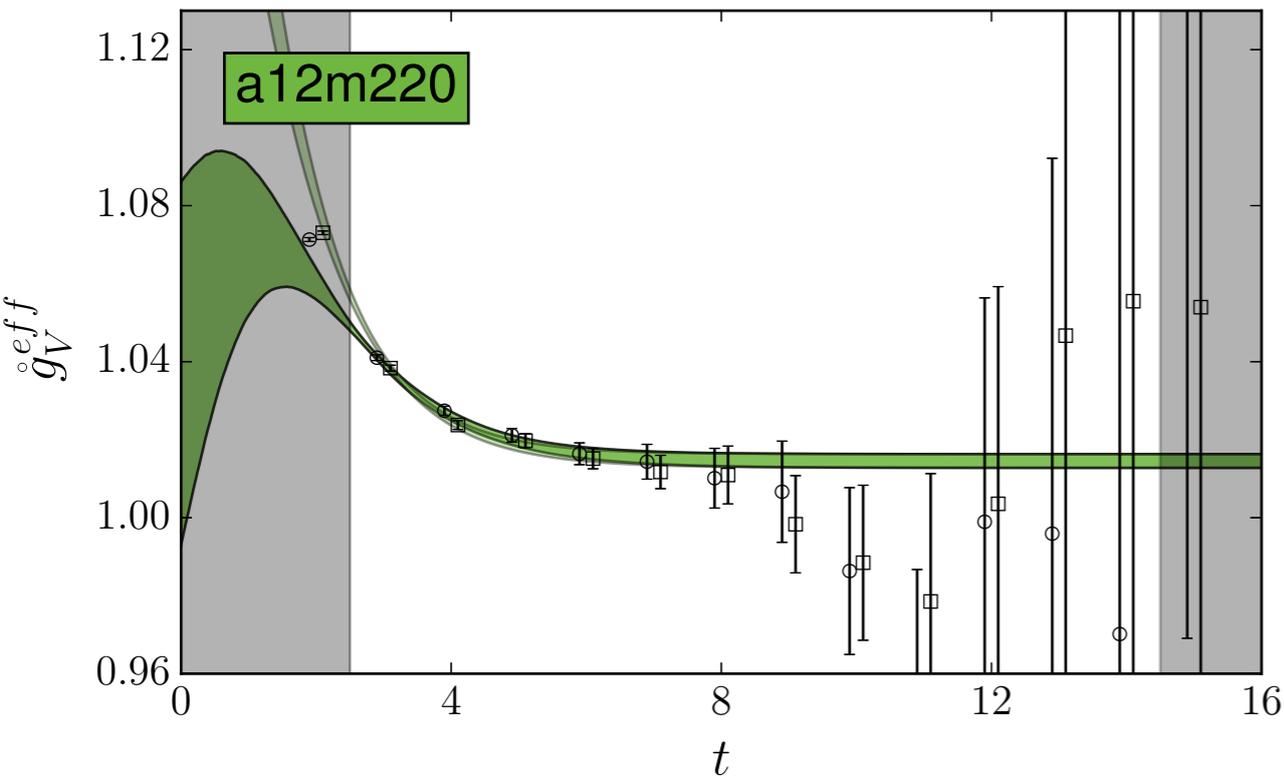
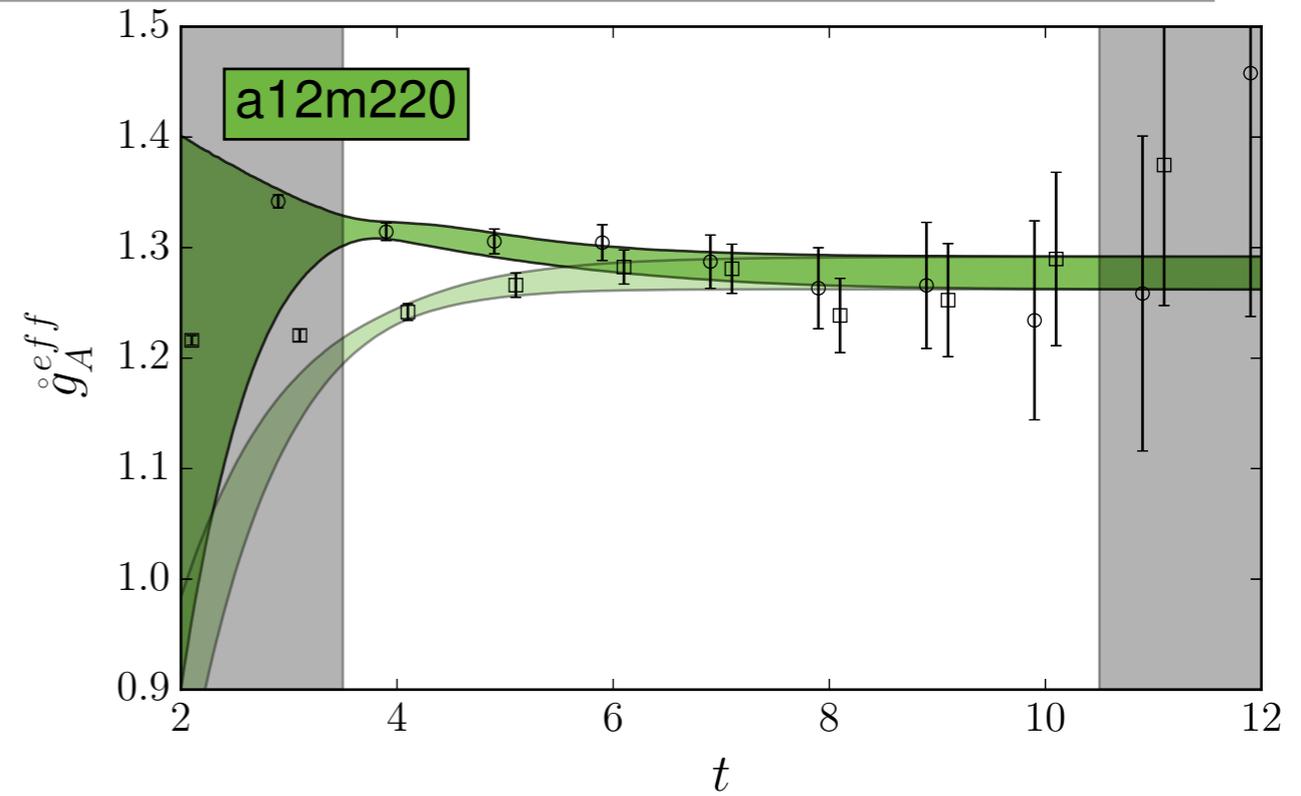
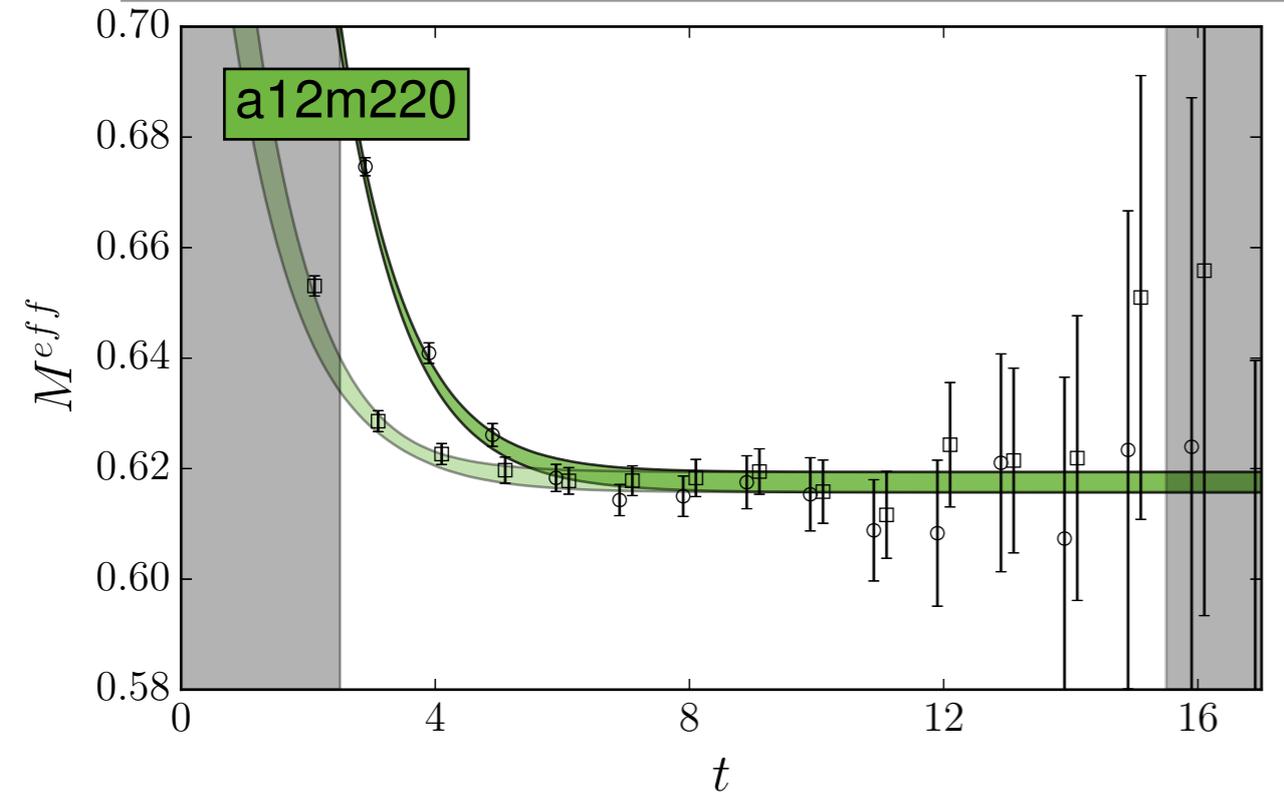
Data + jupyter notebook
available on GitHub

https://github.com/callat-qcd/project_gA



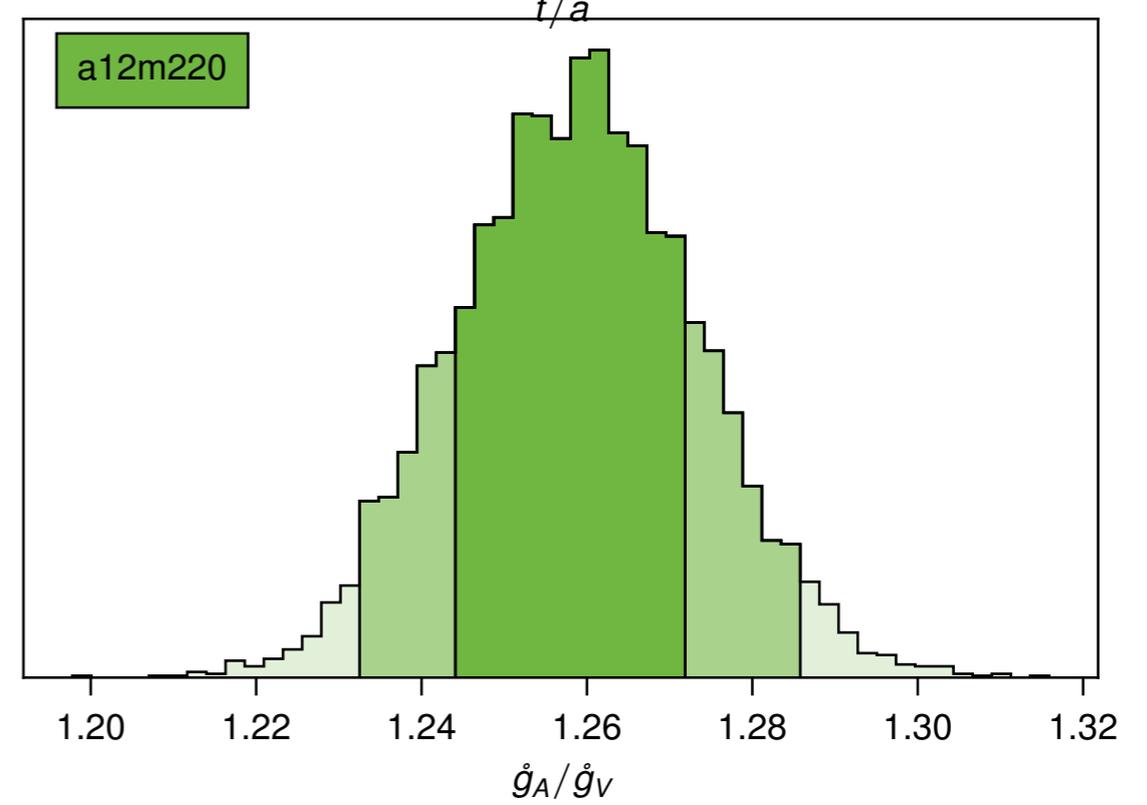
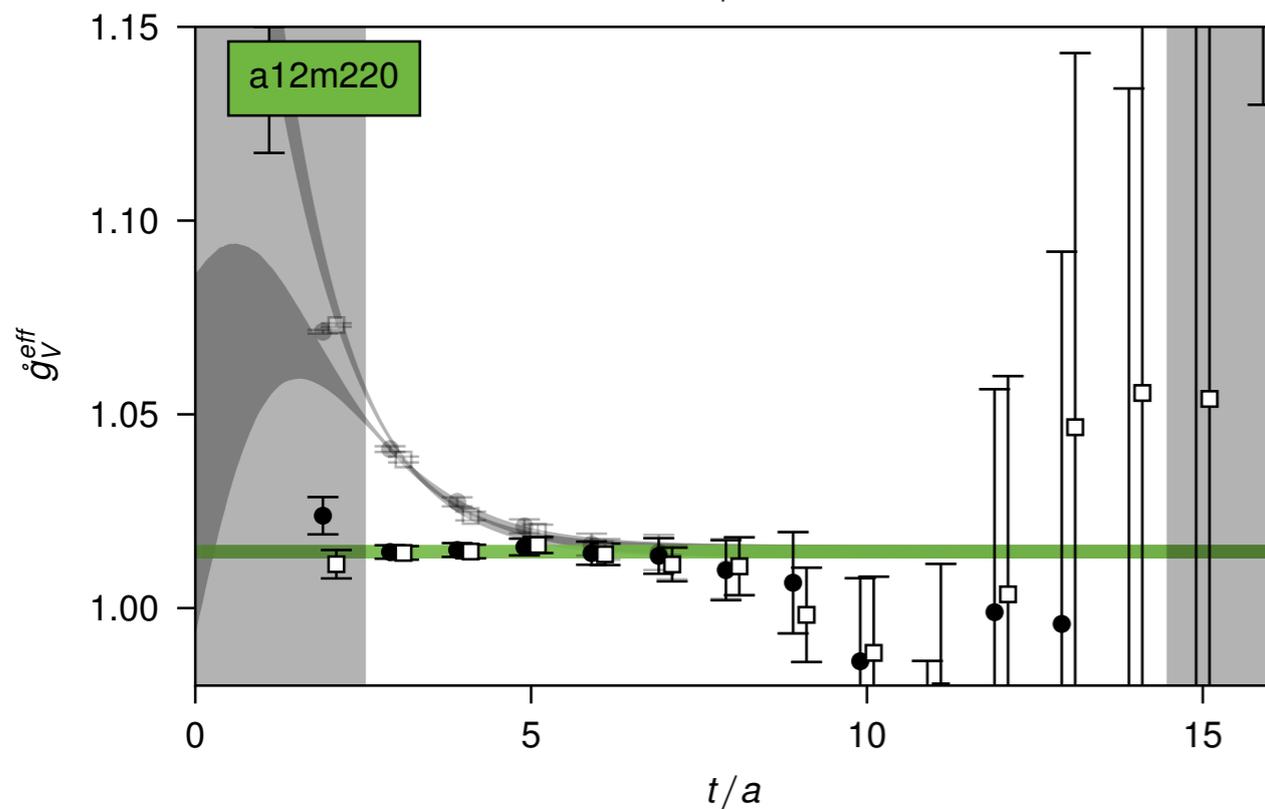
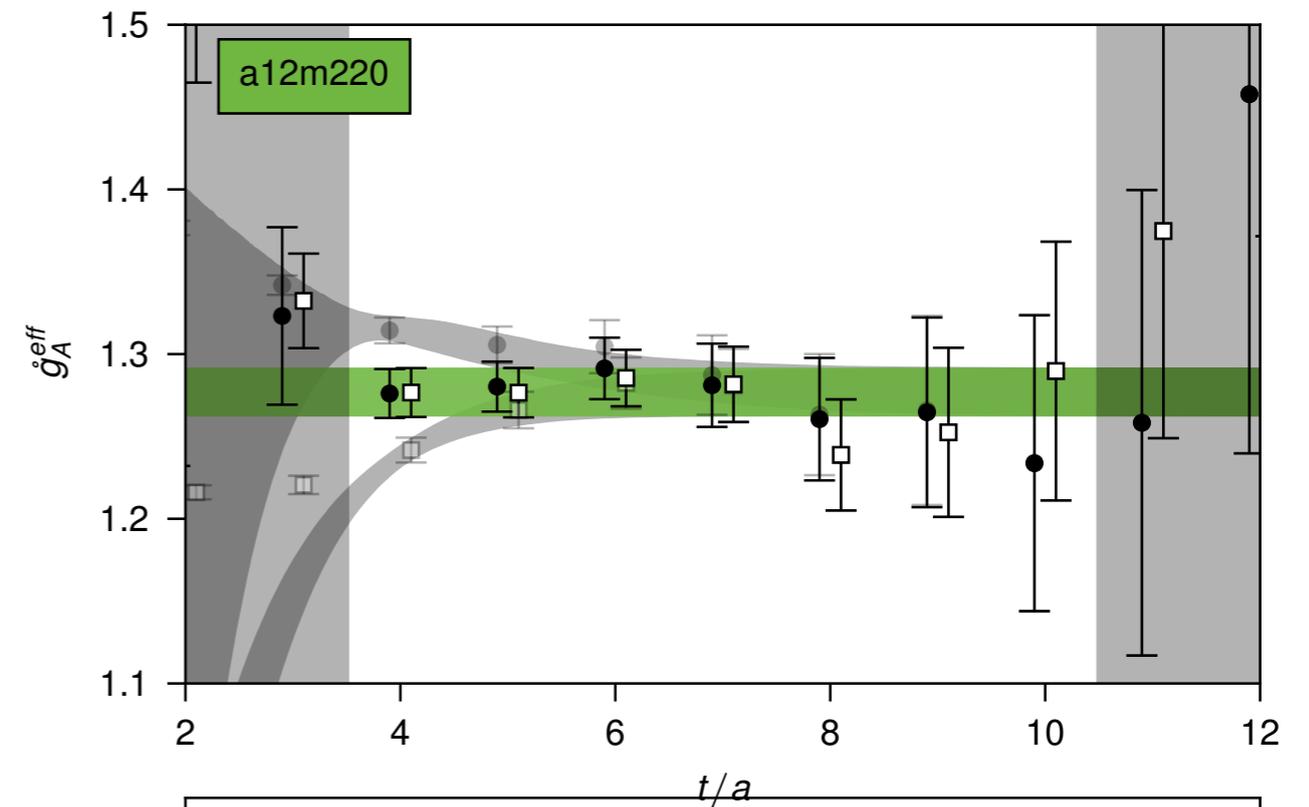
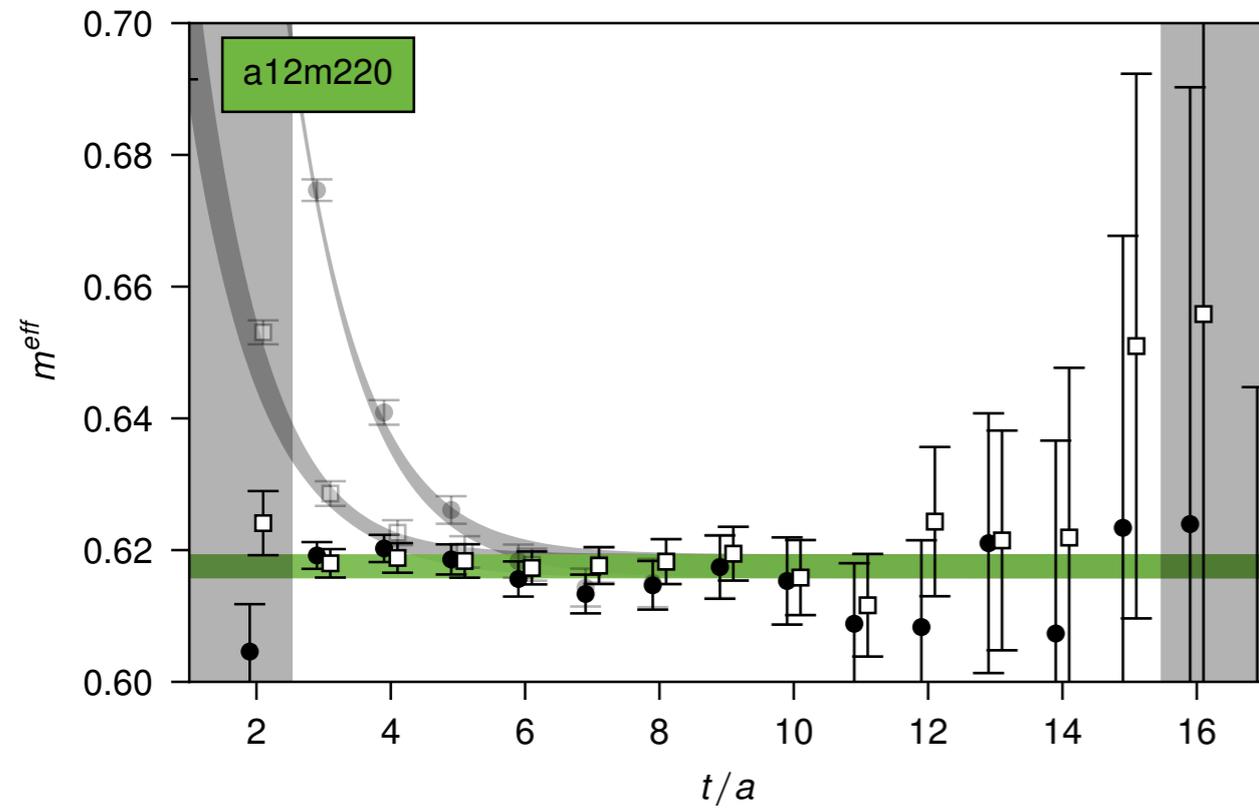
Systematics for an example point

Nature 558 (2018) 91



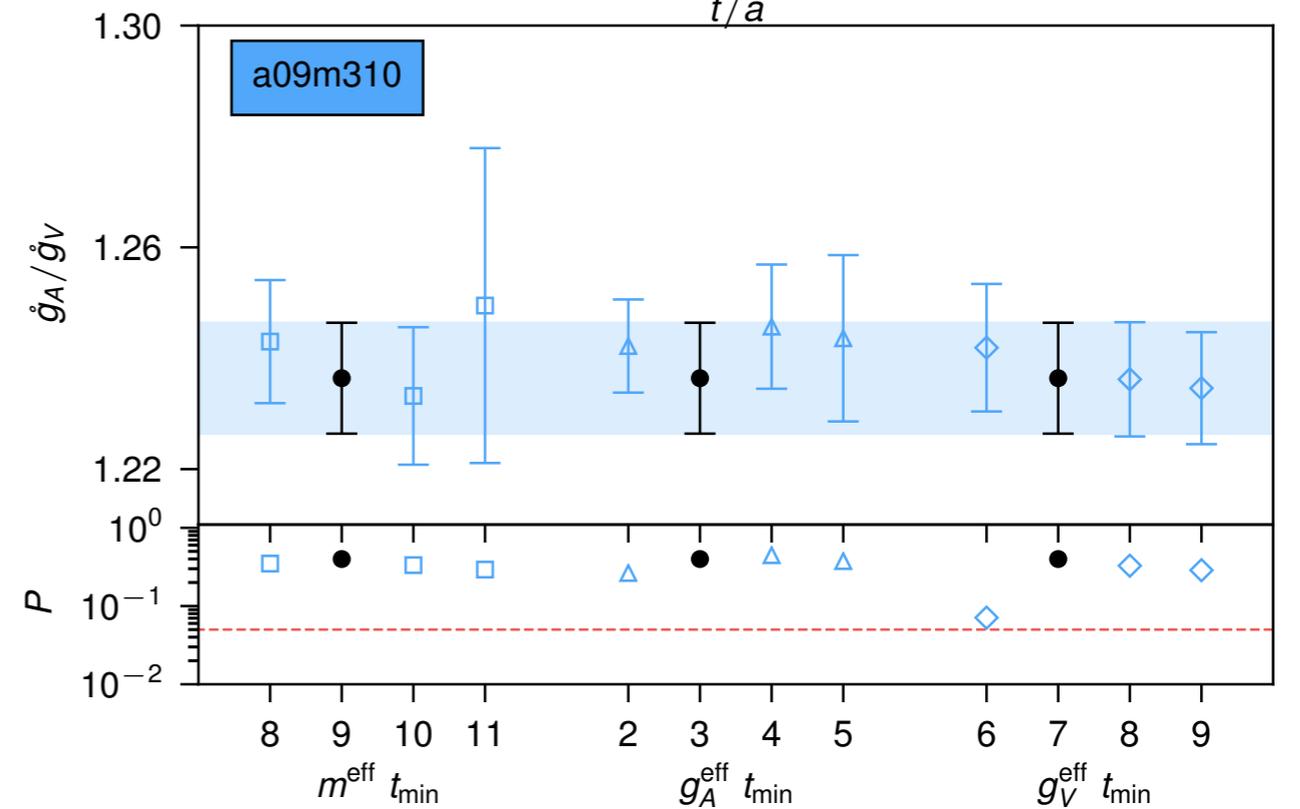
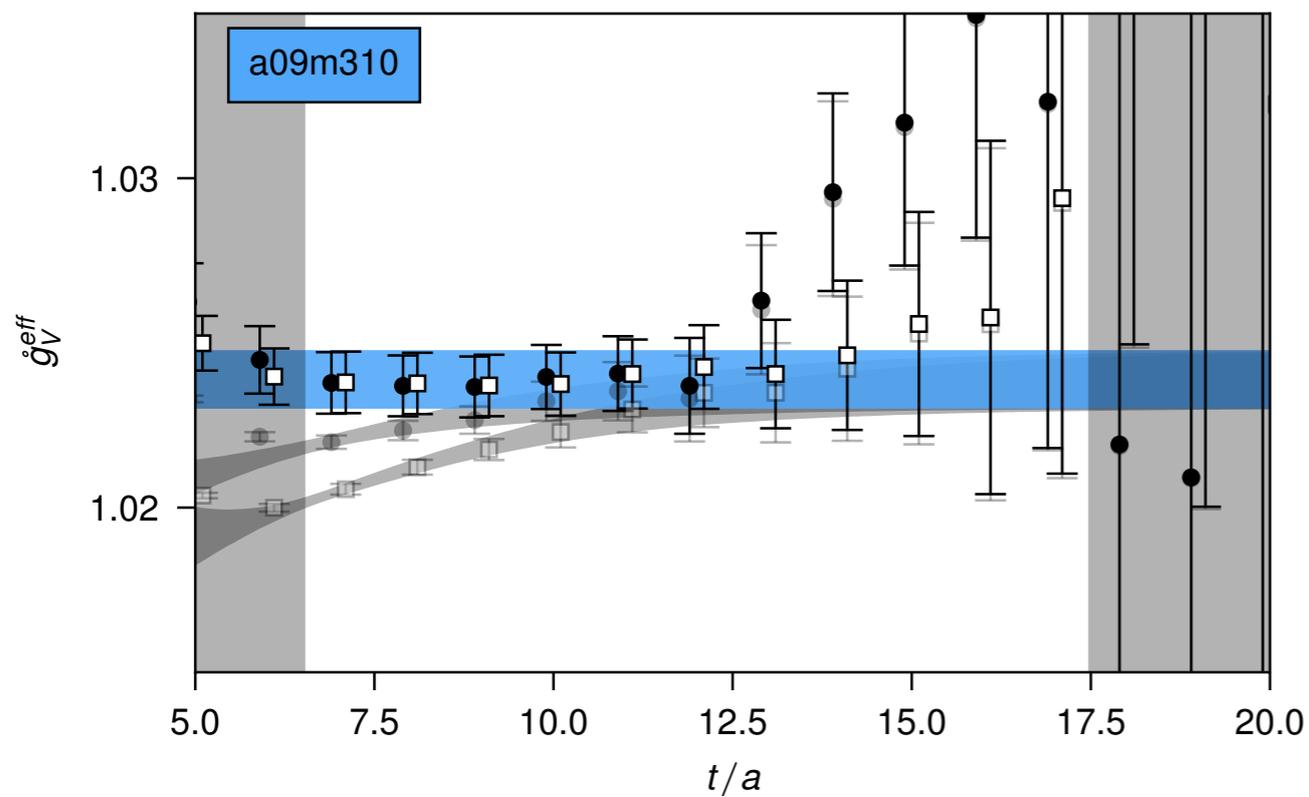
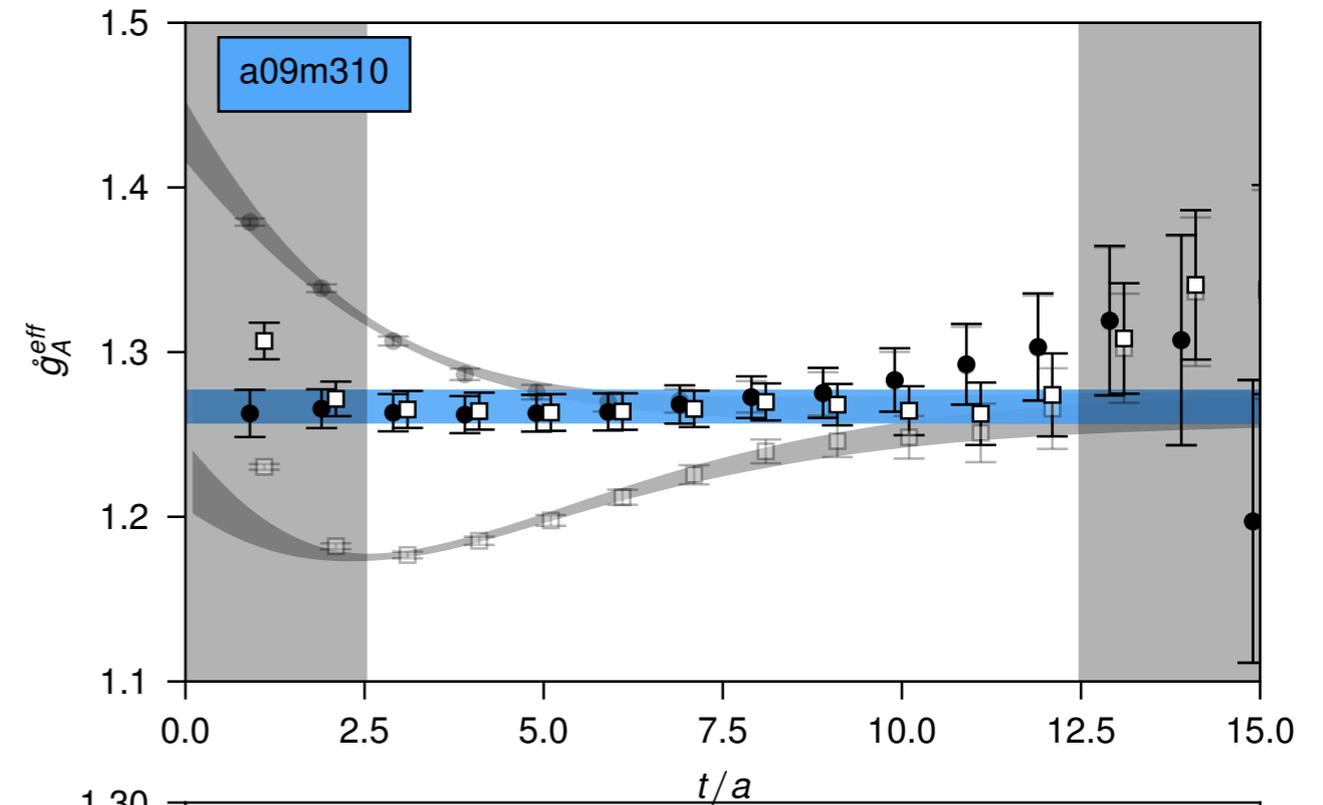
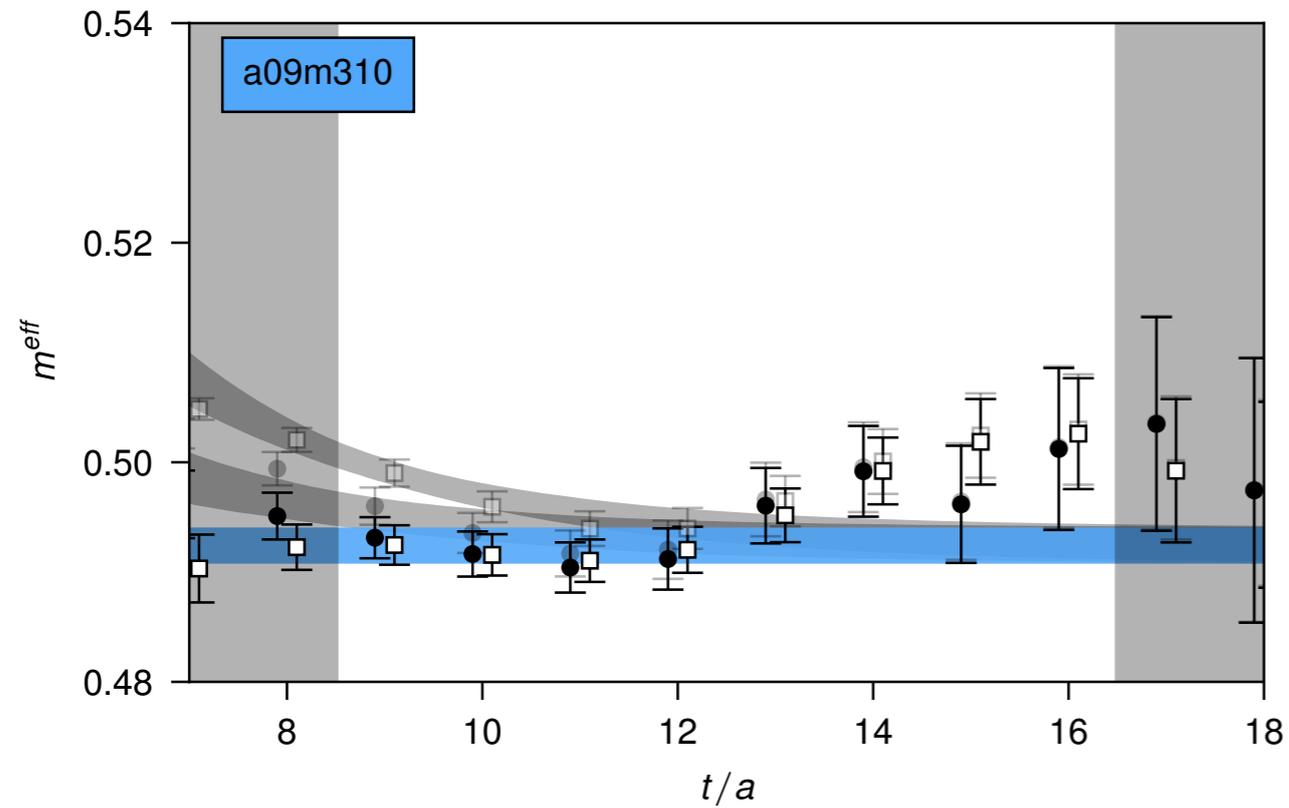
Systematics for an example point

Nature 558 (2018) 91



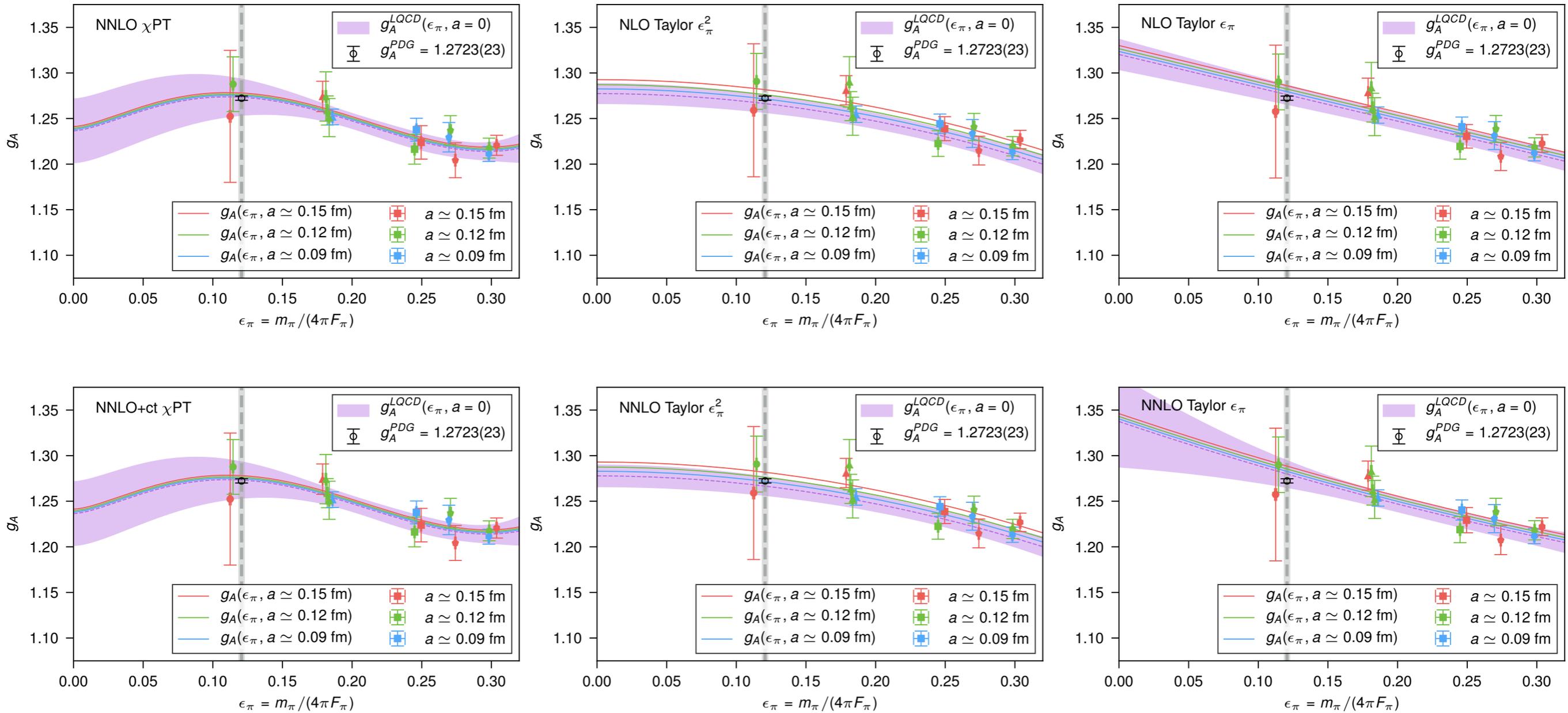
Another example point

Nature 558 (2018) 91



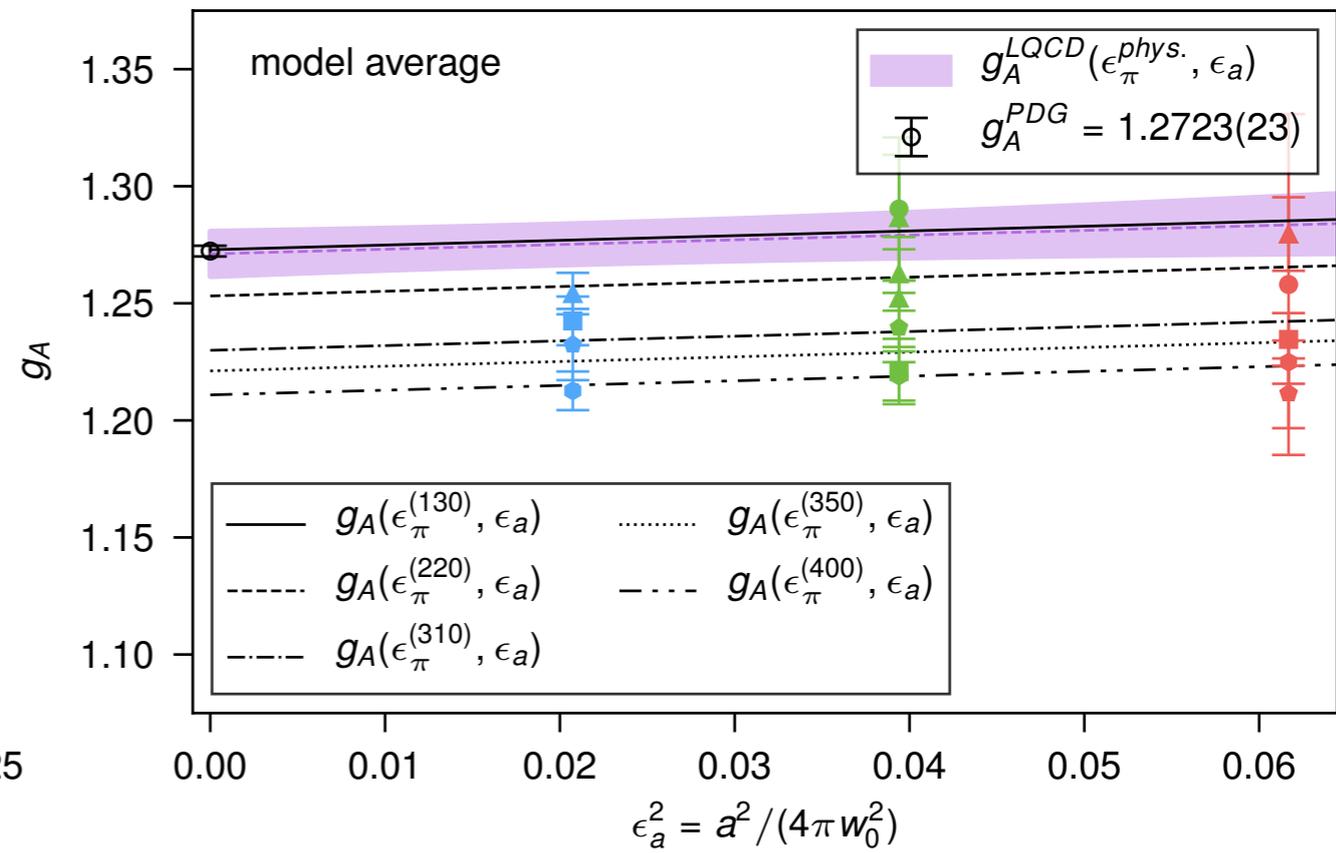
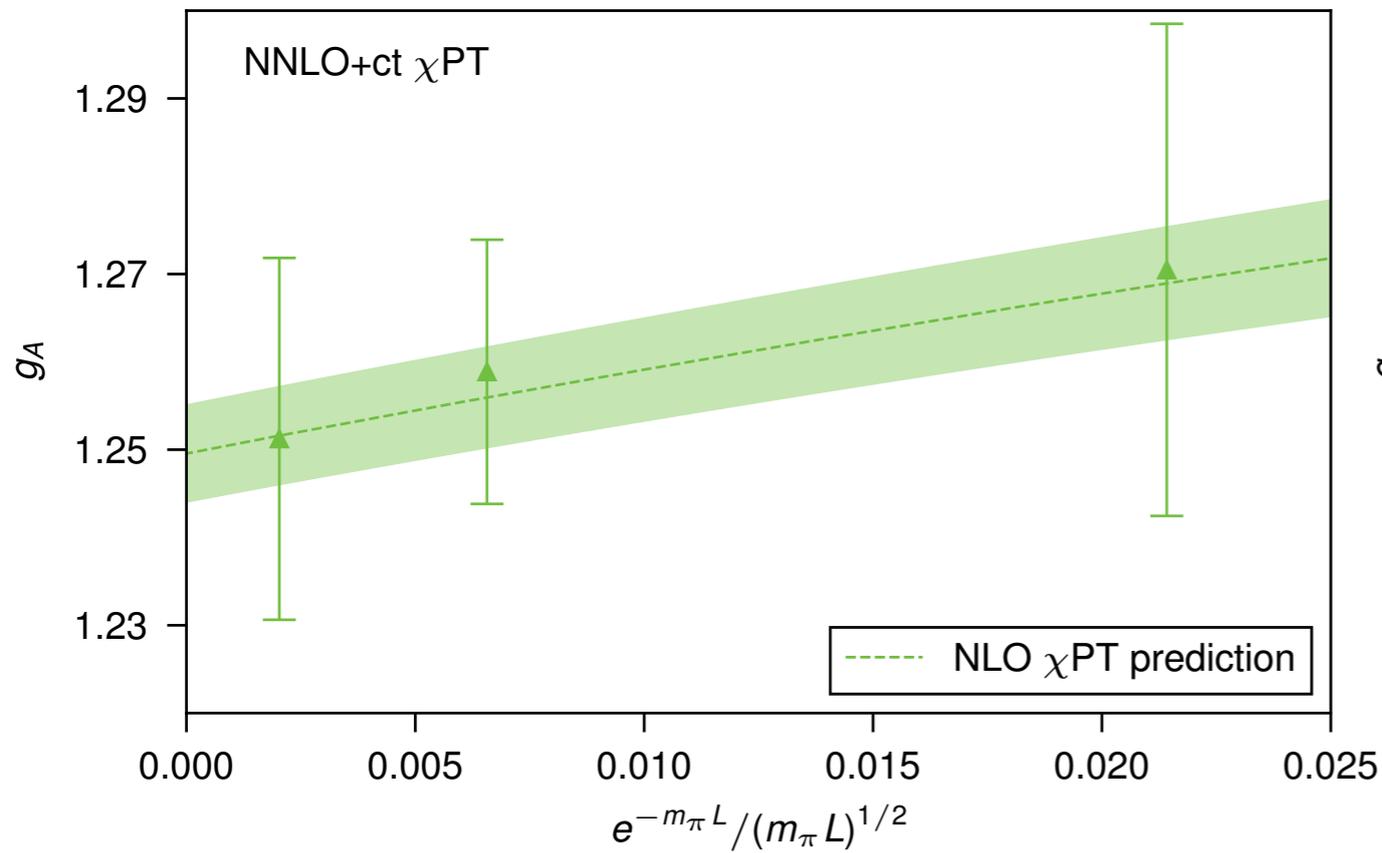
Systematics: m_π

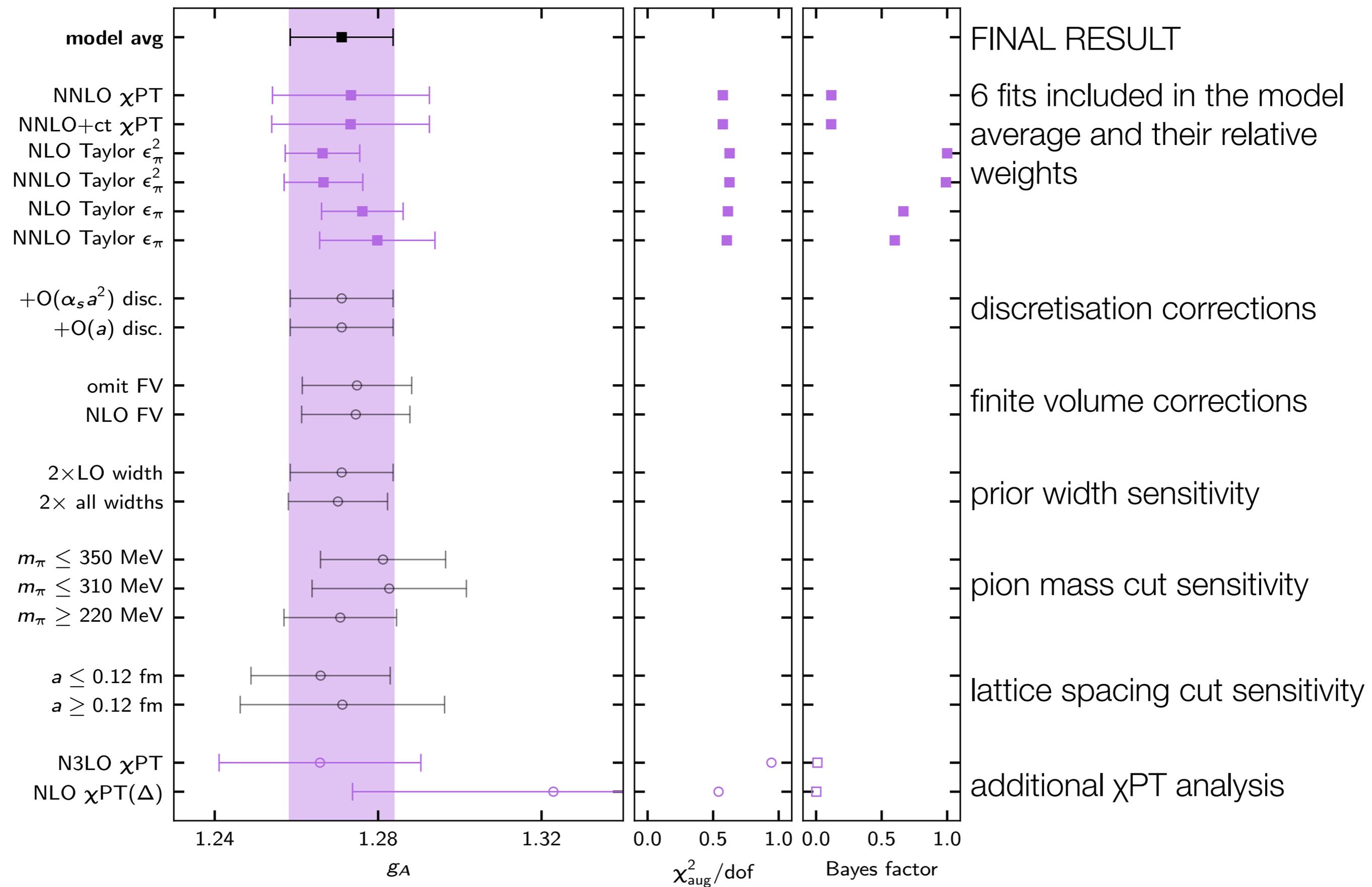
Nature 558 (2018) 91



Systematics: infinite volume, continuum

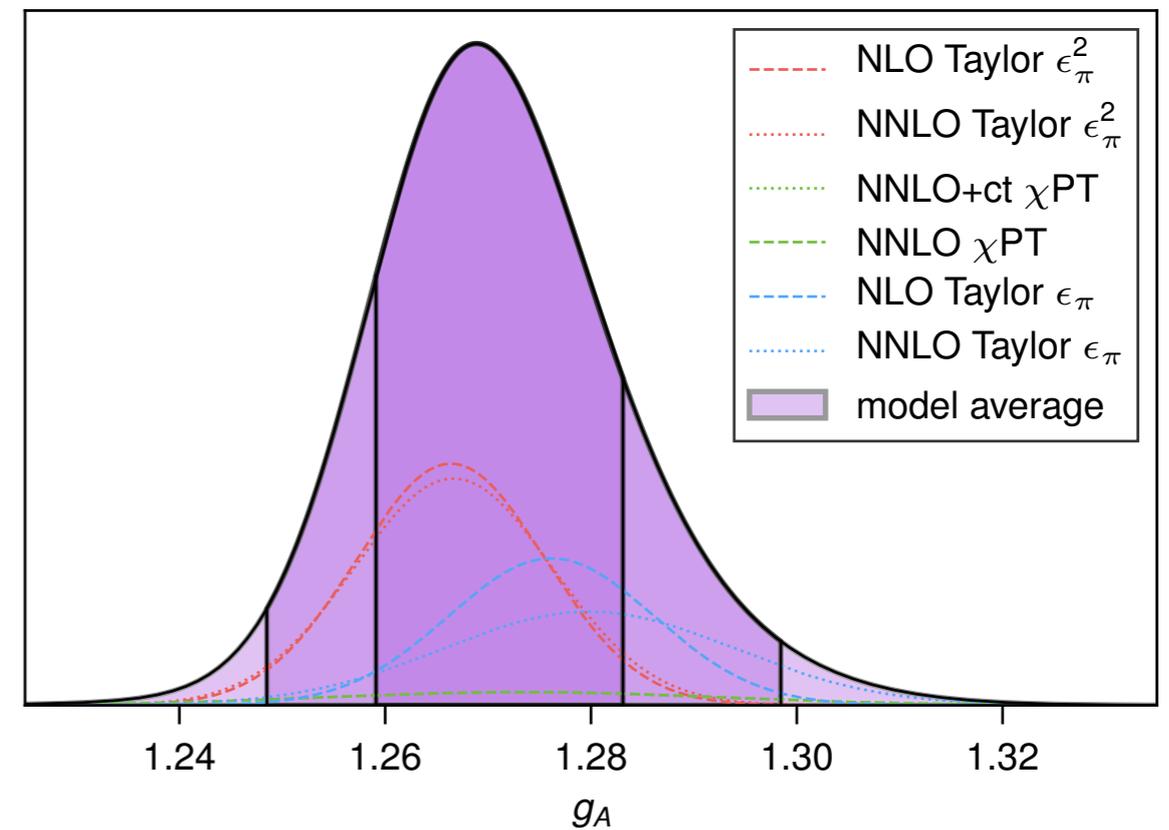
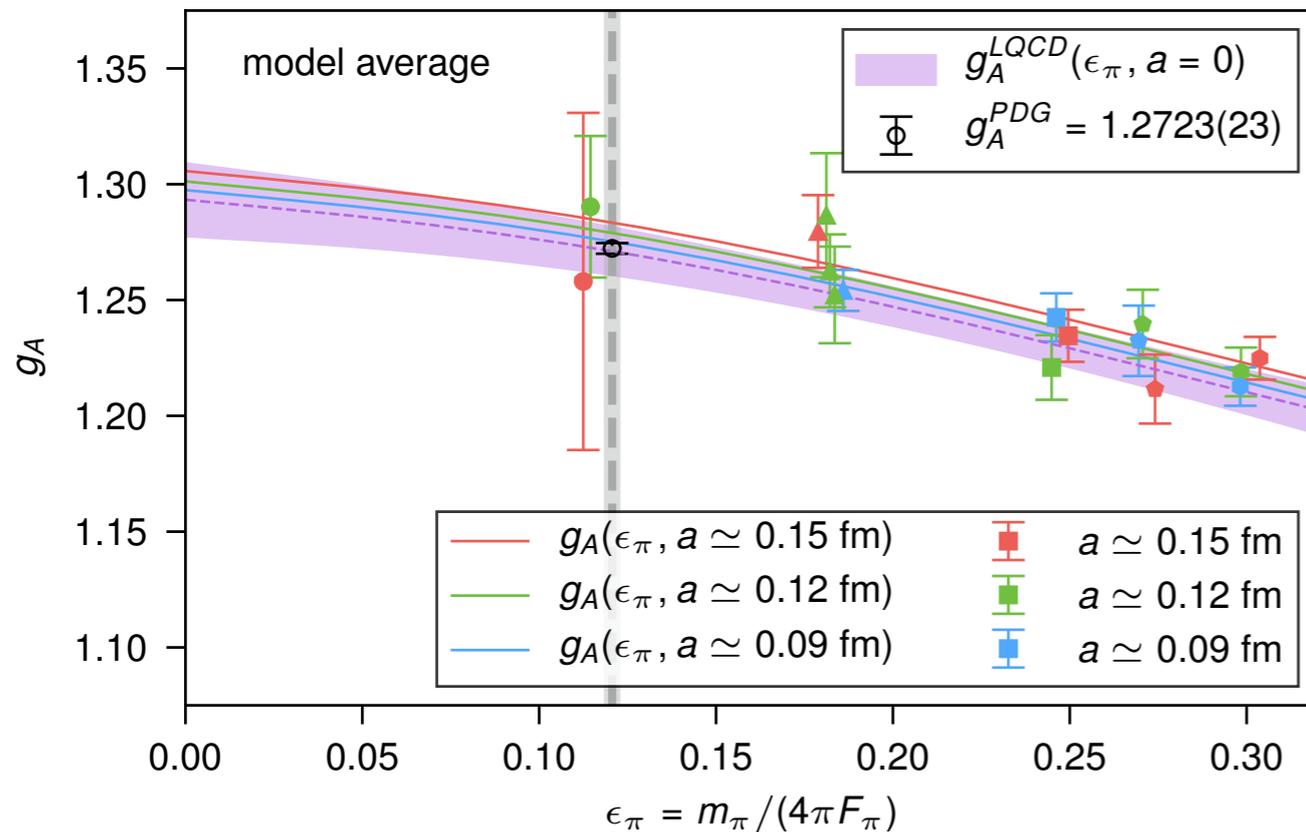
Nature 558 (2018) 91





Model Average $g_A = 1.2711(103)^s(39)^x(15)^a(19)^v(04)^I(55)^M$

Nature 558 (2018) 91



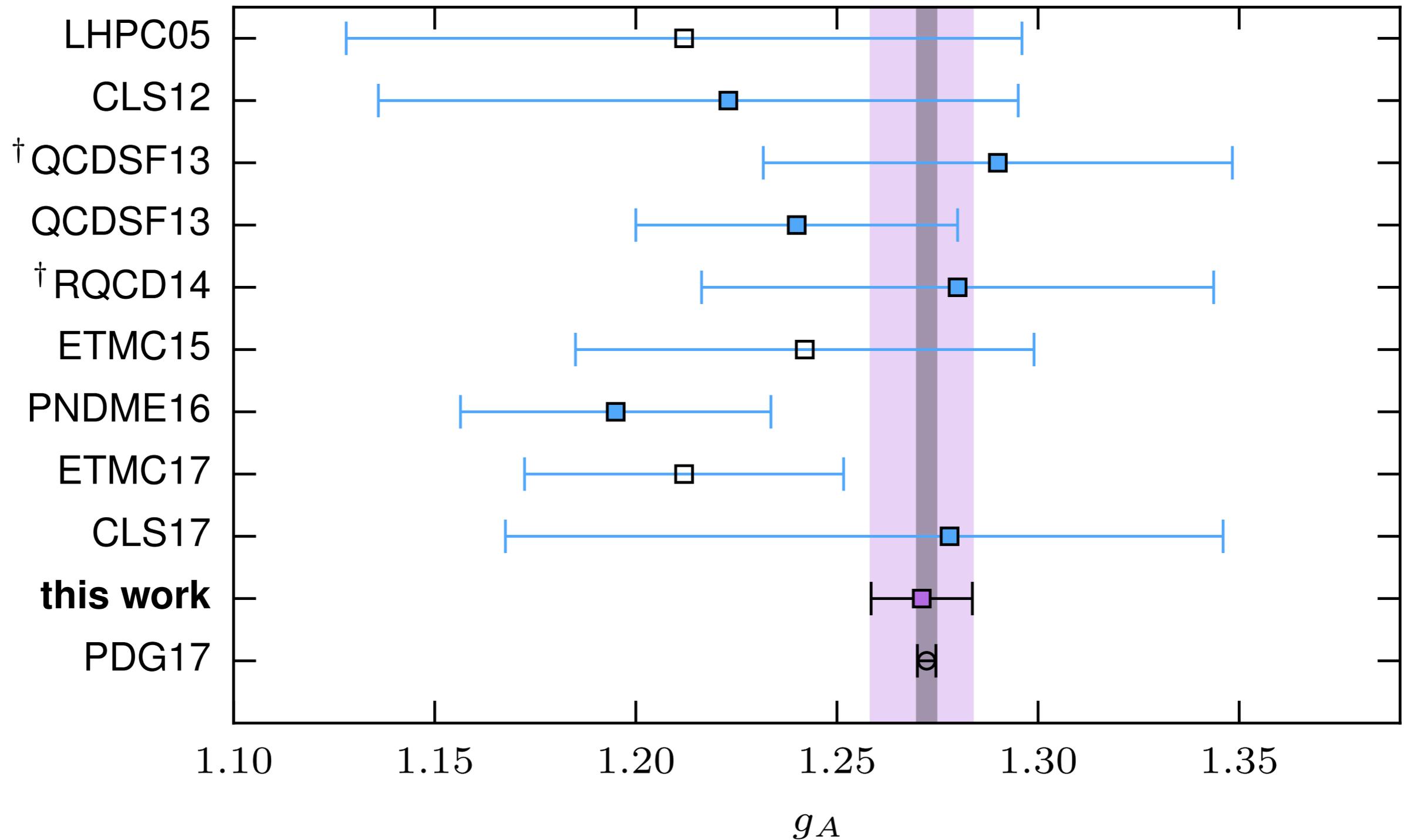
statistical	0.81%
chiral	0.30%
continuum	0.12%
infinite volume	0.15%
isospin breaking	0.03%
model selection	0.43%
total (added in quadrature)	0.98%

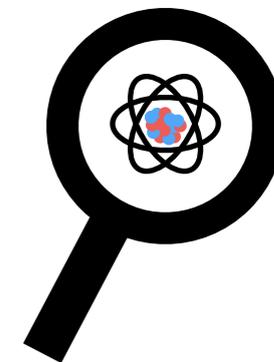
Final uncertainty is dominated by statistics, model selection, and chiral extrapolation.

Better control over the $m_\pi \sim 130$ MeV points will improve all three.

Comparison with previous LQCD results

Nature 558 (2018) 91



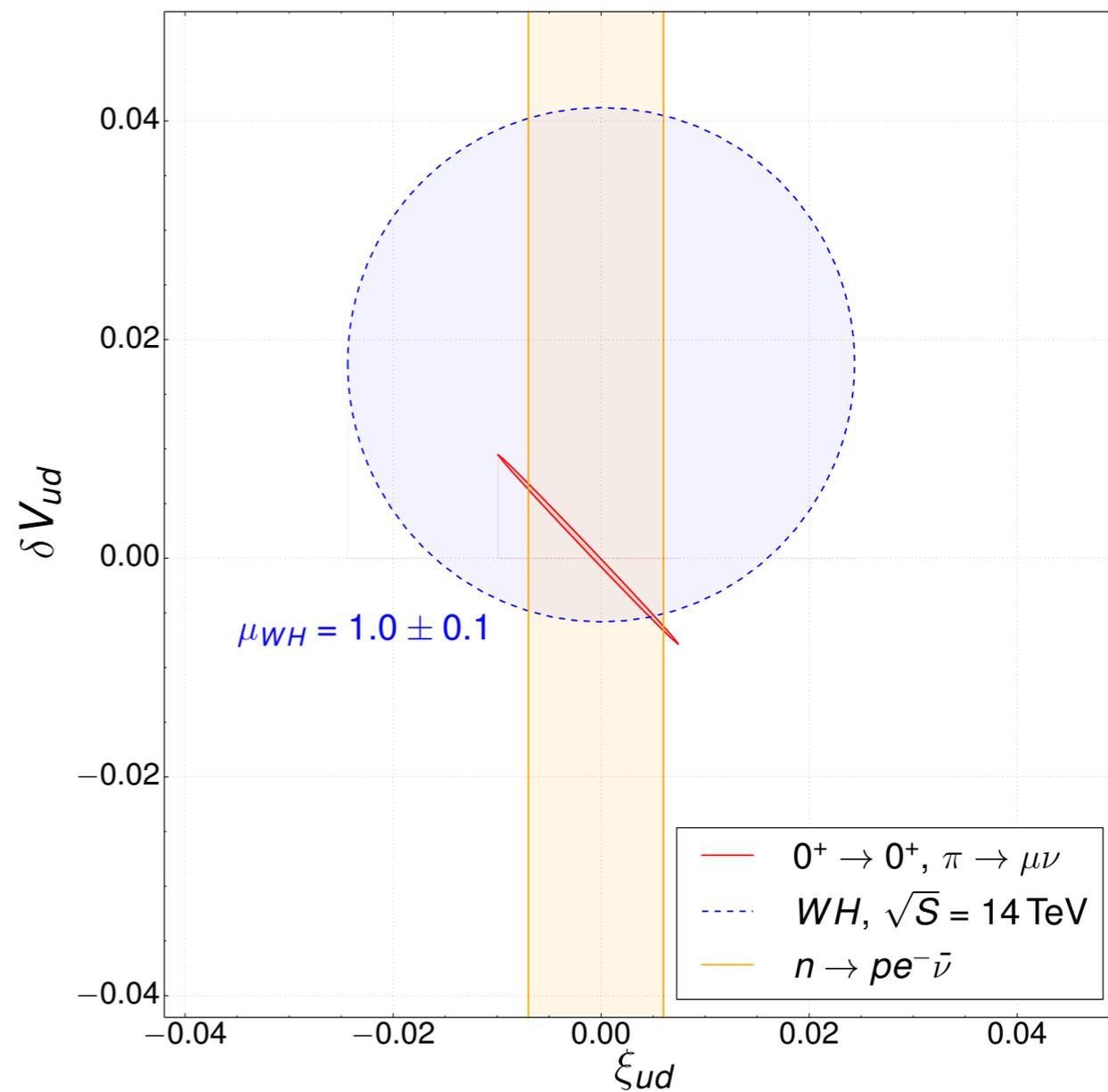
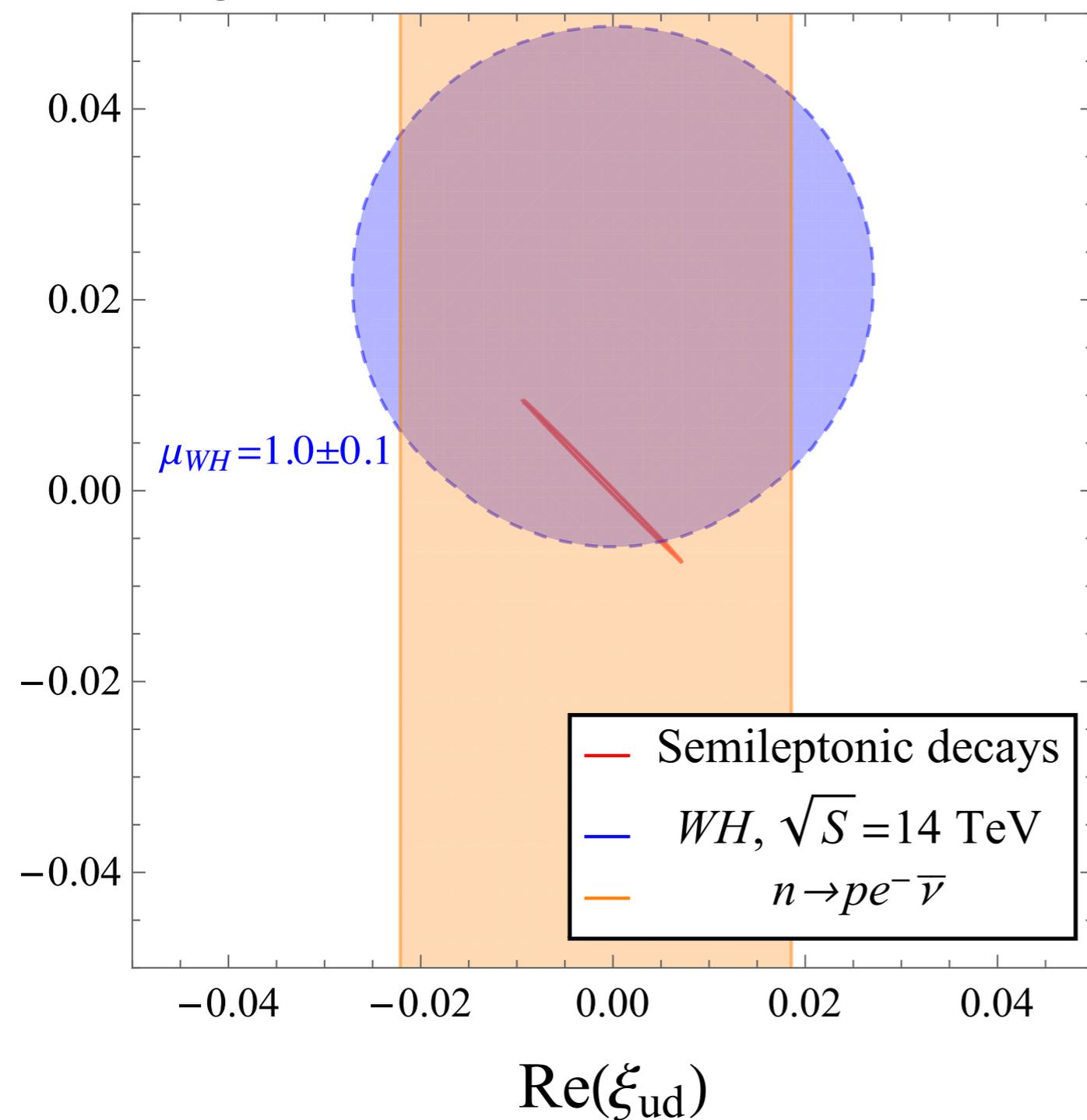


Searches for violations of V–A

Alioli, Cirigliano, Dekens, de Vries, Mereghetti JHEP 1705 (2017) 086 arXiv:1703.04751

$$g_A^{\text{QCD}} = 1.27 \pm 0.05$$

$$g_A^{\text{QCD}} = 1.271(13)$$



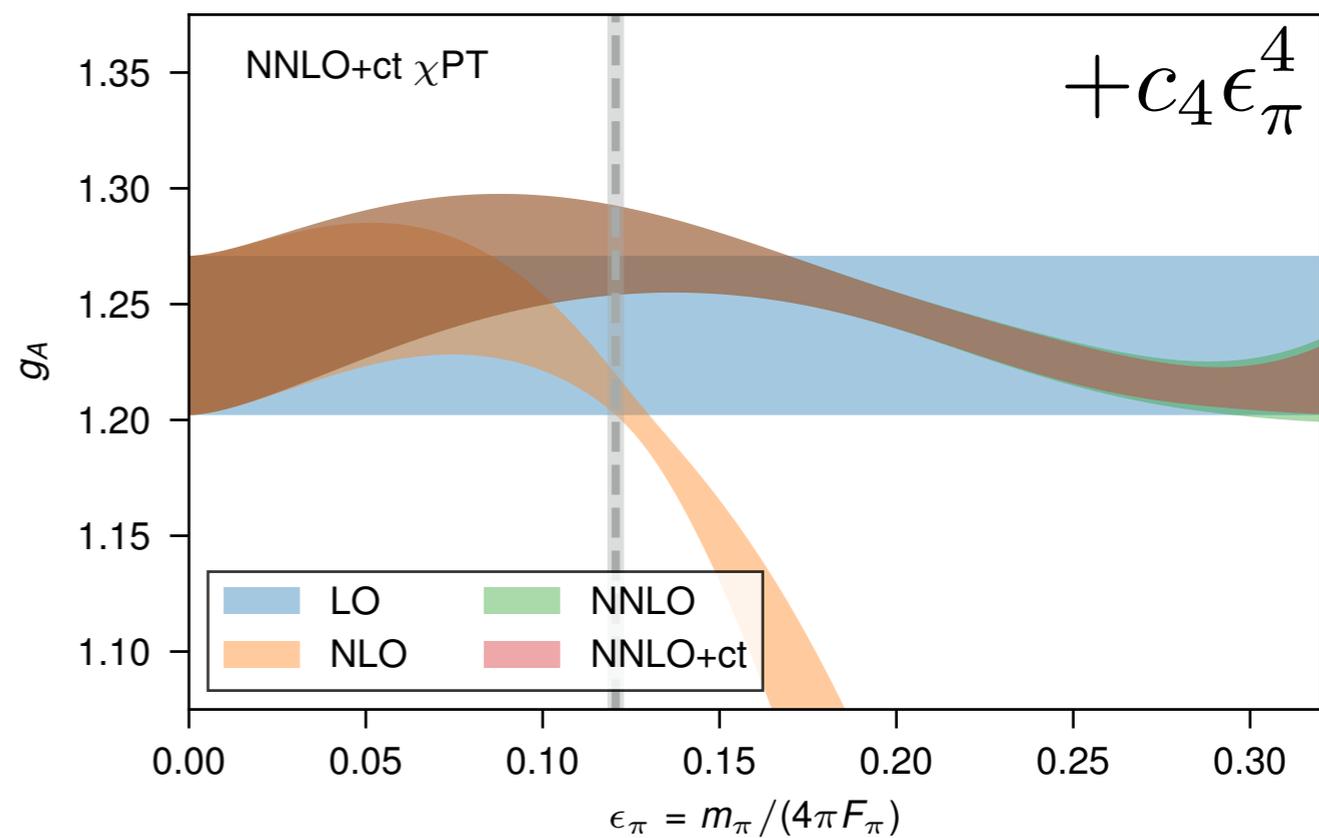
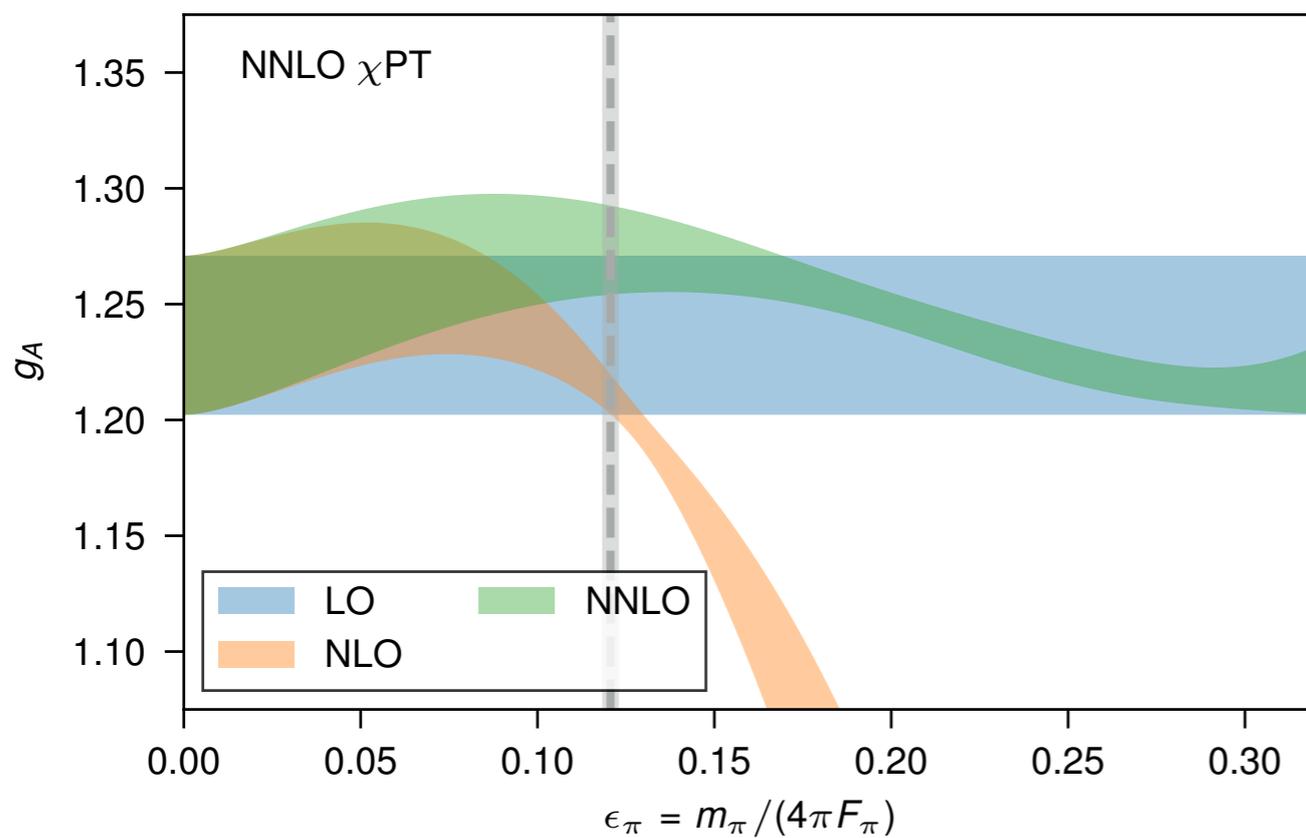
NNLO[+ct] χ PT

Nature 558 (2018) 91

$$g_A = g_0 + c_2 \epsilon_\pi^2 - (g_0 + 2g_0^3) \epsilon_\pi^2 \ln(\epsilon_\pi^2) + g_0 c_3 \epsilon_\pi^3$$

Bernard and Meißner, PLB 639 (2006) 279-282

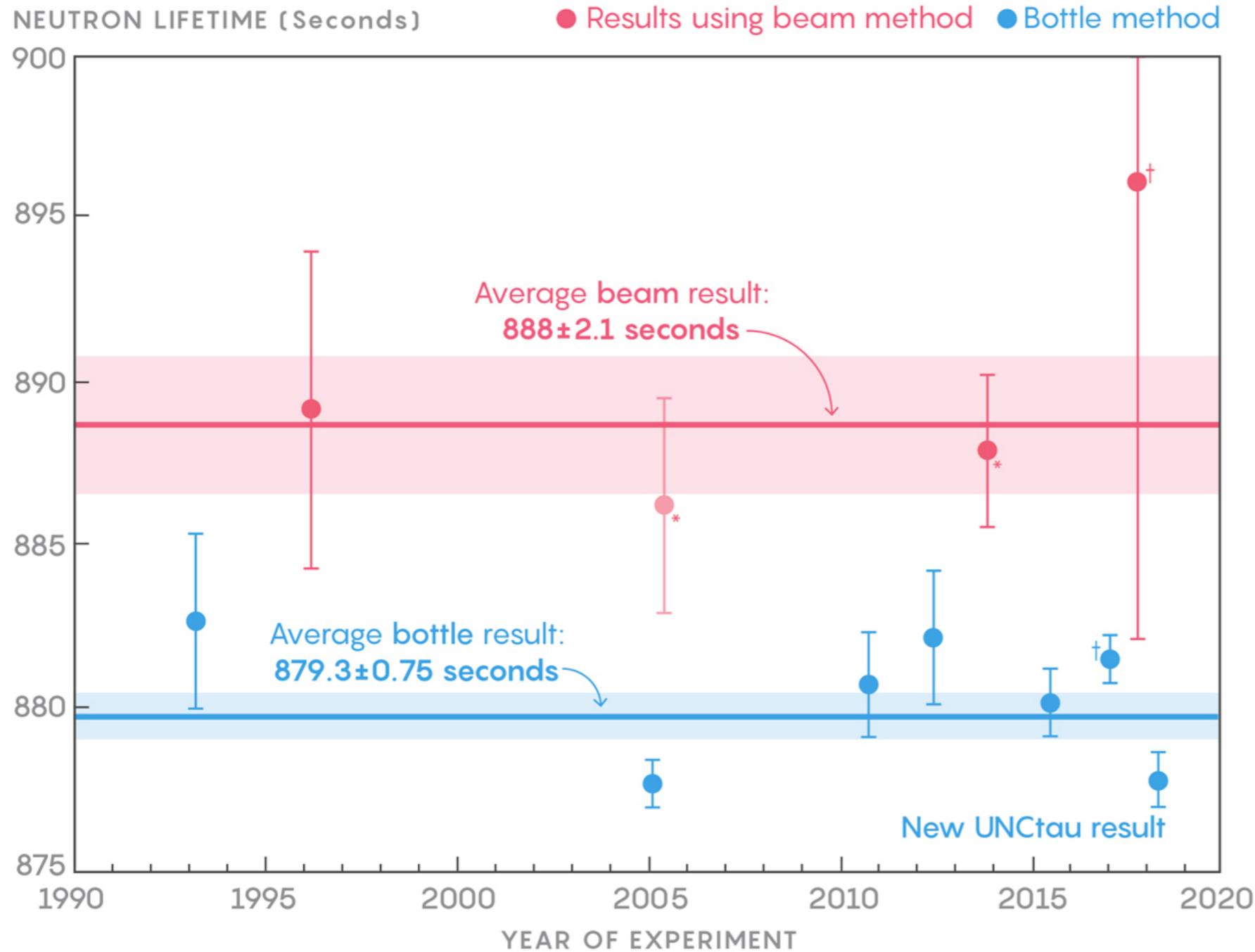
$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi} \quad (F_\pi \sim 92 \text{ MeV})$$



$$g_A = 1.2711(103)^s(39)^x(15)^a(19)^v(04)^I(55)^M$$

Nature 558 (2018) 91

$$\tau_n = \frac{(5172.0 \pm 1.1)\text{seconds}}{1 + 3g_A^2}$$



According to the Standard Model
 $885(14)$ s

*Nico result (2005) was superseded by an updated and improved result, Yue (2013);

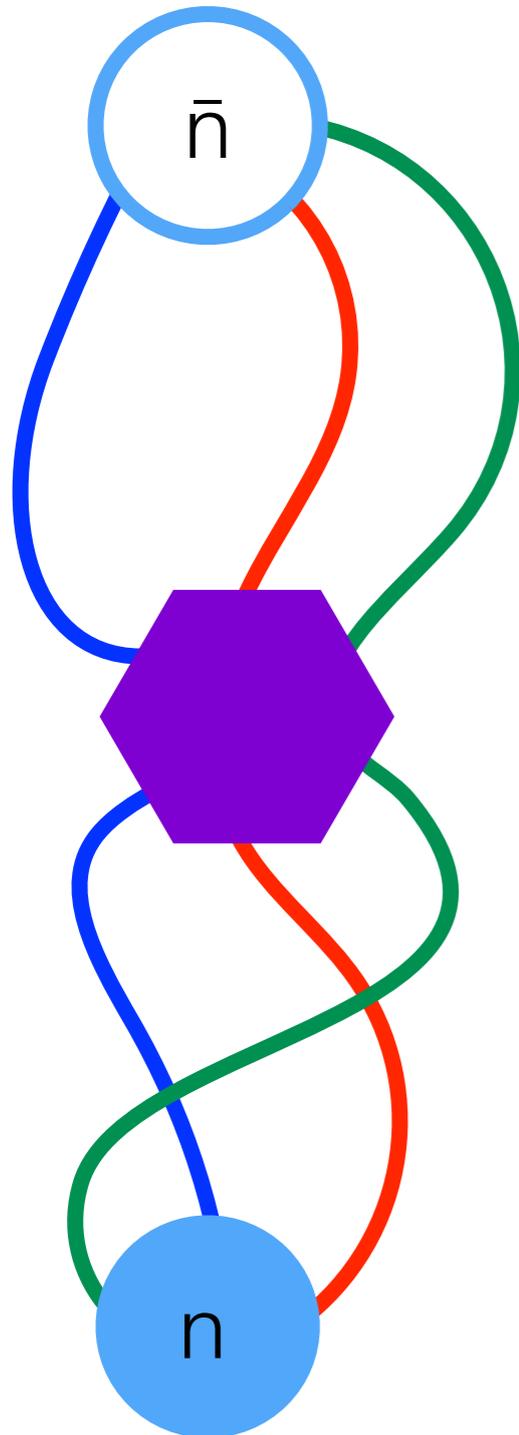
†Preliminary results

Outlook



$\bar{n}n$ Oscillations

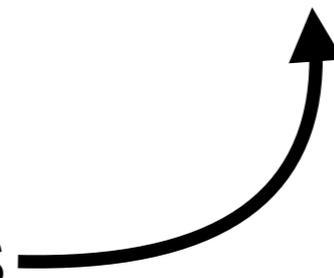
Rinaldi, Syritsyn, Wagman, Buchoff, Schroeder, and Wasem 1809.00246



Physical point, continuum, infinite volume

Operator	$\overline{\text{MS}}(2 \text{ GeV}),$ 10^{-5} GeV^6	$\frac{\overline{\text{MS}}(2 \text{ GeV})}{\text{MIT bag B}}$	Bare, 10^{-5} l.u.	χ^2/dof
Q_1	-44(19)	5.0	-3.7(1.6)	0.75
Q_2	140(40)	12.8	11.8(3.2)	0.69
Q_3	-79(23)	9.7	-6.6(1.9)	0.72
Q_5	-1.43(64)	2.1	-0.096(42)	0.73

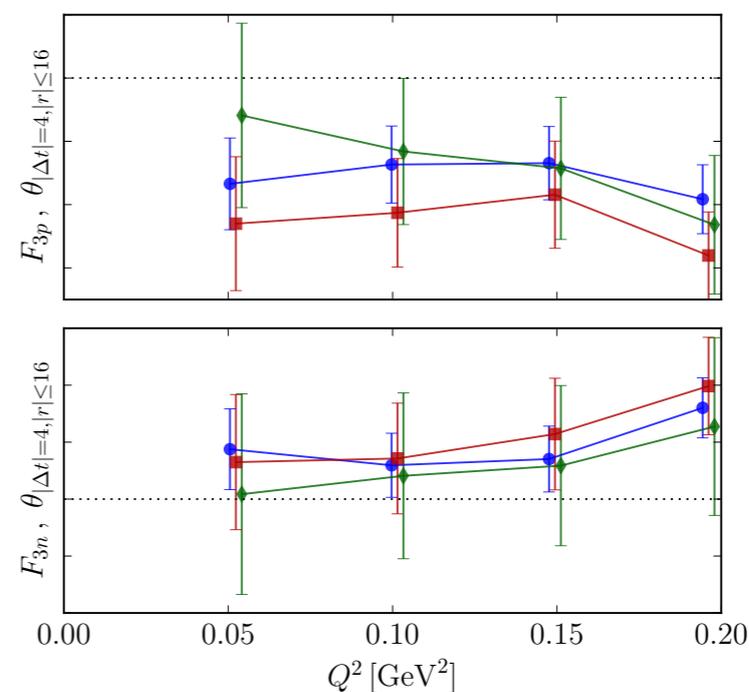
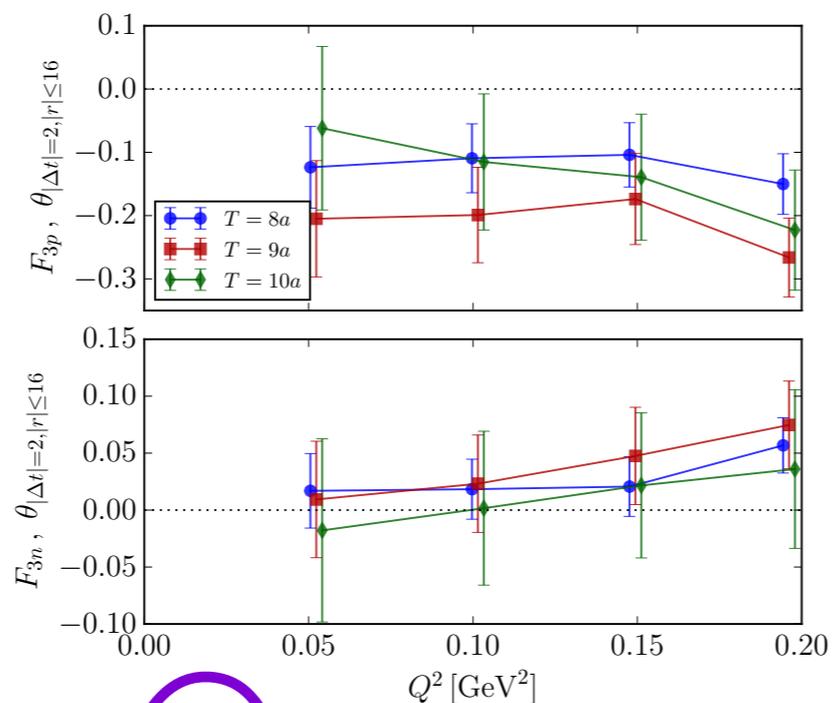
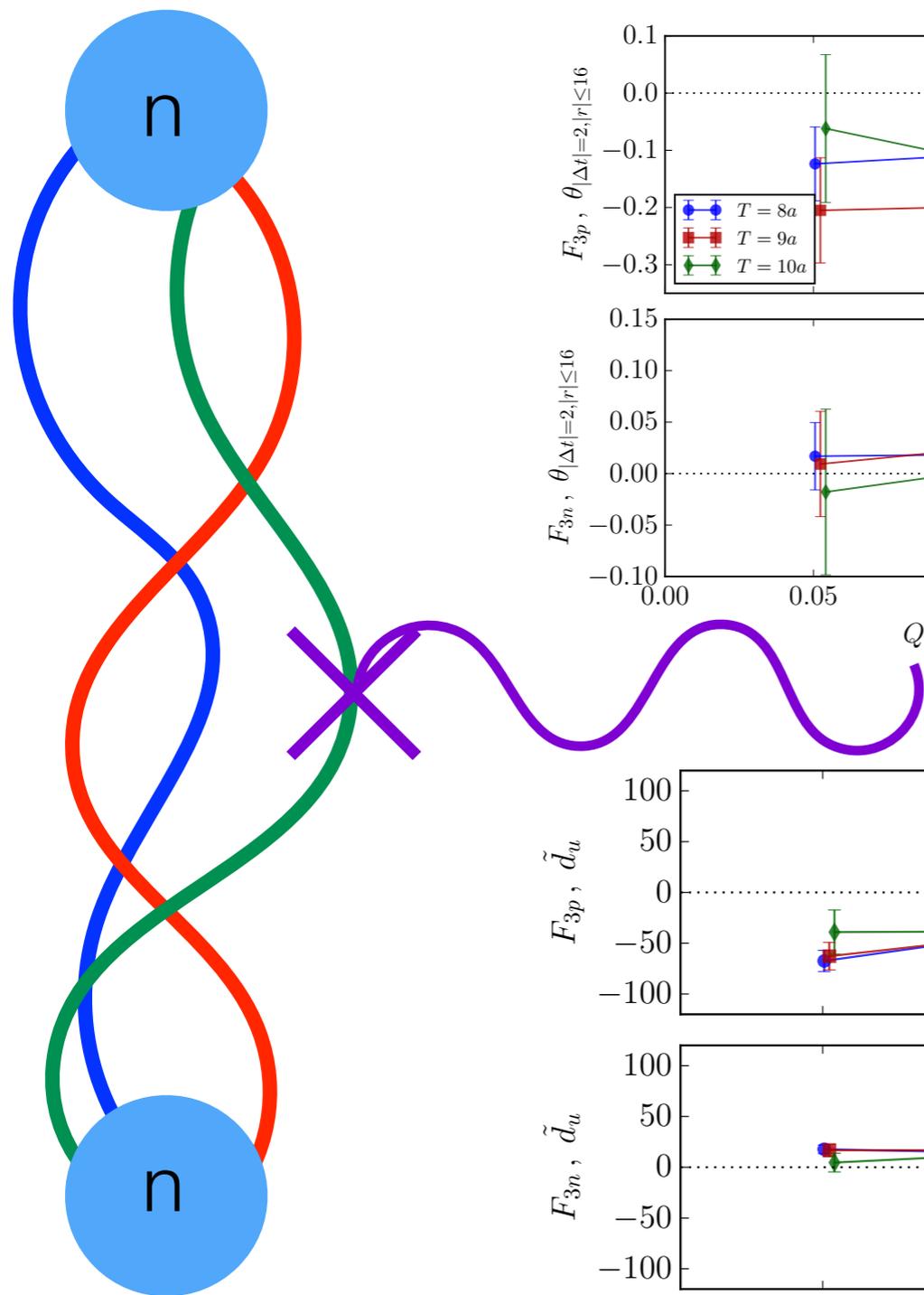
enhancement
means experiments
have greater reach



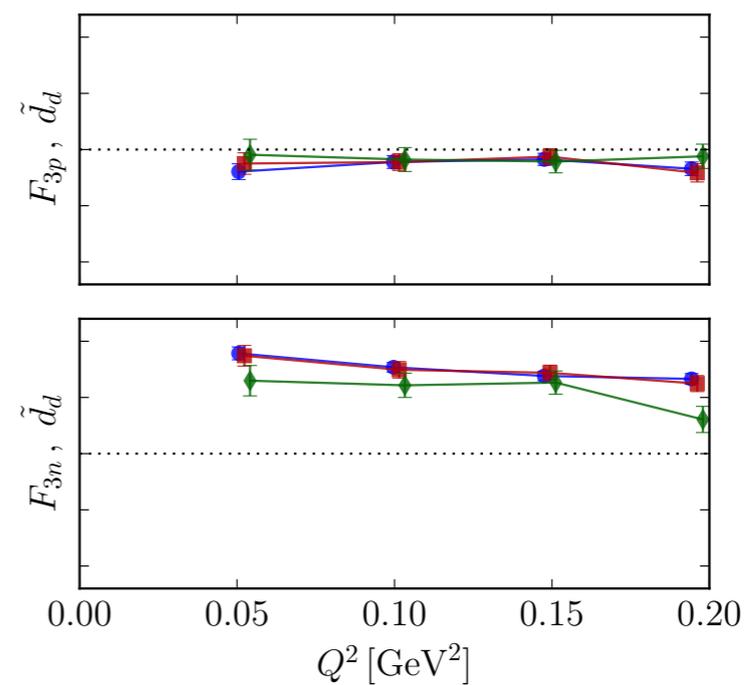
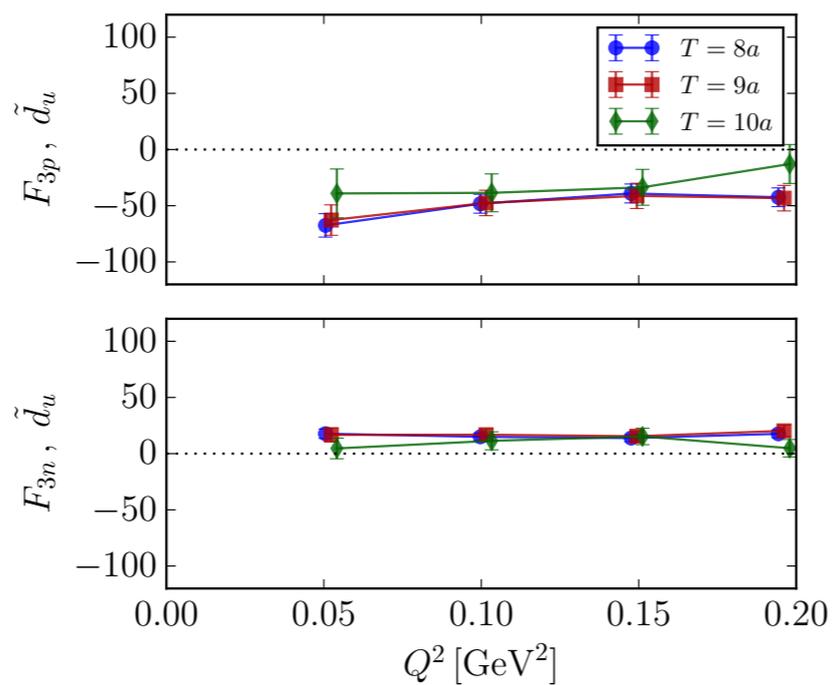
n EDM

Syritsyn, Ohki, Izubuchi CIPANP 2018 1810.03721

$V = 48^3 \times 96$ ($\times 24$ DWF)
 $a = 0.114$ fm
 $m_\pi = 139.2$ MeV



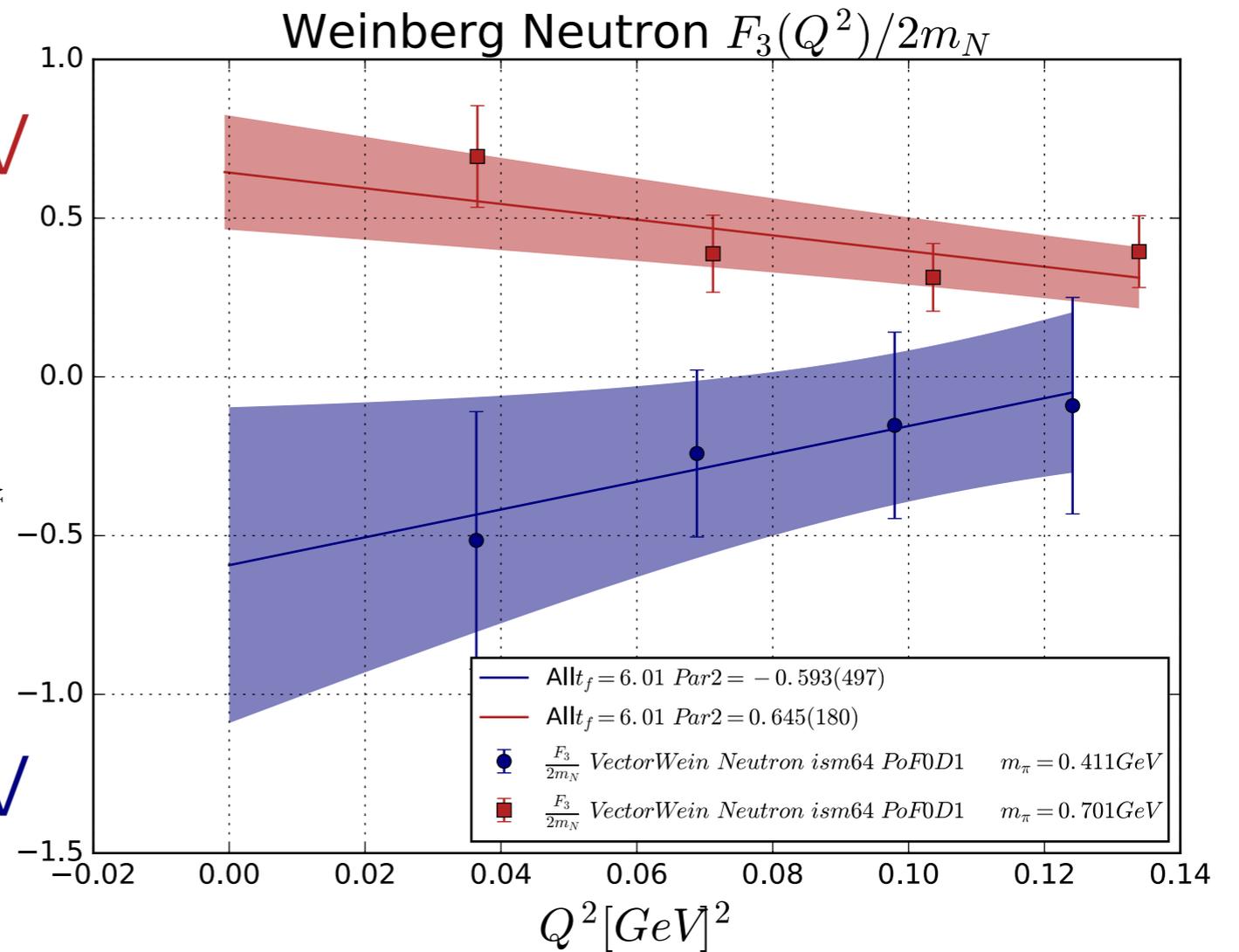
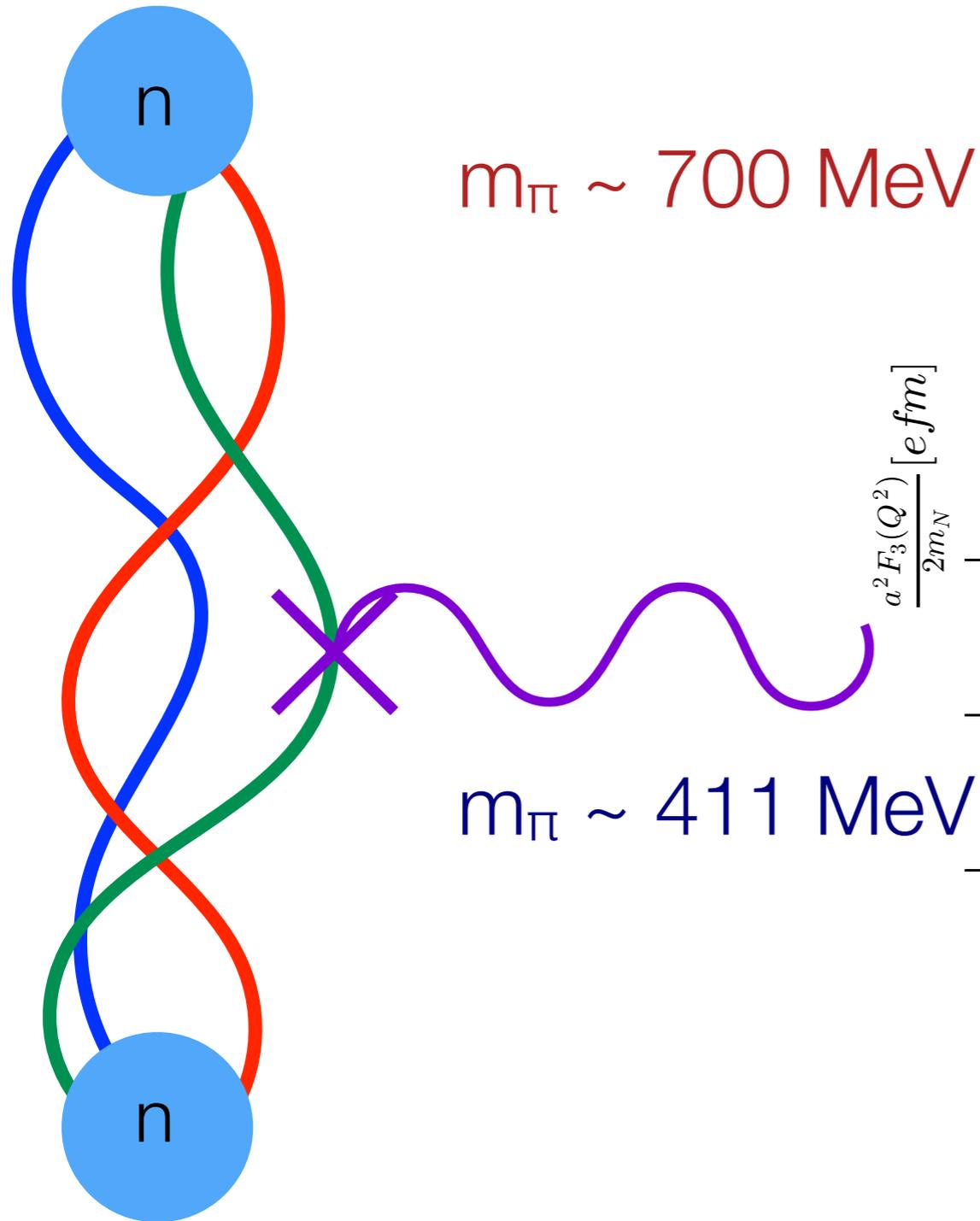
θ_{QCD}



CEDM

n EDM

Dragos, Luu, Shindler, de Vries LATTICE 2017 EPJ Web Conf 175 (2018) 06018 arXiv:1711.04730

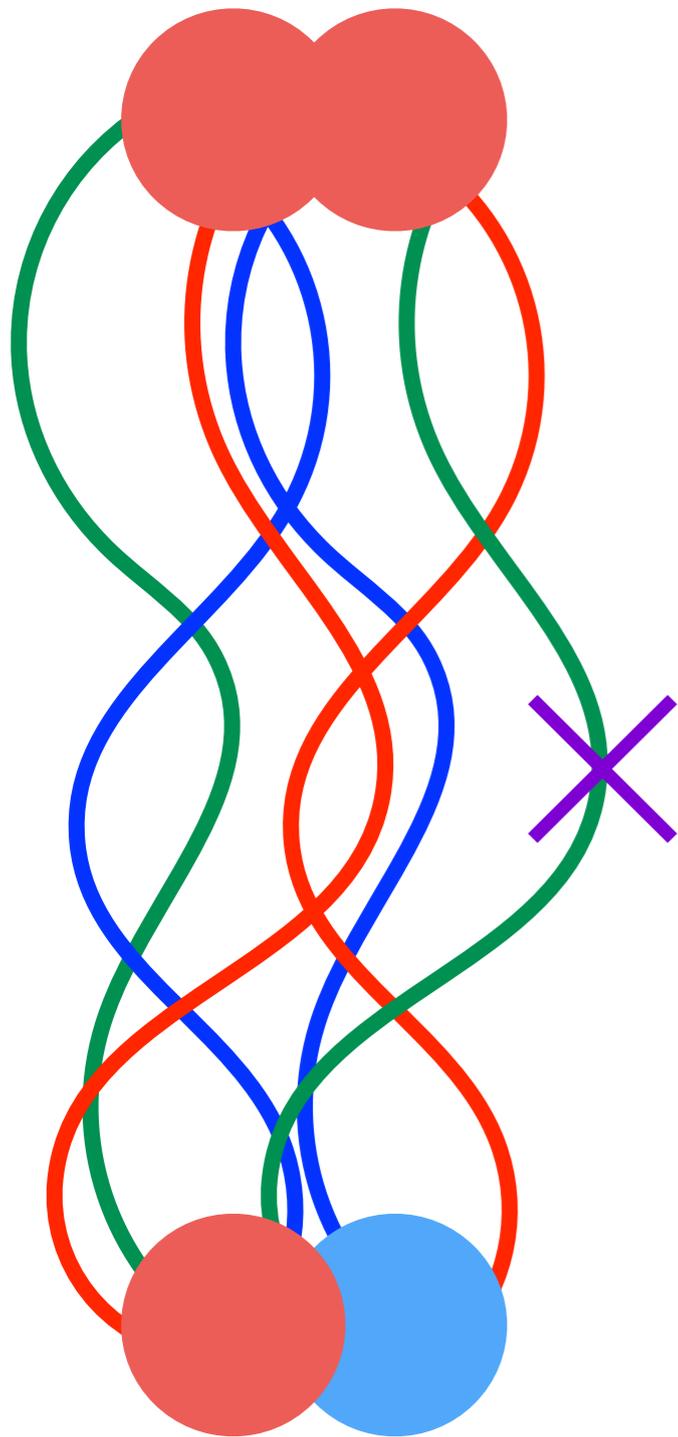


$$-i \frac{\alpha \tilde{G}}{\Lambda^2} \frac{1}{3} f^{ABC} \tilde{G}_{\mu\nu}^A G_{\mu\rho}^B G_{\nu\rho}^C$$

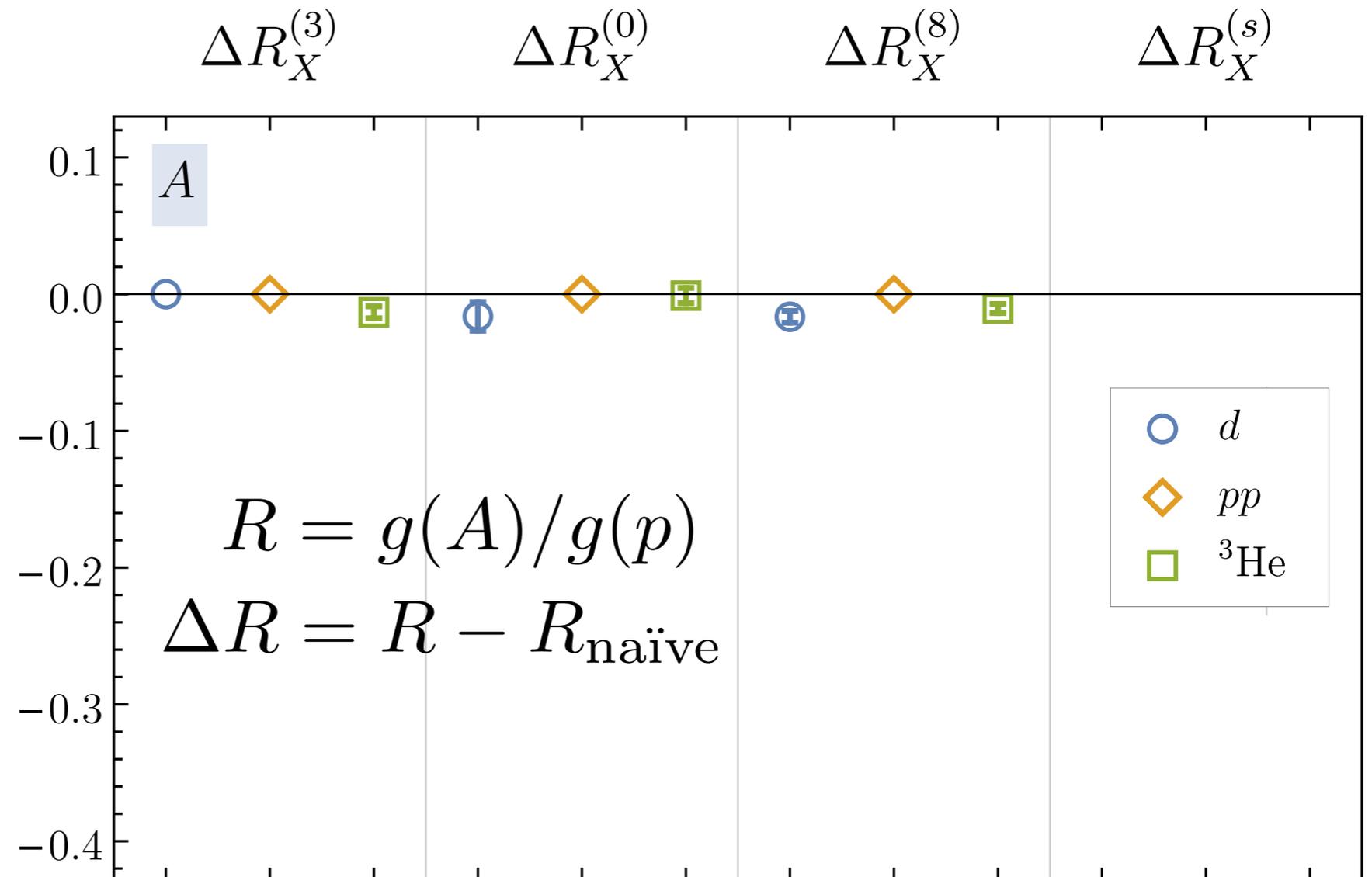
g_A Quenching

NPLQCD PRL 120 (2018) 15 152002 arXiv:1712.03221

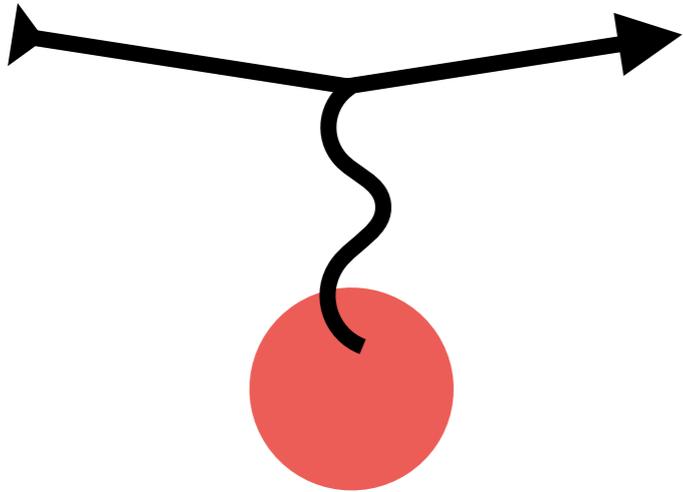
$m_\pi \sim 800$ MeV; $a \sim 0.12$ fm



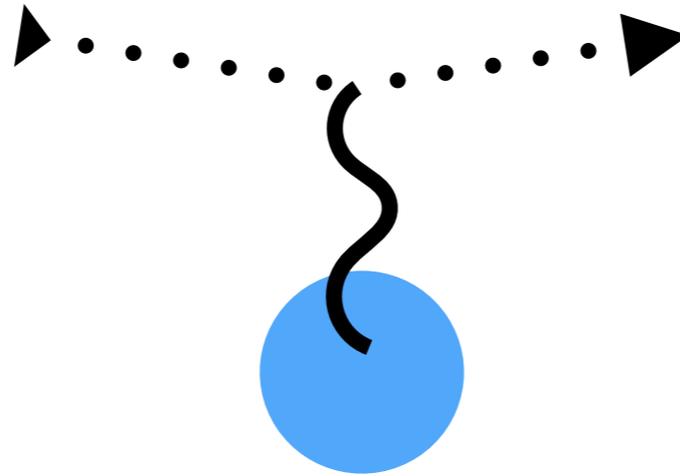
different flavor structures



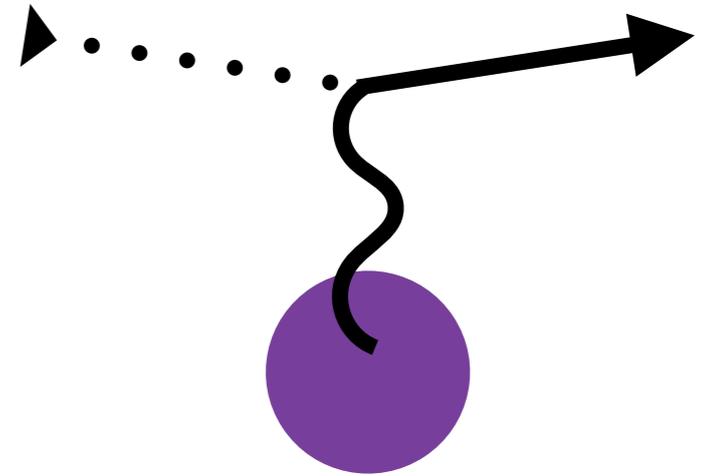
Other Bilinear Matrix Elements



r_E

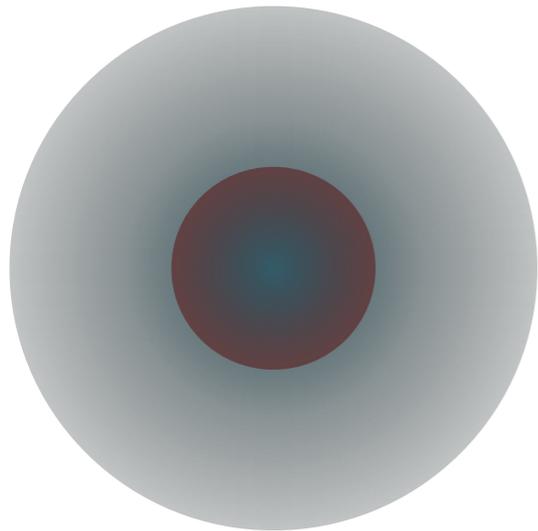


r_A



v_e

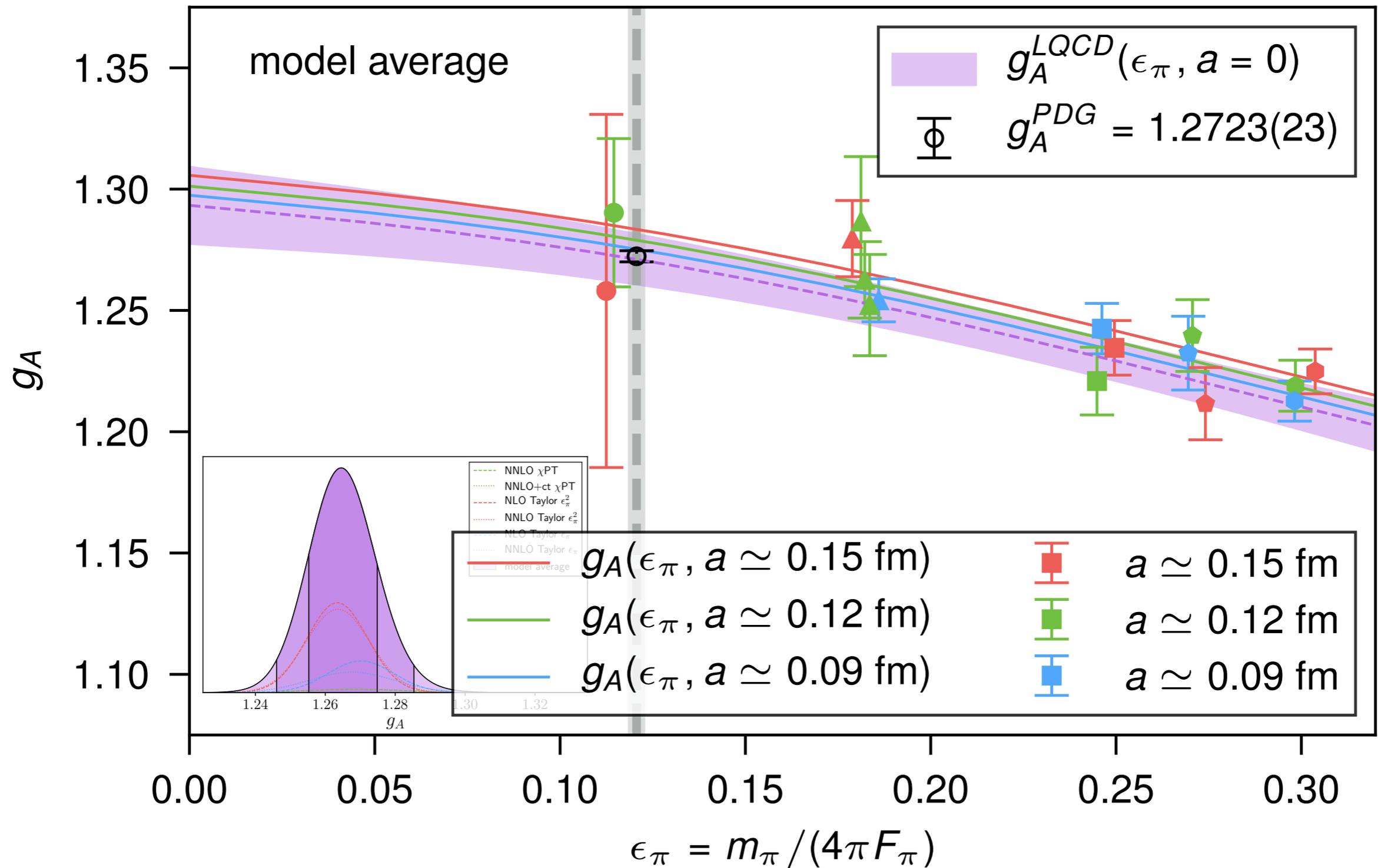
conversion



Form Factors
Radii
 σ term
Polarizabilities
...



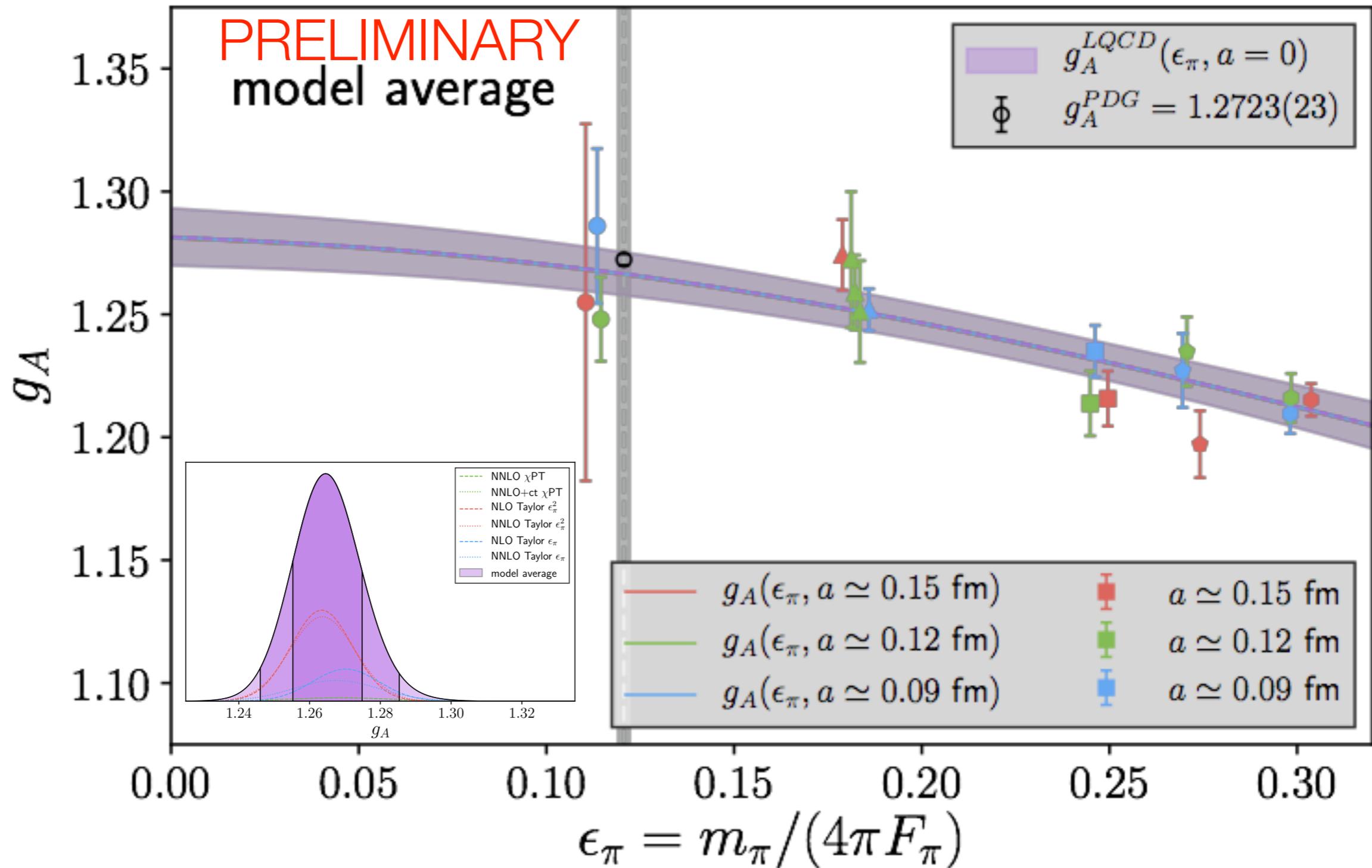
Published $g_A^{\text{QCD}} = 1.271(13)$



Published $g_A^{\text{QCD}} = 1.271(13)$

UPDATE $g_A^{\text{QCD}} = 1.2670(97)$ [0.76%]

PRELIMINARY!!!!

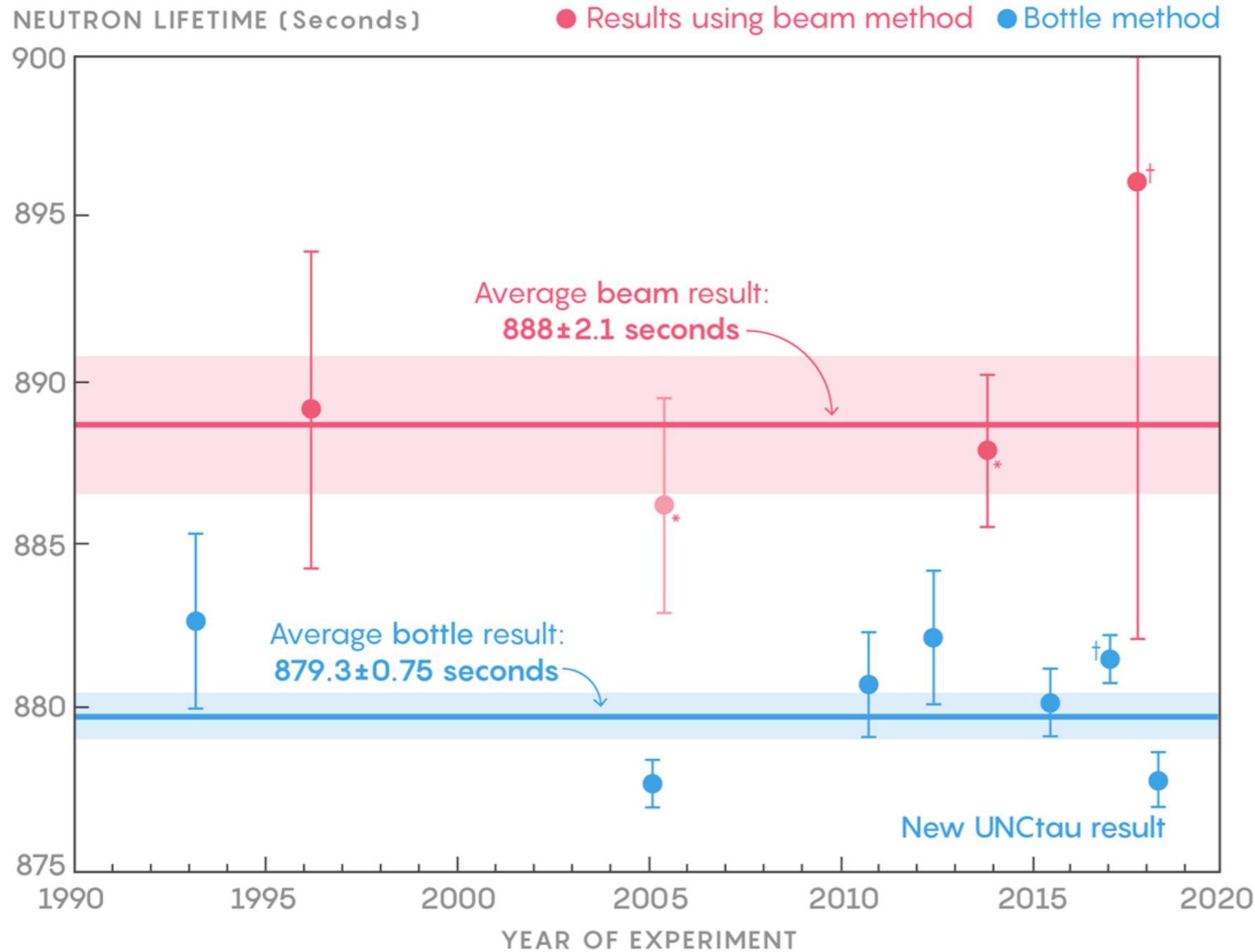


Published $g_A^{\text{QCD}} = 1.271(13)$ [1%]

UPDATE $g_A^{\text{QCD}} = 1.2670(97)$ [0.76%]

Nature 558 (2018) 91

$$\tau_n = \frac{(5172.0 \pm 1.1) \text{seconds}}{1 + 3g_A^2}$$



PRELIMINARY
891(12) s

885(15) s

*Nico result (2005) was superseded by an updated and improved result, Yue (2013);

†Preliminary results

Backup Slides

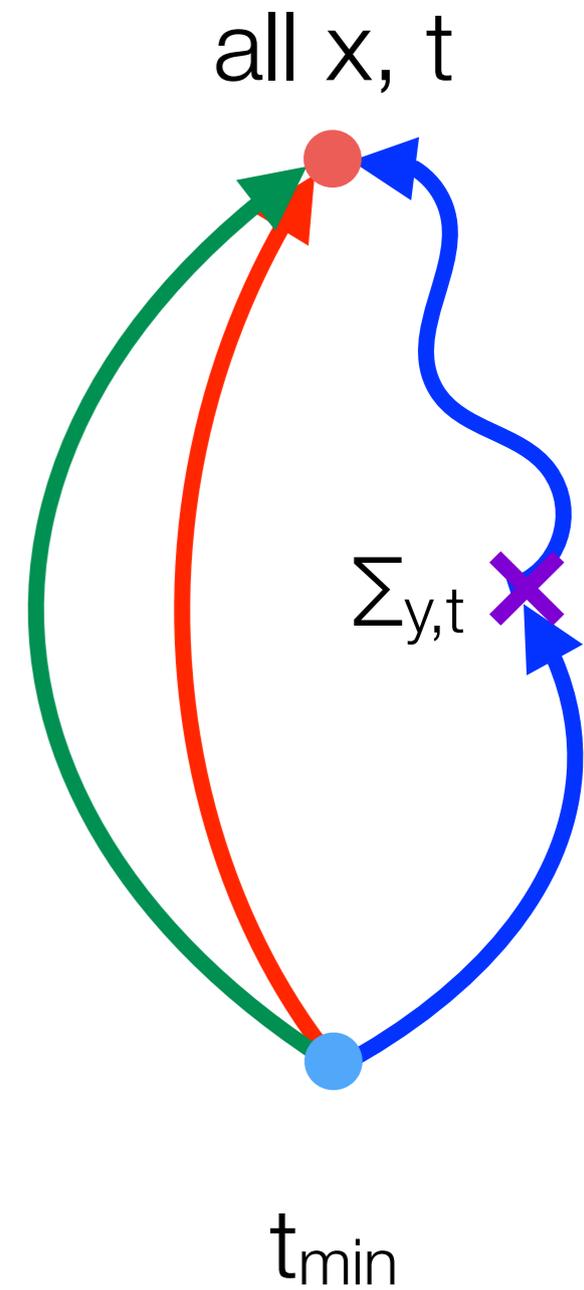
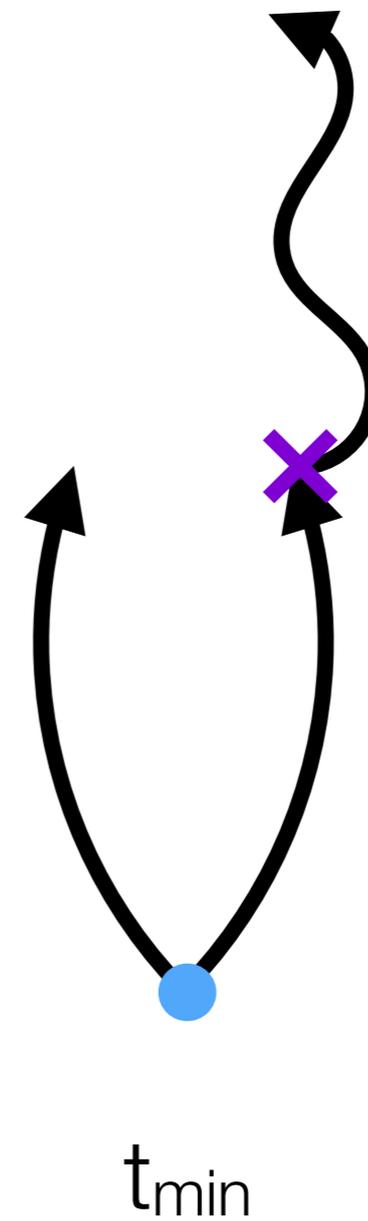
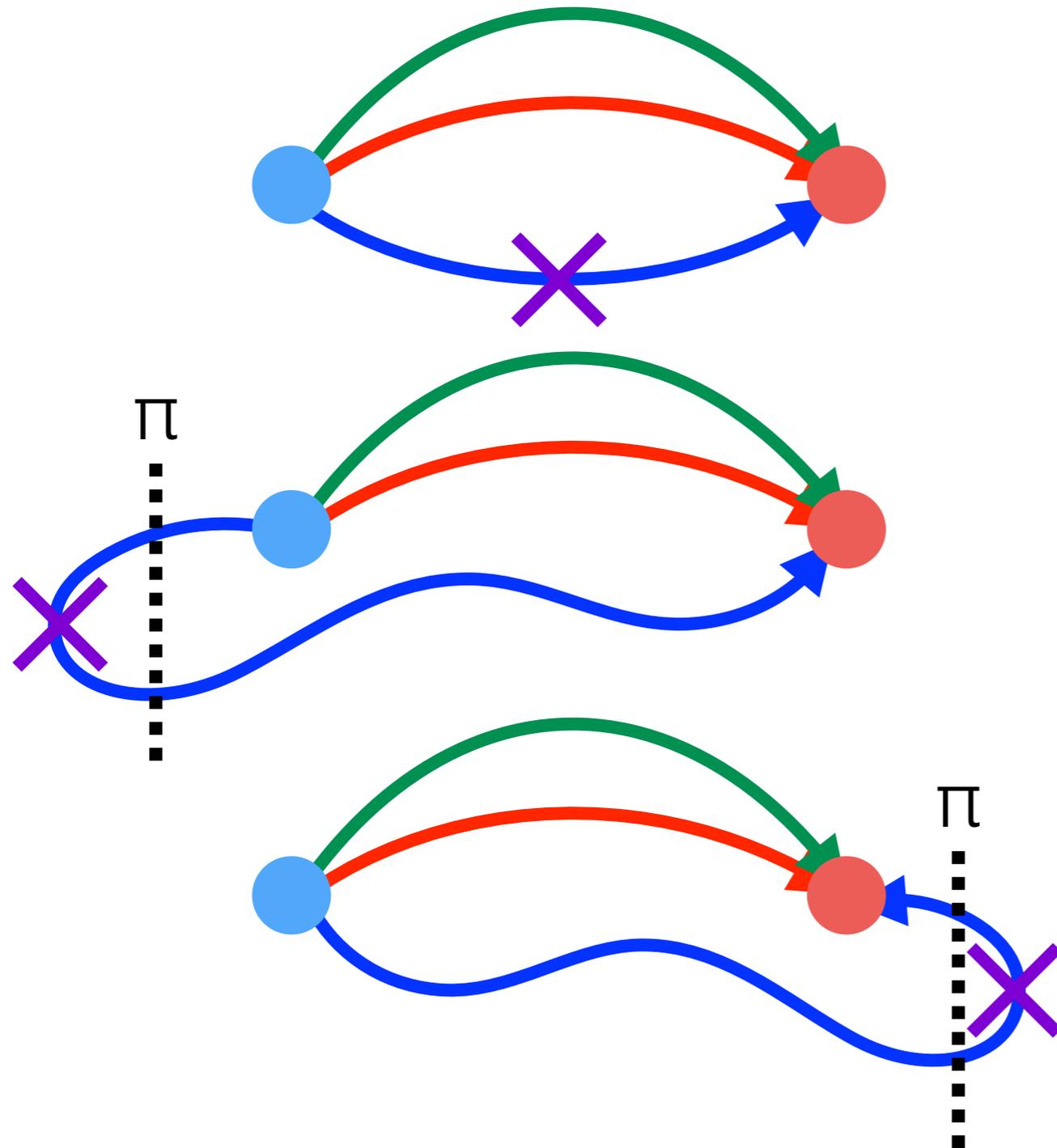


Improved systematics

Bouchard, Chang, Kurth, Orginos, and Walker-Loud arXiv:1612.06963

See also Maiani, Martinelli, Paciello and Taglienti Nucl. Phys. B293 (1987)

NPLQCD 1610.04545, 1611.00344, 1701.03456, 1702.02929



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Bouchard, Chang, Kurth, Orginos, and Walker-Loud arXiv:1612.06963

See also Maiani, Martinelli, Paciello and Taglienti Nucl. Phys. B293 (1987)
 NPLQCD 1610.04545, 1611.00344, 1701.03456, 1702.02929

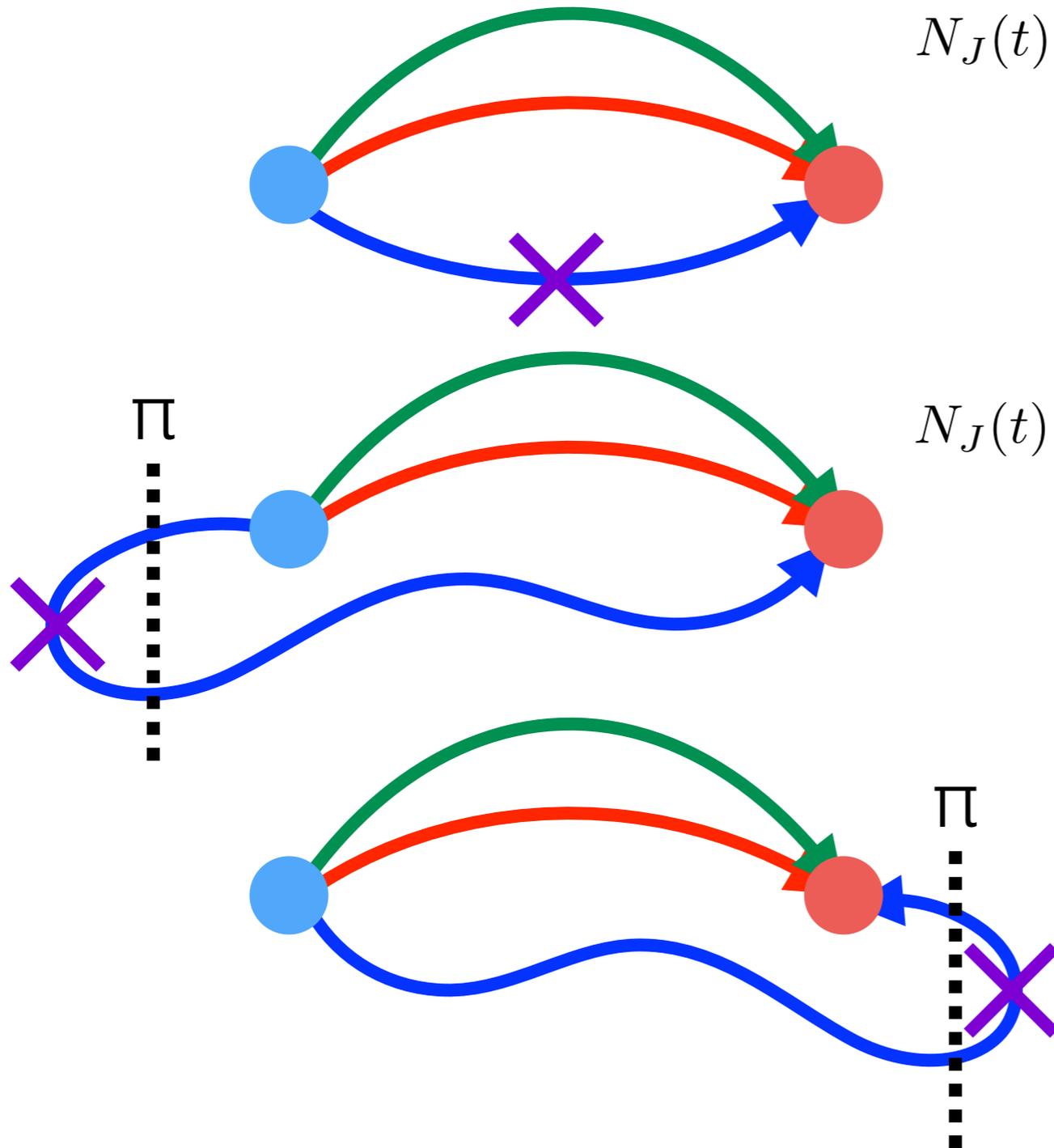


$$N_J(t) = \sum_{t'} \langle \Omega | T \{ O(t) J(t') O^\dagger(0) \} | \Omega \rangle$$

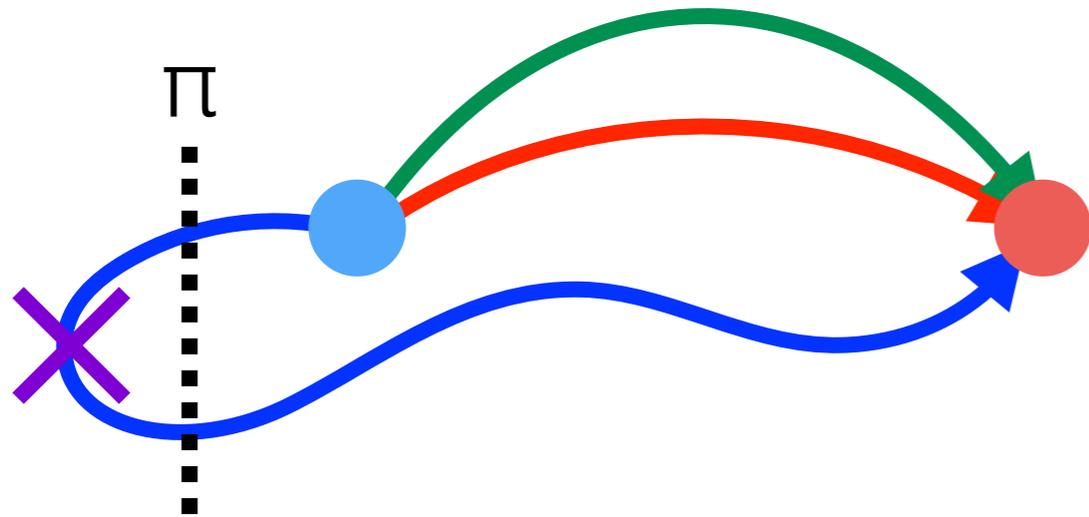
time dependence of
what you want

$$N_J(t) = \sum_n \left[(t-1) z_n g_{nn}^J z_n^\dagger + d_n^J \right] e^{-E_n t} + \sum_{\substack{n \\ m \neq n}} z_n g_{nm}^J z_m^\dagger \frac{e^{-E_n t + \frac{\Delta_{nm}}{2}} - e^{-E_m t - \frac{\Delta_{mn}}{2}}}{e^{\frac{\Delta_{nm}}{2}} - e^{-\frac{\Delta_{mn}}{2}}}$$

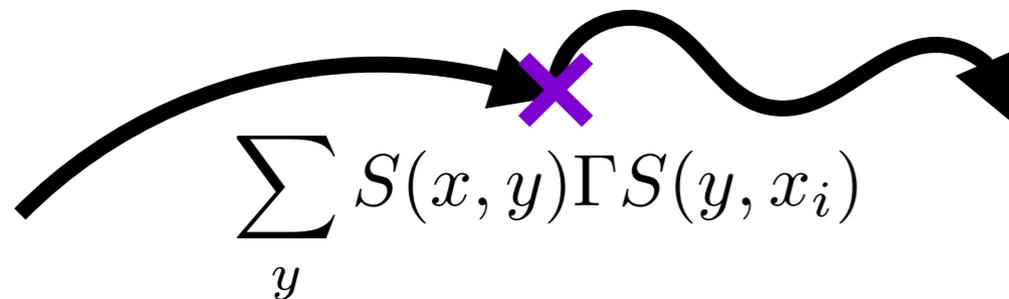
differs from the time dependence of
pieces you don't care about



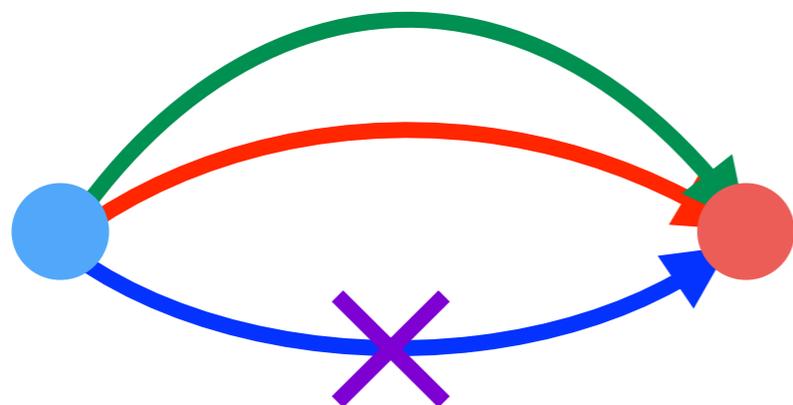
Comparison with the summation method



Summation method doesn't have this contamination

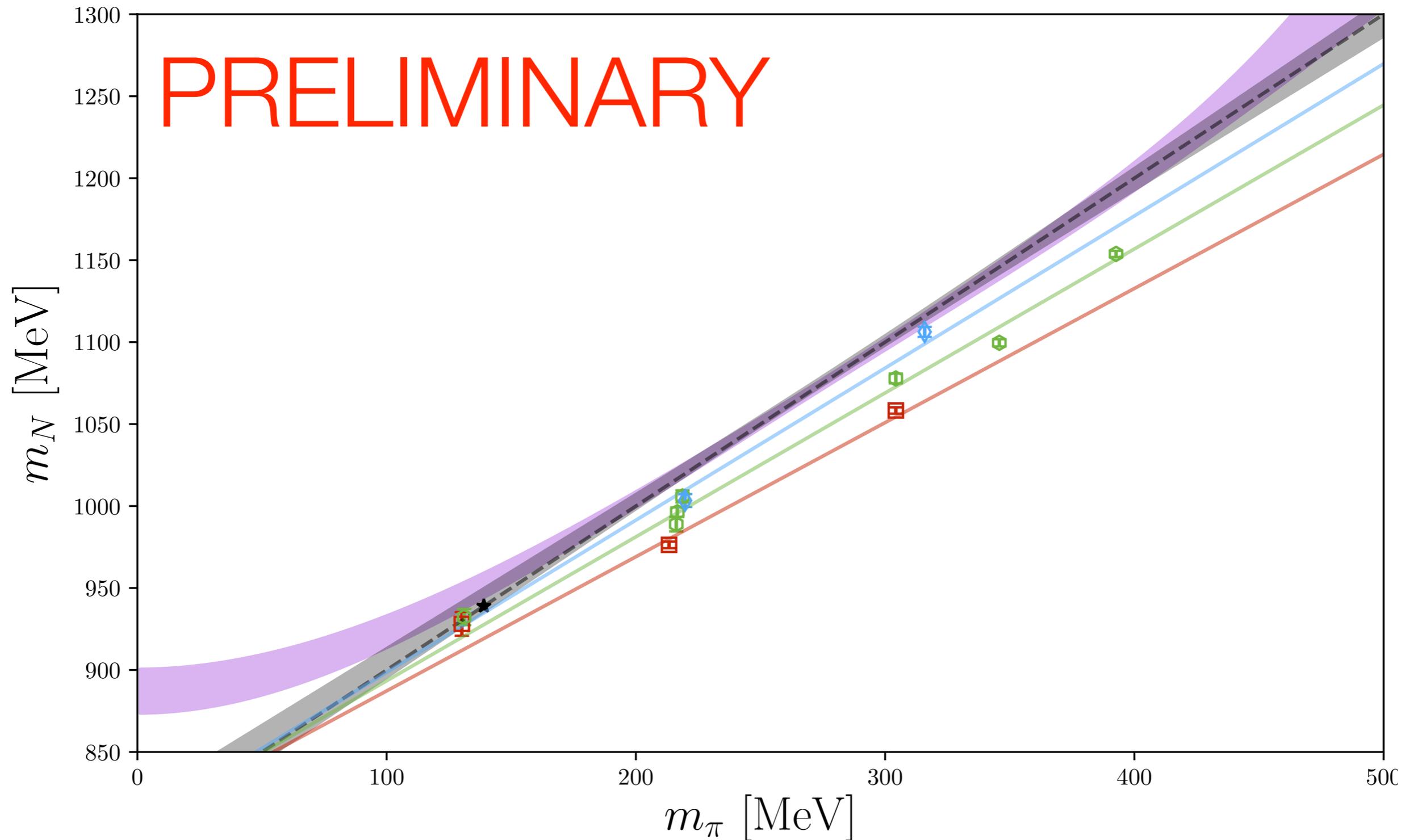


FH method requires new solves to study different insertions



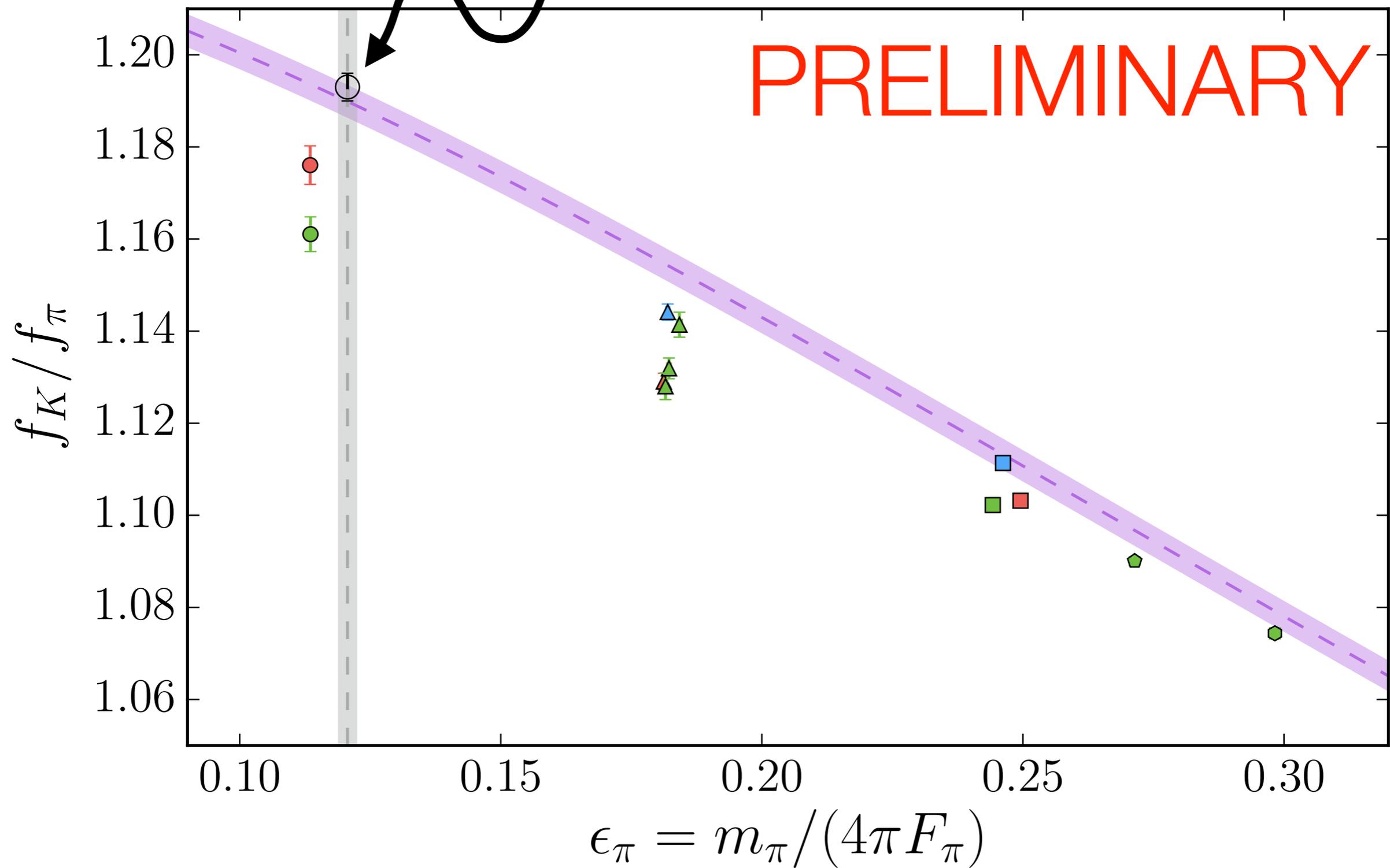
Summation method needs new solves for different source-sink separations

The Ruler still rules: $m_N(m_\pi) \approx 800 \text{ MeV} + m_\pi$

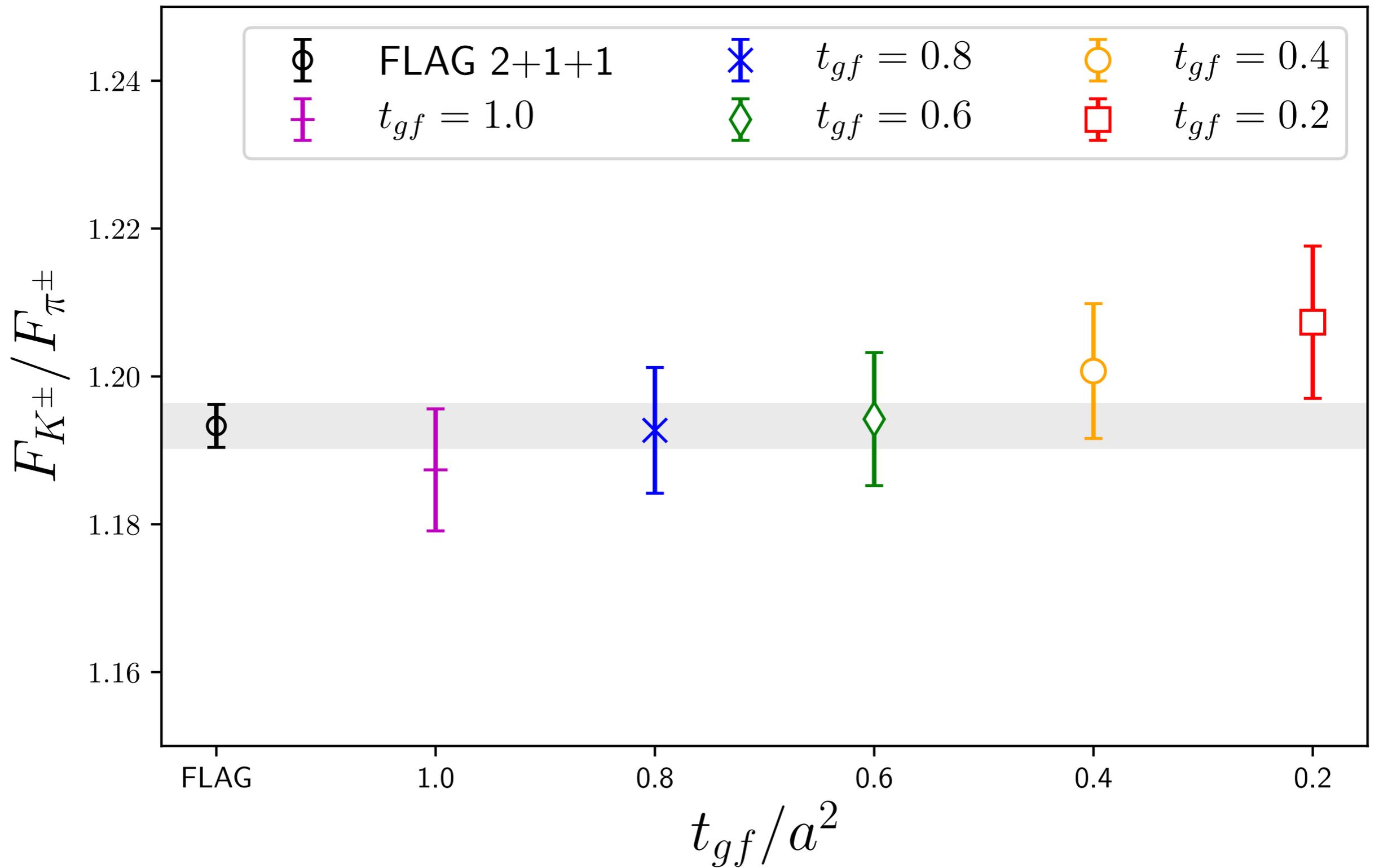


f_K/f_π

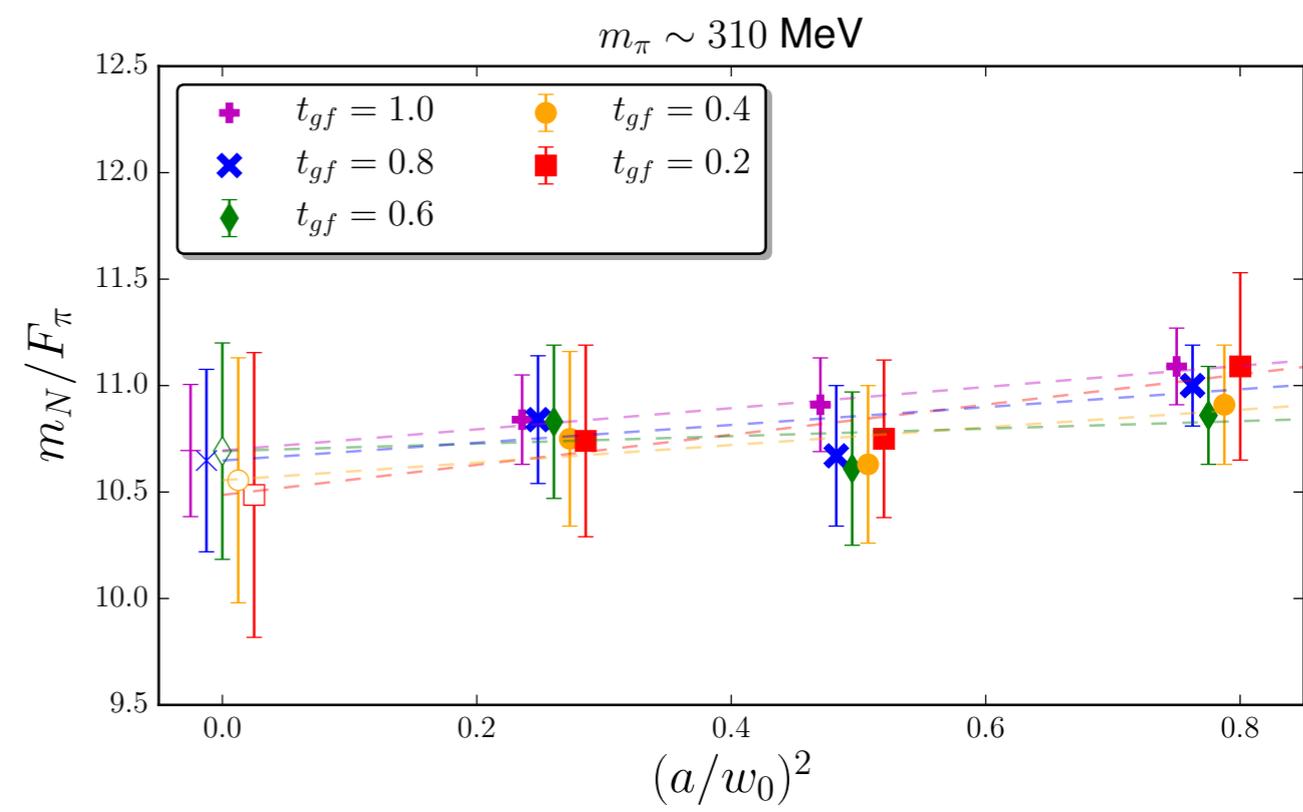
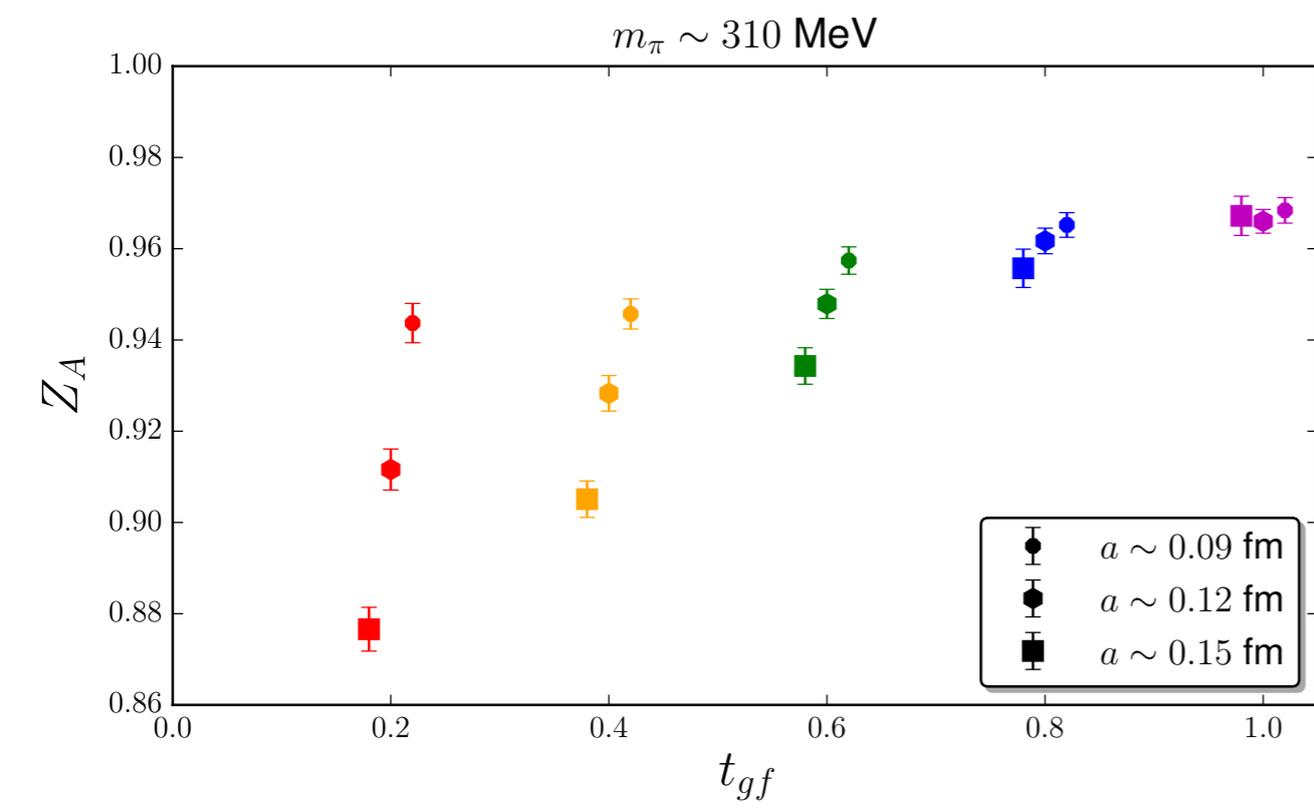
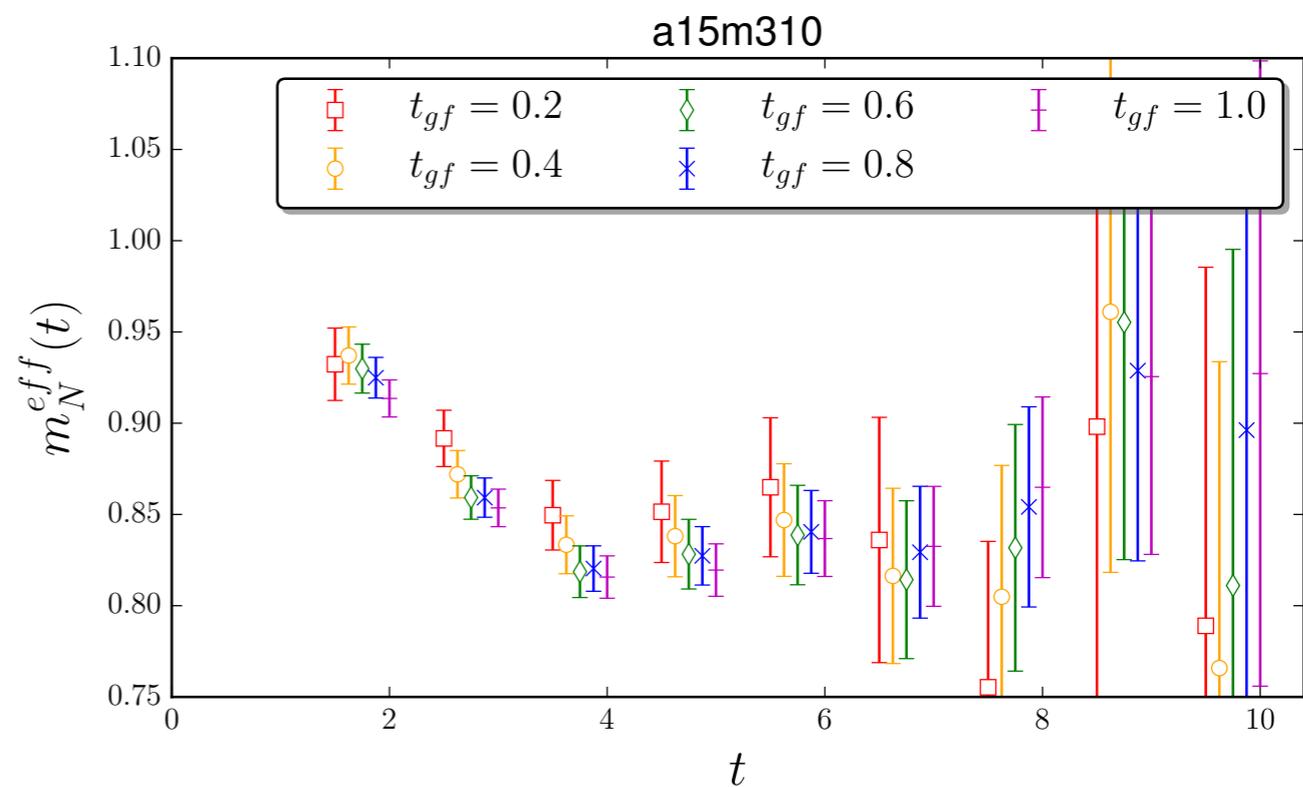
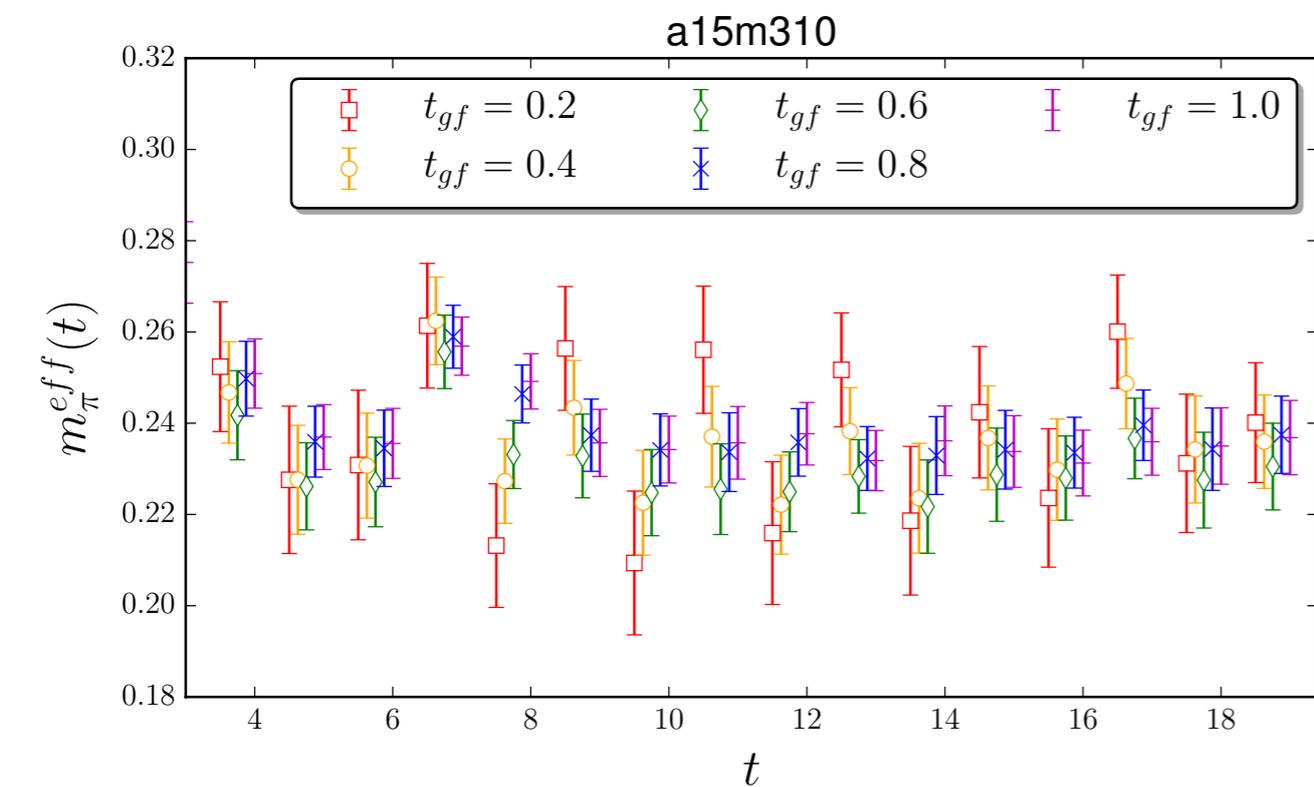
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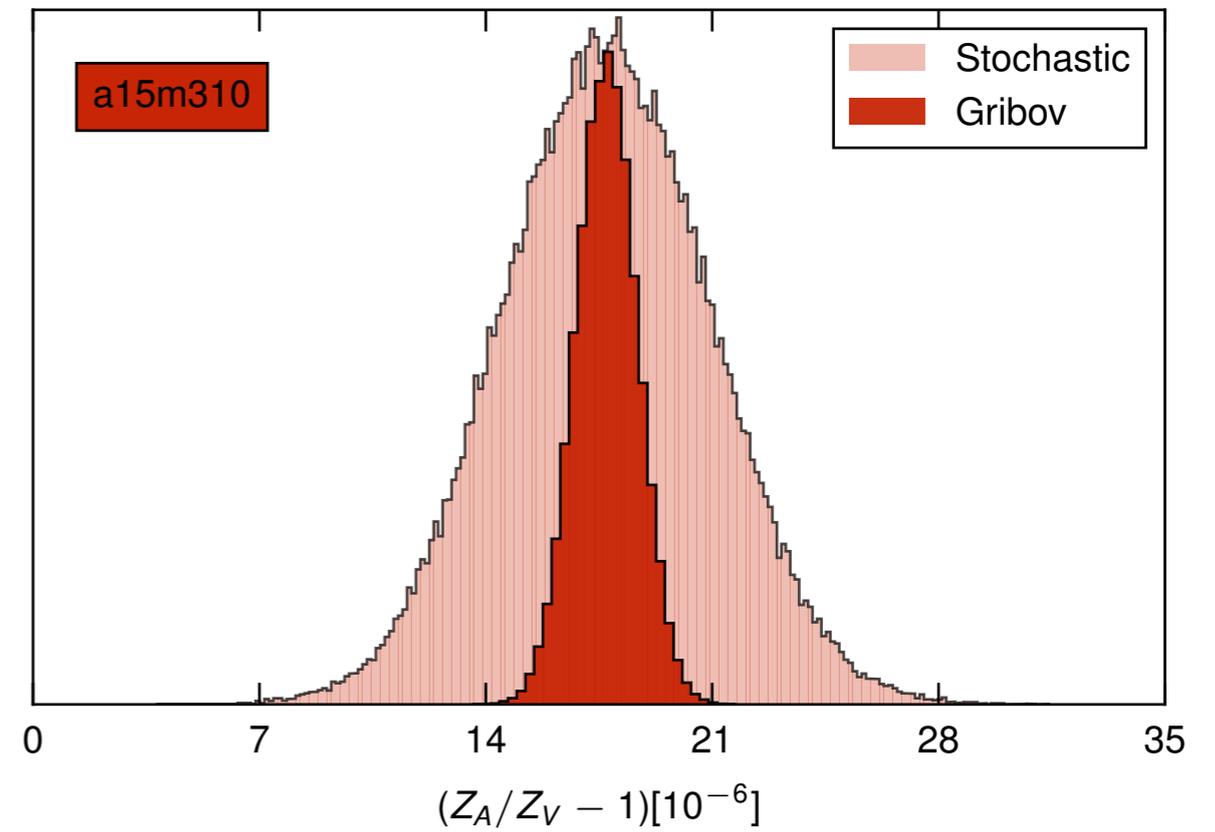
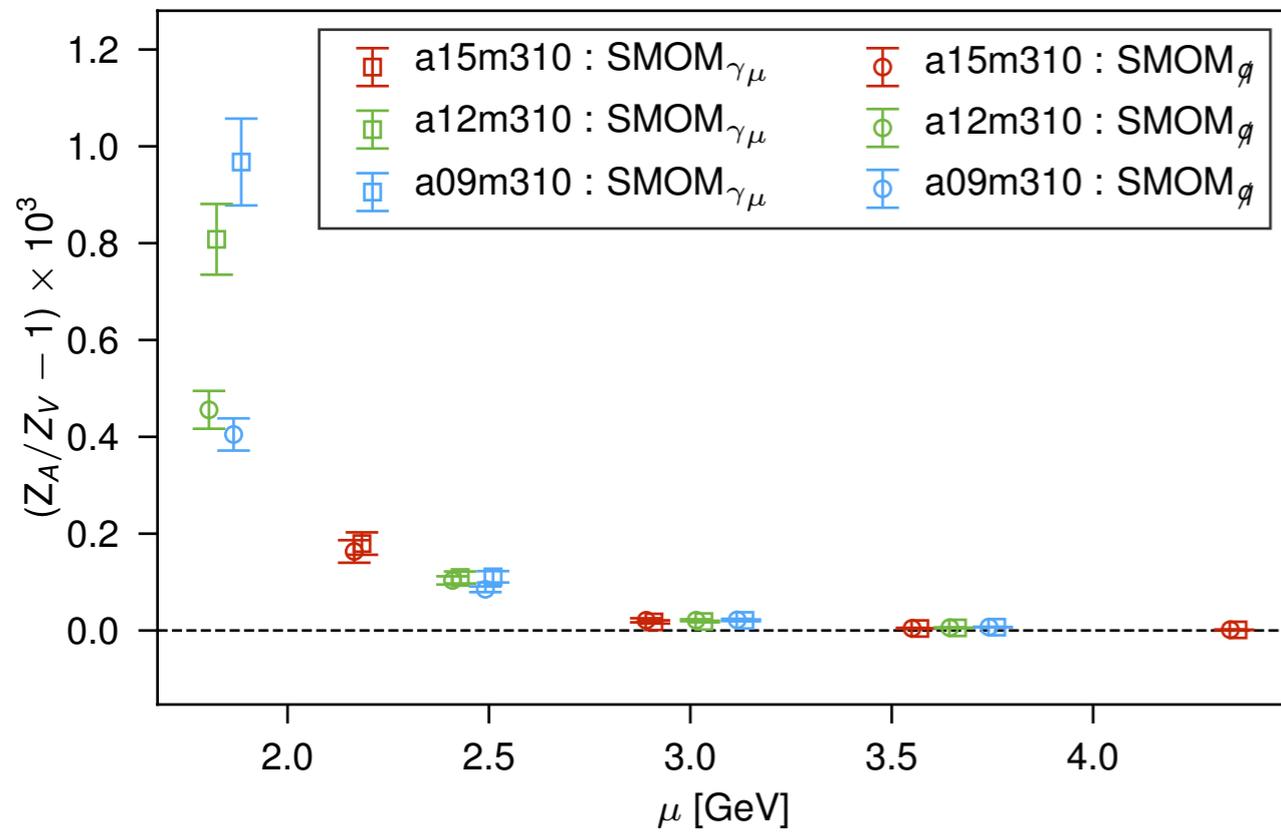
F_K/F_π smearing study ($m_\pi = 135$ MeV)



Smearing Study



Nonperturbative Renormalization



What does this have to do with Feynman-Hellmann?

Bouchard, Chang, Kurt, Orginos, Walker-Loud arXiv:1612.06963

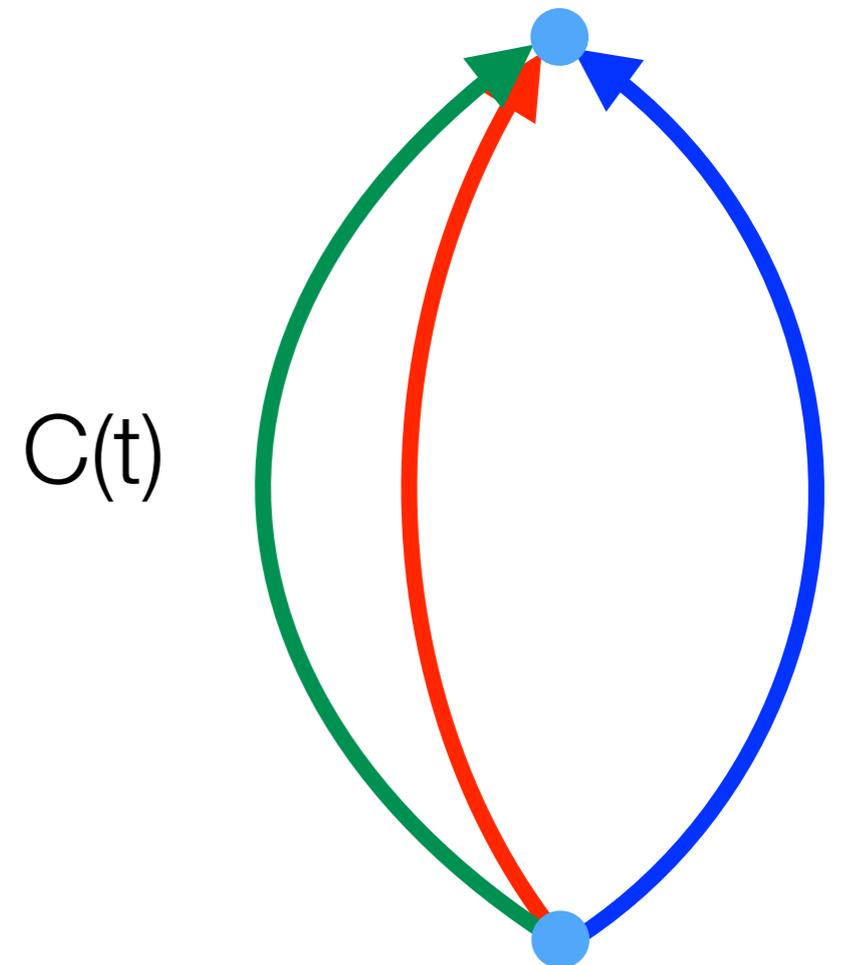
NPLQCD 1610.04545, 1611.00344, 1701.03456, 1702.02929

$$C(t) = \langle \mathcal{N}(t) \bar{\mathcal{N}}(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U \mathcal{N}(t) \bar{\mathcal{N}}(0) e^{-S[U]} = \frac{\text{tr} [\mathcal{N}(t) \bar{\mathcal{N}}(0) e^{-\beta H}]}{\text{tr} [e^{-\beta H}]}$$

$$m_{\text{eff}} = \frac{1}{\tau} \ln \left(\frac{C(t)}{C(t + \tau)} \right) \xrightarrow{t \rightarrow \infty} E_0$$

$$\text{FH: } \frac{\partial E_\lambda}{\partial \lambda} = \left\langle \psi_\lambda \left| \frac{\partial \hat{H}_\lambda}{\partial \lambda} \right| \psi_\lambda \right\rangle$$

$\partial_\lambda E_0$ = a matrix element of interest



What does this have to do with Feynman-Hellmann?

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NPLQCD 1610.04545, 1611.00344, 1701.03456, 1702.02929

$$S[U] \rightarrow S[U] + \lambda \int_x \mathcal{J}(x) \mathcal{O}(x)$$

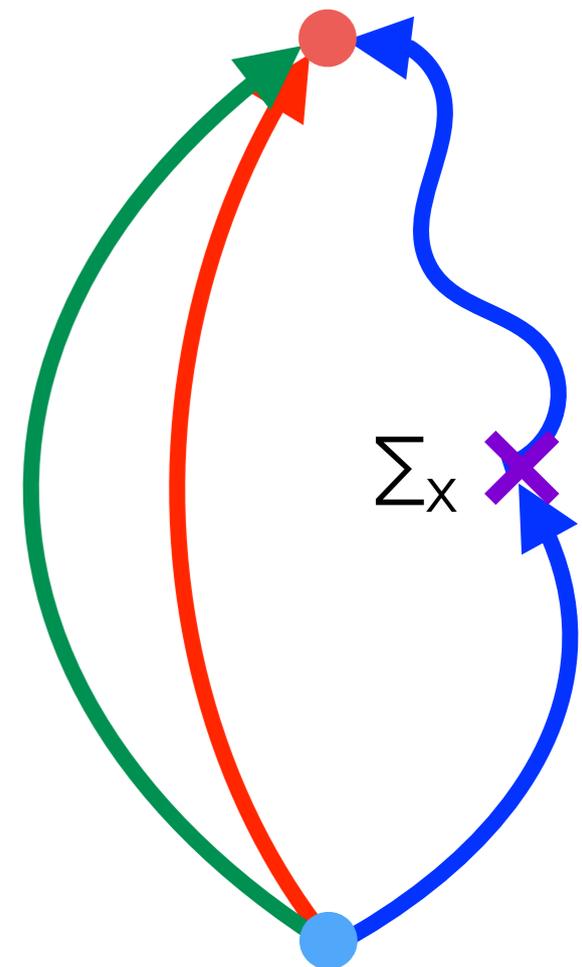
$$\partial_\lambda C(t) = - \left\langle \mathcal{N}(t) \left(\int_x \mathcal{J}(x) \mathcal{O}(x) \right) \bar{\mathcal{N}}(0) \right\rangle$$

$$\left. \frac{\partial m_{\text{eff}}}{\partial \lambda} \right|_{\lambda=0} = \frac{1}{\tau} \left[\left. \frac{\partial_\lambda C(t)}{C(t)} - \frac{\partial_\lambda C(t + \tau)}{C(t + \tau)} \right] \right|_{\lambda=0}$$

$$\mathcal{J}_\mu(x) = 1$$

$$\mathcal{O}^\mu(x) = \bar{q} \gamma^\mu \gamma^5 \tau^+ q$$

$$\xrightarrow{t \rightarrow \infty} g_A + O(e^{-E_n t})$$



Improved systematics

Bouchard, Chang, Kurth, Orginos, and Walker-Loud arXiv:1612.06963

See also Maiani, Martinelli, Paciello and Taglienti Nucl. Phys. B293 (1987)
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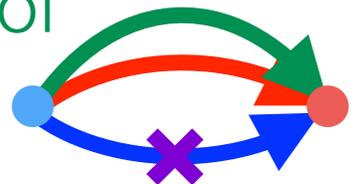
$$g_{nn}^J \equiv \frac{J_{nn}}{2E_n} \quad J_{nn} = \langle n | J | n \rangle$$

$$g_{nm}^J \equiv \frac{J_{nm}}{\sqrt{4E_n E_m}} \quad J_{nm} = \langle n | J | m \rangle$$

$$\Delta_{nm} \equiv E_n - E_m$$

$$N_J(t) = \sum_{t'} \langle \Omega | T \{ O(t) J(t') O^\dagger(0) \} | \Omega \rangle$$

time dependence of
what you want



$$N_J(t) = \sum_n \left[(t-1) z_n g_{nn}^J z_n^\dagger + d_n^J \right] e^{-E_n t}$$

$$+ \sum_{\substack{n \\ m \neq n}} z_n g_{nm}^J z_m^\dagger \frac{e^{-E_n t + \frac{\Delta_{nm}}{2}} - e^{-E_m t - \frac{\Delta_{mn}}{2}}}{e^{\frac{\Delta_{nm}}{2}} - e^{-\frac{\Delta_{mn}}{2}}}$$

differs from the time dependence of
pieces you don't care about

$$d_n^J \equiv Z_n Z_{J:n}^\dagger + Z_{J:n} Z_n^\dagger + Z_n Z_n^\dagger \langle \Omega | J | \Omega \rangle$$

$$+ \sum_j \frac{Z_n Z_{nj}^\dagger J_j^\dagger + J_j Z_{jn} Z_n^\dagger}{2E_j (e^{E_j} - 1)}$$

