• damit d'Alembert'sche Gleichung

$$\boxed{\sum_{i=1}^{N} (m_i \ddot{\vec{r}}_i - \vec{F}_i)\,\delta \vec{r}_i = \sum_{i=1}^{N} \vec{Z}_i \,\delta \vec{r}_i = 0} \qquad (1)$$

2.4 Lagrange Gleichungen (2. Art)

• Schreibe (1) in generalisierten Koordinaten

$$\vec{r}_i = \vec{r}_i(q_1, \dots, q_f, t) \;, \quad i = 1, \dots, N$$

$\hookrightarrow$ (i) $\dfrac{\partial \dot{\vec{r}}_i}{\partial q_\alpha} = \dfrac{d}{dt}\left(\dfrac{\partial \vec{r}_i}{\partial q_\alpha}\right)$ (ii) $\dfrac{\partial \dot{\vec{r}}_i}{\partial \dot{q}_\alpha} = \dfrac{\partial \vec{r}_i}{\partial q_\alpha}$

(iii) $\delta \vec{r}_i = \sum_\alpha \dfrac{\partial \vec{r}_i}{\partial q_\alpha} \delta q_\alpha$ (beachte $\delta t = 0$!)

$\hookrightarrow$ die δq_α sind unabhängig, da Zwangsbed. explizit erfüllt sind!

• i, ii, iii in (1) einsetzen

$$\hookrightarrow \sum_\alpha \left\{ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_\alpha}\right) - \frac{\partial T}{\partial q_\alpha} - Q_\alpha \right\} \delta q_\alpha = 0$$

mit $Q_\alpha = \sum_i \vec{F}_i \dfrac{\partial \vec{r}_i}{\partial q_\alpha}$ und $T = \sum_i \frac{1}{2} m_i \dot{\vec{r}}_i^{\,2} = T(q, \dot{q}, t)$

• da alle δq_α unabhängig

$$\hookrightarrow \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_\alpha}\right) - \frac{\partial T}{\partial q_\alpha} - Q_\alpha = 0 \qquad \alpha = 1, \dots, f$$