Single- & Double-strangeness Hypernuclei in the frame work of the NCSM and  $\chi$ EFT

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Special thanks to:

Dr. Gabriele Erkelenz and the prize committee collaborators: Dr. Johann Haidenbauer, Prof. Ulf-G Meißner, Dr. Andreas Nogga



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#### Outline

- Introduction:
  - Motivations
  - Baryon-Baryon (BB) interactions within chiral EFT theory
- Numerical approach:
  - Jacobi NCSM for nuclear systems with S = -1, -2
  - Similarity Renormalization Group (SRG) in two- & three-body space
- Results:
  - Separation energies and CSB in A=4-8  $\Lambda$  hypernuclei:  ${}^{4}_{\Lambda}$  He,  ${}^{5}_{\Lambda}$  He,  ${}^{7}_{\Lambda}$  Li,  ${}^{8}_{\Lambda}$  Li
  - ► Double-strangeness systems:  ${}^{4}_{\Lambda\Lambda}$  He,  ${}^{5}_{\Lambda\Lambda}$  He,  ${}^{6}_{\Lambda\Lambda}$  He;  ${}^{4}_{\Xi}$  H,  ${}^{5}_{\Xi}$  H,  ${}^{7}_{\Xi}$  H
- Summary & Outlook





#### **Erkelenz' work and modern BB interactions**







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(J. Haidenbauer, U.-G. Meißner, A. Nogga, EPJA 56 (2019) 91)

#### How to further constrain YN

•  $\Lambda$ -separation energies are known with high accuracy





#### Jacobi-NCSM approach

• Idea: represent the A-body translationally invariant hypernuclear Hamiltonian:

$$\mathbf{H} = \mathbf{T}_{rel} + \mathbf{V}^{NN} + \mathbf{V}^{YN} + \mathbf{V}^{NNN} + \mathbf{V}^{YNN} + \Delta M + \cdots$$
$$= \sum_{i < j=1}^{A-1} h_{ij}^{NN} + \sum_{i < j < k=1}^{A-1} \mathbf{V}_{ijk}^{3N} + \sum_{i=1}^{A-1} h_{iY}^{YN} + \sum_{i < j}^{A-1} \mathbf{V}_{ijY}^{YNN} + \cdots$$

in a basis constructed from HO functions

• Jacobi basis: depends on relative Jacobi coordinates of all particles

$$\begin{array}{c} (A-1)N \\ | \bigcirc & \bullet \rangle = | \mathcal{N}JT, \mathcal{N}_{A-1}J_{A-1}T_{A-1}, \\ \Lambda(\Sigma) & \underbrace{\mathcal{N}_{A-1}J_{A-1}T_{A-1}}_{antisym.(A-1)N}, \underbrace{\mathcal{N}_{Y}l_{Y}l_{Y}t_{Y}}_{\Lambda(\Sigma) \ state} (J_{A-1}(l_{Y}s_{Y})I_{Y}) J, (T_{A-1}t_{Y})T \rangle \end{array}$$

- anti-symmetrized (A-1)N states are constructed iteratively:
  - 2N states:  $|\alpha_{12}\rangle \equiv |N_{12}(l_{12}S_{12})J_{12}, (t_1t_2)t_{12}\rangle$  antisymmetric
  - ► 3N Jacobi states:  $|\alpha_3^{*(1)}\rangle \equiv |N_3 J_3 T_3 \alpha_{12} n_3 (l_3 s_3) I_3, t_3; (J_{12} I_3) J_3, (t_{12} t_3) T_3\rangle$  antisymmetric w.t. r. (1)  $\leftrightarrow$  (2)

idea: use antisymmetrizer to project out the (non)antrisymmetric states:  

$$\langle \alpha_3^{'*(1)} | \frac{1}{3} (1 - 2\mathcal{P}_{23}) | \alpha_3^{*(1)} \rangle \langle \alpha_3^{*(1)} | \alpha_3 \rangle = \lambda \langle \alpha_3^{*(1)} | \alpha_3 \rangle \qquad \lambda = (0)1$$





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### Jacobi-NCSM approach



$$\Rightarrow \langle \bigcirc \bullet | \sum_{i < j = 1}^{A-1} h_{ij}^{NN} | \bullet \bigcirc \rangle = \langle \bigcirc \bullet | \downarrow \bigcirc \rangle \langle \bigcirc \downarrow | \sum_{i < j = 1}^{A-1} h_{ij}^{NN} | \downarrow \bigcirc \langle \bigcirc \downarrow | \bullet \bigcirc \rangle$$
$$= \delta_Y \delta_{core(A-3)} \begin{pmatrix} A-1 \\ 2 \end{pmatrix} \langle \bigcirc | \downarrow - \circlearrowright \rangle \langle \alpha_{12} | h_{ij}^{NN} | \alpha'_{12} \rangle \langle \bigcirc - \downarrow | \bigcirc \rangle$$

2-body matrix element

• basis truncation:  $\mathcal{N} = \mathcal{N}_{A-1} + 2n_{\lambda} + \lambda \leq \mathcal{N}_{max} \Rightarrow E_b = E_b(\omega, \mathcal{N}_{max})$ 

- extrapolate in  $\omega$ - and  $\mathcal{N}$ -spaces to obtain converged results

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA (2020)



#### Convergence of E with respect to ${\mathscr N}$





#### Similarity Renormalization Group (SRG)

#### Idea: continuously apply unitary transformation to H to suppress off-diagonal matrix elements

→ observables (binding energies) are conserved due to unitarity of transformation

F.J. Wegner NPB 90 (2000). S.K. Bogner, R.J. Furnstahl, R.J. Perry PRC 75 (2007)

$$\frac{dV(s)}{ds} = \left[ \left[ T_{rel}, V(s) \right], H(s) \right], \qquad H(s) = T_{rel} + V(s) + \Delta M$$
  
$$s = 0 \to \infty \qquad \qquad V(s) = V_{12}(s) + V_{13}(s) + V_{23}(s), \quad V_{123,s=0} \equiv V_{NNN}^{bare}; \quad (V_{YNN}^{bare} = 0)$$

• separate SRG flow equations for 2-body and 3-body interactions:

(S.K. Bogner et al PRC75 (2007), K. Hebeler PRC85 (2012))

$$\frac{dV^{NN}(s)}{ds} = [[T^{NN}, V^{NN}], T^{NN} + V^{NN}]$$

$$\frac{dV^{YN}(s)}{ds} = [[T^{YN}, V^{YN}], T^{YN} + V^{YN} + \Delta M]$$

$$\frac{dV_{123}}{ds} = [[T_{12}, V_{12}], V_{31} + V_{23} + V_{123}]$$

$$+ [[T_{31}, V_{31}], V_{12} + V_{23} + V_{123}]$$

$$+ [[T_{23}, V_{23}], V_{12} + V_{31} + V_{123}] + [[T_{rel}, V_{123}], H_s]$$
Eqs.(1)
$$SRG-induced 3BFs are generated even if V_{123}^{bare} = 0$$

• Eqs.(1) are solved by projecting on a 3N (YNN) Jacobi-momentum basis

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### SRG evolution of NN, YN

•  $\lambda = (4\mu^2/s)^{1/4}$ ,  $[\lambda] = [p]: \lambda \sim$  width of the band-diagonal structure of V in p-space

(S.K. Bogner et al., PRC 75 (2007))

YN: NLO19(500)

V[MeVfm<sup>3</sup>]

0

-10

1.0

0.5 [<sub>€</sub> 0.0 0.0 -0.5 NIMe 0.5

-1.0

V[MeVfm<sup>3</sup>]

10

-10

v[MeVfm<sup>3</sup>]

10

0

2

-2

-10

V[MeVfm³]

V[MeVfm<sup>3</sup>]







### A=3-5 hypernuclei with SRG-induced YNN



R. Wirth, R. Roth PRL117 (2016), PRC100 (2019)

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#### Impact of YN interactions on $B_{\Lambda}(A \leq 7)$

- NLO13 and NLO19 are almost phase equivalent in the 2-body sector
- NLO13 characterised by a stronger  $\Lambda N \Sigma N$  transition potential (especially in  ${}^{3}S_{1}$ )





 $B_{\Lambda}(NLO19) > B_{\Lambda}(NLO13) \longrightarrow contribution$ 

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contribution of chiral YNN force

### Impact of YN interactions on $B_{\Lambda}(A \le 8)$

- NLO13 and NLO19 are almost phase equivalent in the 2-body sector
- NLO13 characterised by a stronger  $\Lambda N \Sigma N$  transition potential (especially in  ${}^{3}S_{1}$ )



→  ${}^{4}_{\Lambda}$ H(1<sup>+</sup>),  ${}^{5}_{\Lambda}$ He,  ${}^{7}_{\Lambda}$ Li,  ${}^{8}_{\Lambda}$ Li are fairly well described by NLO19; NLO13 underestimates these systems HL, J. Haidenbauer, U.-G. Meißner, Andreas Nogga, <u>arXiv:2210.03387</u>

### Charge symmetry breaking (CSB) in A=4



# Charge symmetry breaking (CSB) in A=4

(J. Haidenbauer, U-G. Meißner and A. Nogga FBS 62(2021))







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$$\Delta E(1^+) = B_{\Lambda}(^4_{\Lambda}\text{He}, 1^+) - B_{\Lambda}(^4_{\Lambda}\text{H}, 1^+)$$
  
= -83 ± 94 keV

 $\Delta E(0^+) = B_{\Lambda}(^4_{\Lambda}\text{He}, 0^+) - B_{\Lambda}(^4_{\Lambda}\text{H}, 0^+)$  $= 233 \pm 92 \text{ keV}$ 

- CSB in singlet ( ${}^{1}S_{0}$ ) is much larger than in triplet ( ${}^{3}S_{1}$ )
  - predictions for A=4 are independent of cutoff, same results for NLO13
  - predictions for CSB in A=7,8 multiplets ?



#### Charge symmetry breaking (CSB) in A=4-8



- NLO13 & NLO19 CSB results for A=7 are comparable to experiment.
- two potentials predict a somewhat larger CSB in A=8 doublet as compared to experiment
- → experimental CSB splitting for A=8 could be larger than  $40 \pm 60$  keV?
  - CSB estimated for A=4 could still be too large or have different spin-dependence?



#### **Fitting LECs to new Star measurement**

Recent Star measurement suggests somewhat different CSB in A=4:

 $\Delta E(1^+) = B_{\Lambda}(^4_{\Lambda}\text{He}, 1^+) - B_{\Lambda}(^4_{\Lambda}\text{H}, 1^+)$   $= -83 \pm 94 \text{ keV} \Rightarrow (\text{CSB})$   $= -160 \pm 140 \pm 100 \text{ keV}^* \Rightarrow (\text{CSB}^*)$   $\Delta E(0^+) = B_{\Lambda}(^4_{\Lambda}\text{He}, 0^+) - B_{\Lambda}(^4_{\Lambda}\text{H}, 0^+)$   $= 233 \pm 92 \text{ keV} \Rightarrow (\text{CSB1})$   $= 160 \pm 140 \pm 100 \text{ keV}^* \Rightarrow (\text{CSB}^*)$ \* STAR Collaboration PLB 834 (2022)

	NLO19(500)	CSB	CSB*
$a_s^{\Lambda p}$	-2.91	-2.65	-2.58
$a_s^{\Lambda n}$	-2.91	-3.20	-3.29
$\delta a_s$	0	0.55	0.71
$\boxed{a_t^{\Lambda p}}$	-1.42	-1.57	-1.52
$a_t^{\Lambda n}$	-1.41	-1.45	-1.49
$\delta a_t$	-0.01	-0.12	-0.03

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga arXiv:2210.03387

→ How does the STAR measurement affect the predictions of CSB in A=7,8 multiplets ?



#### Impact of Star measurement on CSB in A=7,8





CSB\* fit predicts reasonable CSB in both A=7 and A=8 systems



## S-shell $\Lambda\Lambda$ hypernuclei:



- ${}^{12}C + \Xi^{-} \rightarrow {}^{6}_{\Lambda\Lambda}He + {}^{4}He + t$  ${}^{6}_{\Lambda\Lambda}He \rightarrow {}^{5}_{\Lambda}He + p + \pi^{-}$
- $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} ({}_{\Lambda\Lambda}^{6} \text{He}) 2B_{\Lambda} ({}_{\Lambda}^{5} \text{He})$  $= 0.67 \pm 0.17 \text{ MeV}$ 
  - 22 (K. Nakazawa et al., NPA 835 (2010))



 ${}^{5}_{\Lambda\Lambda}$ He/ ${}^{5}_{\Lambda\Lambda}$ H?

- bound in few-body calculations using phenomenological potentials
- predicted  $\Delta B_{\Lambda\Lambda}$  is model-dependence
- experimental searching is on going at J-PARC E75





#### **Predictions of chiral YY potentials?**



 $\mathcal{R}$ ddelts, for  $^{6}_{\Lambda\Lambda}$ He,  $^{5}_{\Lambda\Lambda}$ He

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(HL, J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA 57 7(2021))



- $\Delta B_{\Lambda\Lambda}, B_{\Lambda\Lambda}$ Effect of SRG-induced YYN forces is negligible
  - NLO results are comparable to the Nagara, LO potential overestimates  ${}^{6}_{\Lambda\Lambda}$  He  $\Delta B_{\Lambda\Lambda}({}^{5}_{\Lambda}$  He) <  $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}$  He)  $\Delta B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}$  He) <  $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}$  He)  $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}$  He)  $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}$  He)  $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}$  He) >  $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}$  He) (K. S.
  - (K. S. Myint et al EPJ (2003))

 $\Delta B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\text{He}) < \Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\text{He}) \text{ (I. Filikhin, A. Gal NPA 707 (2002))}_{AA}$  $P_{\Xi}(^{6}_{\Lambda\Lambda} \text{He}) < P_{\Xi} \not\in \bigvee_{\Lambda} \text{galculation}$ : Suppression of  $\Lambda\Lambda \leftrightarrow \Xi N$  in  ${}^{6}_{\Lambda\Lambda}$  He  $?_{P_{\Xi}}$  $P_{\Xi}$  $B_{\Lambda\Lambda}$ 

 $\Lambda$  **Reset**, for  $^{6}_{\Lambda\Lambda}$  He,  $^{5}_{\Lambda\Lambda}$  He

(HL, J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA 57 7(2021))



- $\Delta B_{\Lambda\Lambda}, B_{\Lambda\Lambda}$ Effect of SRG-induced YYN forces is negligible
  - NLO results are comparable to the Nagara, LO potential overestimates  ${}^{6}_{\Lambda\Lambda}$  He Large difference between  $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}$ He)  $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}$ He) (K. S.
  - (K. S. Myint et al EPJ (2003))

 $\Delta B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\text{He}) < \Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\text{He}) \text{ (I. Filikhin, A._5Gal NPA 707 (2002))}_{AA}$  $P_{\Xi}(^{6}_{\Lambda\Lambda}\text{He}) < P_{\Xi}(^{5}_{\Lambda\Lambda}\text{eal}\text{culation})$ 24 Suppression of  $\Lambda\Lambda \leftrightarrow \Xi N$  in  ${}^{6}_{\Lambda\Lambda}$  He  $?_{P_{\Xi}}$  $P_{\Xi}$  $B_{\Lambda\Lambda}$ 

# Is ${}^{4}_{\Lambda\Lambda}{\rm H}$ stable against the breakup to ${}^{3}_{\Lambda}{\rm H}$ + $\Lambda$ ?

- H. Nemura et al., PRL 94 (2005) employ an effective YY potentials
  - →  $^{6}_{\Lambda\Lambda}$ He,  $^{5}_{\Lambda\Lambda}$ H/He are strongly bound,  $B_{\Lambda\Lambda}(^{4}_{\Lambda\Lambda}$ H)  $\approx 2 \text{ keV}$
- L. Contessi et al., PLB 797 (2019) use pointless EFT interactions at LO

 $\rightarrow$  the existence of  ${}^{4}_{\Lambda\Lambda}$  H is incompatible with the Nagara result for  ${}^{6}_{\Lambda\Lambda}$  He

• Chiral YY interactions at LO & NLO: (HL, J. Haidenbauer, U.-G. Meißner, A. Nogga, EPJA 57 7(2021))



→ NLO leads to a particle unstable  ${}^{4}_{\Lambda\Lambda}$ H. Existence of  $A = 4 \Lambda\Lambda$  hypernucleus is very unlikely





#### lighter systems?

using  $\Xi N$  NLO potential we predict the existence of A=4-7  $\Xi$  hypernuclei:



#### **Results for** $A = 4 - 7 \Xi$ **-hypernuclei**



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- Coulomb interaction contributes ~ 200, 600 and 400 keV to  $NN\Xi$ ,  ${}_{\Xi}^{5}H$ ,  ${}_{\Xi}^{7}H$
- $\frac{7}{\Xi}$ **H** is expected to be produced and studied in  $^{7}$ Li( $K^{-}$ ,  $K^{+}$ ) at J-PARC (H. Fujioka et al., FBS 69(2021)

### Summary & outlook

#### At our disposal we have 2 tools to tackle light (hyper)nuclear systems:

- s-shell (hyper)nuclei: Faddeev-Yakubovsky method (Andreas Nogga)
- s-shell & light p-shell: Jacobi No-core Shell Model approach
- establish direct link between underlying YN (YY) interactions and observables (A<9)</p>

#### YN at NLO yields reasonable predictions for A = 3 - 8 hypernuclei:

- include chiral YNN forces in order to properly describe light hypernuclei
- use neural network to perform extrapolation to infinite model space for A=7,8 systems

#### Chiral YY ( $\Xi N$ ) potentials predict the existence of ${}^{5}_{\Lambda\Lambda}$ He/ ${}^{5}_{\Lambda\Lambda}$ H, $NN\Sigma$ , ${}^{5}_{\Xi}$ H and ${}^{7}_{\Xi}$ H:

• include SRG-induce  $\Xi$ NN forces to obtain more quantitive estimates



# Thank you!





